Elements of Strength of Materials

S Timoshenko

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30

OF MATERIALS /*fifth edition*

Professor of Engineering Mechanics, Emeritus, Stanford University

Silas H. Palmer Professor of Civil Engineering, Stanford University

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ELEMENTS OF STRENGTH

S. TIMOSHENKO

D. H. YOUNG

Princeton, New Jersey   Toronto   London   Melbourne

PREFACE

This textbook is an outgrowth of Timoshenko's two-volume *Strength of Materials*, first published in 1930. Whereas the two-volume edition presents both elementary and advanced topics, the present volume is considerably abridged and is designed primarily for undergraduate courses in elementary strength of materials in American colleges and engineering schools.

In this fifth edition, *Elements of Strength of Materials* represents a recasting and rewriting of the original abridgment, although an attempt has been made to retain the same general approach to the subject that characterized the original work. This consists primarily in proceeding gradually from the simplest cases to the more complex ones and relying on physical and geometrical considerations of deformation to establish the patterns of stress distribution under various types of loading. This, of course, characterizes the "strength of materials approach" as contrasted with that of the "theory of elasticity." Such an approach may seem old-fashioned to some, but the authors firmly believe that, for the beginner, it represents a sounder pedagogy. We must all learn to walk before we attempt to run.

New examples have been added and new sets of problems have been substituted for the old ones throughout the book. Answers are given to all problems.

In the first chapter, the ideas of stress and strain within the elastic range of behavior are treated thoroughly before introduction of the complications associated with nonlinear stress-strain behavior. The second chapter begins with a discussion of the stress conditions on an oblique section of a bar in tension in order that the complete stress-strain diagram with proportional limit, yield point, ultimate strength, etc., may be better appreciated. This chapter also contains a section on Plastic Analysis or Limit Design.

Chapter 3 begins with a discussion of stresses in thin-walled pressure vessels which serves to introduce the problem of biaxial stress. Analysis of biaxial stress is then developed in detail and Mohr's circle is introduced. This leads logically to a discussion of pure shear, which is essential to a proper treatment of torsion as taken up in Chapter 4. In the next two chapters (5 and 6), the question of bending stresses and shearing stresses in beams is taken up. The first of these contains the fundamentals of bending theory, and the second treats a number of special topics in bending of beams. Chapter 7 deals with the general problem of plane stress and the notion of principal stresses. Applications to principal stresses in beams and stresses due to combined bending and torsion are fully treated. Chapter 7 ends with sections on the analysis of plane strain and the use of strain rosettes.

Chapter 8 is devoted to methods of calculating deflections of beams. These include the differential equation of the elastic line, the moment-area theorems, and the method of superposition. Statically indeterminate beams are discussed in Chapter 9. Since the concept of strain energy has been developed in earlier chapters, it is natural at this point to discuss Castigliano's theorem and its application to statically indeterminate problems. This chapter ends with a section on limit analysis of statically indeterminate beams, using the concept of the plastic hinge.

Chapter 10 deals with the theory of columns and has been written so as to emphasize the rational approach and minimize the attention given to empirical column formulas. The text proper ends with a chapter on the mechanical properties of materials. It is hoped that the inclusion of such material in an undergraduate textbook will serve to give the student a better appreciation of the importance of the experimental side of the subject of Strength of Materials.

S. TIMOSHENKO
D. H. YOUNG

NOTATION

A	area
a, b, c	dimensions
c	distance from neutral axis to extreme fiber
d	diameter
E	modulus of elasticity
e	eccentricity
F	force
G	shear modulus
g	gravitational acceleration constant
h	height; depth of a beam
hp	horsepower
I	moment of inertia of area
i	radius of gyration
J	polar moment of inertia of area
K	stress concentration factor
k	symbol for $\sqrt{P/EI}$; spring constant; factor
l	length
M	bending moment
N	normal force
n	factor of safety; r.p.m.; number
P	force; load
p	pressure per unit area; pitch
Q	force; statical moment of area
q	load per unit length
R	reaction, radius
r	radius; radius of gyration
S	stress resultant
s	arc length
T	torque; temperature
t	thickness
U	strain energy
u	strain energy per unit volume
V	shearing force; volume
v	velocity
W	weight; total load

w	load per unit length; weight per unit volume
X, Y, Z	forces
x, y, z	coordinates
Z	section modulus
α	temperature coefficient of expansion; angle
β	angle
γ	shearing strain; weight density
δ	deflection; total elongation
ϵ	tensile or compressive strain
θ	slope of elastic line; angle of twist per unit length
μ	Poisson's ratio
ρ	radius of curvature; radial coordinate
σ	normal stress
τ	shearing stress
ϕ	angle of twist; angular coordinate
ω	angular velocity

CONTENTS

1
Tension, Compression, and Shear: I

1.1 Introduction

Various structures and machines — bridges, cranes, airplanes, ships, etc. — will be found, upon examination, to consist of numerous parts or *members* connected together in such a way as to perform a useful function and to withstand externally applied loads. Consider, for example, the simple press shown in Fig. 1.1a. The function of this press is to test specimens of various materials in compression. To accomplish this, the specimen is placed on the floor of the base A and the end of the screw is forced down against it by turning the handwheel at the top. This action subjects the specimen as well as the lower portion of the screw to *axial compression* (Fig. 1.1d) and the side members N to *axial tension* (Fig. 1.1b). It will be observed also that the crosshead M is subjected to *bending* (Fig. 1.1c) and the upper part of the screw to twist or *torsion* (Fig. 1.1e). These four basic types of loading of a member are frequently encountered in both structural and machine design problems. They may be said to constitute essentially the principal subject matter of Strength of Materials. In subsequent chapters we will consider them in the order of their complexity: tension,

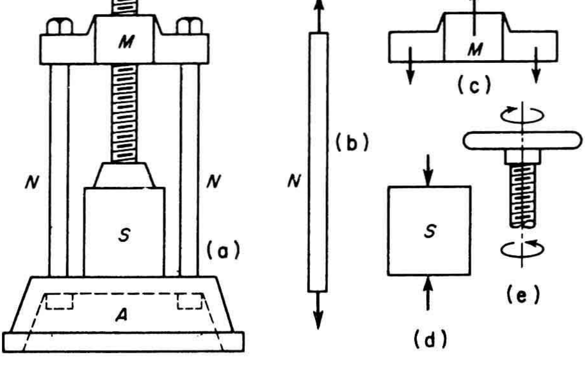

FIG. 1.1

compression, torsion, and bending. We will also see later that in many cases a particular member of a structure or machine may be simultaneously subjected to the action of two or more of these basic types of loading in combination. In such cases, the problems of analysis and design of the member can become somewhat more involved.

Analysis and design of any structure or machine like the press in Fig. 1.1 involve two major questions: (a) Is the structure strong enough to withstand the loads applied to it and (b) is it stiff enough to avoid excessive deformations and deflections? In Statics, the members of a structure were treated as *rigid bodies;* but actually all materials are deformable and this property will henceforth be taken into account. Thus Strength of Materials may be regarded as the statics of deformable or *elastic bodies.* For example, it is clear that compression of the specimen in Fig. 1.1a can be increased only by advancing the screw of the press downwards through the crosshead M. This relative displacement between two parts of the machine is partly accounted for by shortening of the specimen and the lower part of the screw and partly by extension of the side bars N as well as some bending deflection of the crosshead M. Thus, the amount of compressive force on the specimen that will correspond to one turn of the handwheel will depend upon the relative stiffness of the various members of the machine.

Both the *strength* and *stiffness* of a structural member are functions of its size and shape and also of certain physical properties of the material from which it is made. These physical properties of materials are largely determined from experimental studies of their behavior in a testing machine. The study of Strength of Materials is aimed at predicting just how these geometric and physical properties of a structure will influence its behavior under service conditions. The applications of the subject are broad in scope and will be found in all branches of engineering. We begin with a study of the simplest type of loading, namely, axial tension or compression of a straight prismatic bar.

1.2 Internal Force; Stress

In Fig. 1.2, a prismatic bar AB is subjected to axial tension by the action of a vertical load P applied at B and acting along the axis AB of the bar, the proper weight of which is neglected. This action on the bar stretches it slightly and also tends to pull it apart, i.e., to produce *rupture.* This tendency to rupture is resisted by internal forces within the bar, i.e., by actions and reactions between its various particles. To visualize these internal forces, imagine that the bar is cut by a section mn perpendicular to its axis and that the lower portion is isolated as a *free body* (Fig. 1.2b). At the lower end of this portion of the bar, the external force P is applied. On the upper end are the internal forces representing the actions of the particles of the upper part of the bar on those of the lower part. These forces are continuously distributed over the cross-section mn. In dealing with such distributed forces, the *intensity of force*, i.e., the force per unit area, is of great importance. Visualizing the bar as made up of a bundle of longitudinal fibers, each of which carries its fair share of the load, it appears reasonable to assume, in this case, that the distribution of forces over the cross-section will be *uniform.** From the condition of equilibrium of the free body (Fig. 1.2b), it is seen that the resultant of this uniform distribution of internal forces must be equal to the external load P. Thus, if A denotes the cross-sectional area of the bar and σ, the force per unit area, we have $S = \sigma A = P$, from which

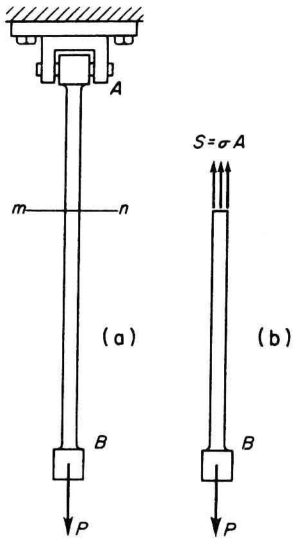

FIG. 1.2

$$\sigma = \frac{P}{A}. \tag{1.1}$$

This force per unit area is called the *stress* in the bar; the total tension $S = \sigma A$ is sometimes called the *stress resultant*. Force is usually measured in pounds and area in square inches so that stress has the dimension of pounds per square inch, denoted by "psi."

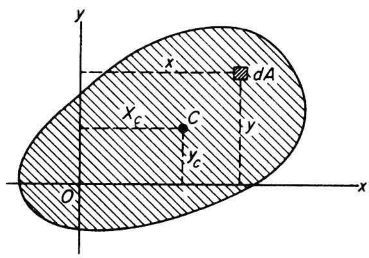

FIG. 1.3

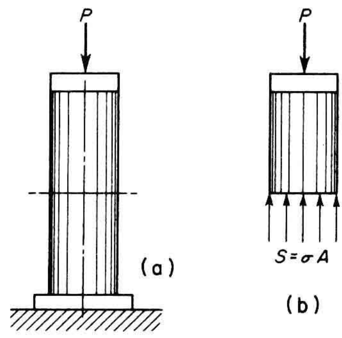

FIG. 1.4

*At cross-sections near the junction points A and B, the distribution may be somewhat non-uniform; but this effect is very localized and will be ignored for the present. For further discussion see Art. 2.5, p. 46.

In order that the applied load P in Fig. 1.2 will actually induce a uniform stress σ over each cross-section of the bar as assumed above, its line of action must pass through the centroid of each cross-section, i.e., P must act along the centroidal axis of the bar. To prove this, consider an arbitrary shape of cross-section as shown in Fig. 1.3 and let dA be any element of area therein. Then for the assumed uniform stress distribution, σ is constant over the cross-section and the element of force acting on dA is σdA, normal to the plane of the section. The resultant of these parallel forces is

$$S = \int \sigma dA = \sigma \int dA = \sigma A \,, \tag{a}$$

also normal to the section.

The point of application of the stress resultant S can be found from the theorem of moments: namely, the moment of the resultant about either of the coordinate axes x or y must equal the algebraic sum of moments of the elemental forces σdA about the same axis. Thus, denoting by $\bar{x}$ and $\bar{y}$ the coordinates of the point of application of the resultant, we have

$$\left.\begin{aligned} \sigma A\bar{x} &= \int x\sigma dA = \sigma \int x dA = \sigma A x_c, \\ \sigma A\bar{y} &= \int y\sigma dA = \sigma \int y dA = \sigma A y_c, \end{aligned}\right\} \tag{b}$$

where x_c and y_c are the coordinates of the centroid C of the cross-section. From eqs. (b), it is seen that $\bar{x} = x_c$ and $\bar{y} = y_c$. Thus for a uniform stress distribution, the stress resultant S acts through the centroid of the cross-section. Furthermore, it can be seen from Fig. 1.2b that the force S must be collinear with the applied force P. Therefore, P can produce a uniform stress distribution over each cross-section only if it acts through their centroids.*

All of the foregoing discussion applies also to the case of a short post or *strut* subjected to a compressive load P as shown in Fig. 1.4. Here also the

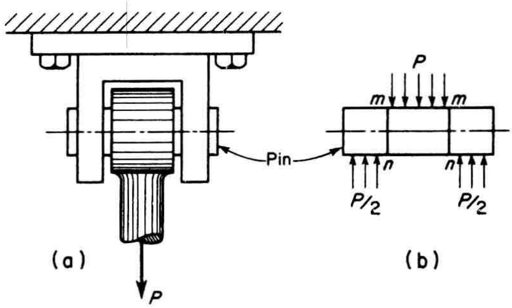

FIG. 1.5

*A tensile load P that does not act along the centroidal axis of a bar will produce bending as well as tension of the bar. This case is discussed in Art. 10.1, p. 264.

load P must act along the centroidal axis of the post to produce the uniform compressive stress σ indicated in Fig. 1.4b. In the case of compression members, this condition is sometimes difficult to fulfill, so that the compression of long slender struts or *columns* requires special consideration which will be taken up later in Chapter 10.

Direct Shear. Referring again to Fig. 1.2, let us consider now in some detail the connection between the tension member AB and the ceiling at its upper end. Clearly, in the interests of good design, this connection should be strong enough to develop the full load-carrying capacity of the bar AB itself. An enlarged detail of this connection is shown in Fig. 1.5a, where it is seen that the load P on the tension member must be transmitted to the fork by the horizontal pin connecting the two parts. A free-body diagram of this pin is shown in Fig. 1.5b and it is seen that the pin is primarily in a condition of *shear* which tends to cut it across the sections *mn*.* Assume now that the internal shearing forces resisting this tendency are uniformly distributed over each of the cross-sections *mn*. Then denoting by τ_{av} the shear force per unit area, i.e., the *average shear stress*, we see that equilibrium conditions of the middle portion of the pin require that $\tau_{av}A_s = P$, from which

$$\tau_{av} = \frac{P}{A_s}, \tag{1.2}$$

where A_s is the total area in shear — in this case, twice the cross-sectional area A of the pin.

Since shearing conditions are never as simple as assumed above, it must be realized that the average shear stress as calculated from eq. (1.2) may be only a rough approximation to the actual stresses that exist in the material. Nevertheless, lacking any more exact knowledge of the true stress distribution, the designer is often forced to use this simple concept of average shear stress as a basis for design.

In dealing with various kinds of machines and structures, the engineer frequently encounters members subjected to simple direct tension, compression, or shear as discussed above. The general problem of design of such members consists in proportioning them so that they can safely and economically withstand the loads that they have to carry. As a basis of doing this, many materials have been tested in the laboratory to establish their strength or resistance to rupture under various types of loading and thereby establish allowable or safe *working stresses*† to be used in design.

*There is also some bending of the pin, but if the clearances are small this will be of secondary importance. Only the shearing action will be considered in the present discussion.

†The establishment of working stresses is a very complex question which will not be discussed in any detail at this point. For further discussion, see Art. 2.2, p. 32.

An allowable working stress is usually taken as $1/n$ times the value of the stress at which failure of the test specimen took place. Thus in using such a working stress, the designer has a so-called *factor of safety n* to allow for overloading or other unforeseen adverse effects. Using these somewhat arbitrarily assigned working stresses together with eqs. (1.1) and (1.2), the designer can determine the proper dimensions for the various members of a machine or structure subjected to the action of given loads. Or, if the structure has already been built, he can establish safe values for the allowable loads in a similar manner.

EXAMPLE 1. A vertical load $P = 5000$ lb is supported by two inclined steel wires AC and BC as shown in Fig. 1.6. Determine the required cross-sectional area A of each wire if the allowable working stress in tension is $\sigma_w = 10{,}000$ psi and the angle $\theta = 30°$.

SOLUTION. When the load P is applied to the ring, each wire is subjected to tension and therefore exerts on the ring a force S directed along the axis of that wire as shown in Fig. 1.6a. Actually, under tension, the wires stretch slightly, so that after the load P is applied, the angles of inclination θ will be slightly greater than 30°. However, in computing the magnitudes of the forces S, we will neglect this

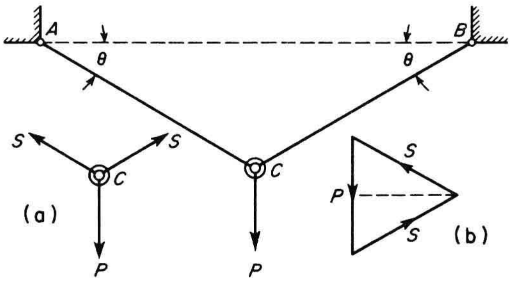

FIG. 1.6

slight change in configuration of the system due to deformation and assume in Fig. 1.6a that each force S is inclined to the horizontal by 30°. Thus, the corresponding closed triangle of forces in Fig. 1.6b is equilateral, and we conclude that $S = P = 5000$ lb. Then, from eq. (1.1), the necessary cross-sectional area of each wire is

$$A = \frac{S}{\sigma_w} = \frac{5000}{10{,}000} = 0.5 \text{ sq in.}$$

EXAMPLE 2. The piston of a deep-well pump is operated by a vertical prismatic steel rod of length $l = 320$ ft attached to a crank at its upper end as shown in Fig. 1.7. Determine the extreme values of tensile and compressive stress σ in the rod if the resistance on the piston during the downstroke is 200 lb and during the upstroke is