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# TURBULENCE IN POROUS MEDIA

MODELING AND APPLICATIONS

SECOND EDITION

MARCELO J. S. DE LEMOS

# **Turbulence in Porous Media**

## **Modeling and Applications**

***Second Edition***

***Marcelo J.S. de Lemos***

*Instituto Tecnológico de Aeronáutica—ITA, Brazil*



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## **Turbulence in Porous Media**

*to the memory of Floriano Eduardo de Lemos  
to Magaly, Pedro and Isabela*

# Preface to the First Edition

The main idea of this monograph is to introduce the reader to the possible characterizations of turbulent flow, and heat and mass transport in permeable media, including analytical, numerical and a review of available experimental data. Such transport processes, occurring at relatively high velocity in permeable media, are present in a number of engineering and natural flows. Therefore, a book like this one that compiles, details, compares and evaluates the available methodologies in the literature for modeling and simulating such flows can be useful to different fields of knowledge, covering engineering (mechanical, chemical, aerospace and petroleum) and basic sciences (thermal sciences, chemistry, physics, applied mathematics, and geological and environmental sciences).

This work has as its primary audience graduated students, engineers and scientists involved in design and analyses of: (a) modern engineering system such as fuel cells, porous combustors and advanced heat exchangers, (b) studies of bio-mechanical problems such as air flow in lungs and (c) modeling of natural and environmental flows such as atmospheric boundary layer over vegetation, fire in forests and spread of contaminants near rivers and bays.

In regard to already published works, it looks to the author that a book on the specific topic of "turbulence in porous media" is still not yet available as this topic is still quite new and unexplored. Only a few years ago papers on this topic have started to appear in scientific journals.

This book was written for use in graduate courses on turbulent transport phenomena in engineering, chemical and environmental programs which deal with research areas involving this theme.

My former and present graduate students have all contributed to the completion of this monograph. In the book, I also tried to collect, review and compile some of our joint publications and reports, which were completed during the last fifteen years or so. My sincere thanks to all of them, which includes Drs Pedras, M.H.J., Rocamora Jr., F.D., Assato, M., Braga, E.J., Mesquita, M.S., doctoral candidates Silva, R.A., Saito, M.B., Tofaneli, L.A., master graduates Magro, V.T., Graminho, D.R. and Santos, N.B.

Our research sponsors, CNPq and FAPESP (Brazilian Federal and São Paulo State Research Funding Agencies, respectively) have also greatly contributed to our "existence" as a research group. Most of my graduate students so far have been funded by them. To CNPq and FAPESP, my sincere thanks and my hopes that they keep on believing in our work.

In my nearly twenty years of teaching at Instituto Tecnológico de Aeronáutica (ITA), Brazil, I have always had support from colleagues and the Institute

administration. None of our research goals would have been accomplished if we had been exposed to an “adverse pressure gradient” during these years. ITA, consistently and continually, has provided a fruitful and open environment where new ideas find fertile ground to grow and develop. I wish to express my sincere thanks to ITA and all the colleagues of the Mechanical Engineering Division.

A prominent professor in Brazil recently said that in the post-war era the international language spoken in most meetings and conferences around the world is “Bad English”. Transposing this notion to written material, I wish to convey my deep apologies to all native English speakers for doing so much damage to Shakespeare’s language.

The idea of writing this book came from a conversation I had with the senior publishing editor of Elsevier Arno Schouwenburg, who trusted that I had adequate scholarship for putting forward this project. I am grateful to him and to Vicki Wetherell, Editorial Assistant, for their invaluable help during the preparation of this work.

Finally, I hope that this work does not disappoint the reader who wishes to keep abreast with the latest developments in the interesting and innovative field of modeling turbulence in permeable structures.

Marcelo J.S. de Lemos  
São José dos Campos, September 2005



This photo, with my father<sup>1</sup> and son, is to me unmistakable evidence that, in our short life, uncertainty and fear lie in between youth hopefulness and old age wisdom.

<sup>1</sup> During the production stage of the first edition of this book, Prof. Dr. Floriano Eduardo de Lemos passed away.

# Preface to the Second Edition

Five years after publication of the first edition of this book it is about time to revise and expand the information presented in this text. During this period, several groups around the world started working on double-averaging techniques and we, at ITA, were led to believe that somehow the book played a role in the dissemination of new ideas and in the overwhelming response that those thoughts have had within the community. Not a single month has passed without a paper on such theme to be reviewed by us. On the overall, authors have worked on similar treatments for turbulence in porous media and at several levels of complexity, sometimes combining what was already understood and detailed, some other instances dividing the turbulence spectrum into bands, each of which handled by its own transport equation. In most works, however, the time-volume or volume-time sequence of integration has played always a role in setting up the overall modeling strategy.

We, at ITA, have also worked on some additional progress in extending the original model to situations including reactive systems, where an exothermic reaction rate in the porous material was allowed to take place in the space occupied by a flowing gaseous phase. This additional development is documented in Chapter 8. Combustion in inert porous materials could then be tackled and important technological applications, such as porous burners, hydrogen production systems and fuel cells, can now benefit from the ideas first described in the original book. Governing equations were rewritten allowing for burning rates in the fluid phase and, in the iterative cycle for numerical solution of the equations, density updates were considered as temperature increases due to release of heat.

Other important development discussed in this second edition is that concerned with the movement of the porous matrix. In a number of engineering flows of practical relevance, as in gasification processes and in manufacturing of advanced materials, one can identify a permeable bed that is moving, either parallel to the working fluid or in the opposite direction to it. Transport equations were then extended to account for the motion of the porous bed and both fields analyzed, the time-volume-averaged and the statistical one, had their governing equations modified in order to cope with the movement of the solid matrix. Chapter 9 documents this development.

My former graduate students R.A. Silva, M.B. Saito and L.A. Tofaneli, doctoral candidates at the time when the first edition came out, have all concluded their Doctoral research programs. I am thankful to them for contributing to our overall goal of developing a tool with an ever-wider range of application. My former Master students C. Fischer and F.T. Dórea have also made an excellent job in



applying and testing the developed model. Currently, doctoral candidate A.C. Pivem is working hard at evaluating the treatment of moving beds and their application to practical situations. Master student J.E.A. Coutinho is also helping to evaluate the combustion model in porous burners. To all my former and current students, my sincere thanks and appreciation for their dedication and hard work.

As in the first edition of the book, I would like to cite the names of our research funding agencies in Brazil, FAPESP and CNPq, for their continuous support. One additional agency that unfortunately was missing in the first edition is CAPES, which is directly under the Ministry for Education and Culture (MEC) in Brazil. To all three of them, CAPES, FAPESP and CNPq, my sincere thanks and my hopes for a continuous support on our work in the years to come.

The writing of this second edition was motivated by the very many e-mails and personal communications in meetings and conferences during the last five years since the introduction of the book. Apparently, colleagues worldwide have praised our work and the many review requests mentioned above indicated to us that, somehow, the book has had a positive impact in the literature. A second edition was then due in order to expand the earlier ideas and to further exploit the theme. Then, after deciding to write this second edition, I contacted Elsevier and learned that sales figures of the printed book did not justify a second printed edition. Surprised by this outcome, I have done some research on what could be the reason for such apparent contradiction. Or say, even though the book had, to a certain extent, motivated new research by others in addition to provoking a number of revision requests, e-mails and a couple of (fully covered) keynote lectures in conferences and meetings, the truth of the matter was that the first edition was, at the end, a financial disaster to the publisher! With the help of a student and after “surfing” for some time on the Internet, I realized the existence of many unauthorized websites from where the book could be freely downloaded. Needless to say, the same happens nowadays with whatever material that can be made digital. My conclusion was that, if for one side profits were not attractive enough to justify a second edition, the overwhelming response that the book seems to have had from the community was unquestionable, even though (or perhaps because) the digital material was made available for free to many. Finally, I wish to express my gratitude to Mrs. Stefani Montemagni for her careful and skilful typing of the manuscript.

Marcelo J.S. de Lemos  
São José dos Campos, December 2011

# Overview

The main focus of this book is to present, in an organized, self-contained, and systematic format, new engineering techniques and novel applications of turbulent transport modeling for flow, heat, and mass transfer in porous media. The motivation for writing this text is the fact that modern engineering equipment design and environmental impact analyses can benefit from the appropriate modeling of turbulent flow in permeable structures. Examples span from flow in advanced porous combustors to atmospheric boundary layers over thick and dense rain forests. This is a very new topic because it involves disciplines that, traditionally, have been developed separately, such as *turbulence*, mostly associated with clear (unobstructed) flows, and *porous media*, which is usually related to low-speed currents, in the laminar regime, through beds or materials containing interconnected pores.

Accordingly, a number of natural and engineering systems can be characterized by some kind of porous structure through which a working fluid permeates in the turbulent regime. That is the case of many transport systems in environmental, petroleum, chemical, mechanical, and aerospace technologies, including flows and fires in rain forests, for example. Here, forests are modeled as porous layers through which atmospheric air flows. In fact, forest fires devastate huge amounts of land and occur frequently in many countries such as Canada and Brazil. The ability to predict the spread of fire fronts more precisely, for example, might help authorities to better manage resources to minimize risk to human life. For analyzing such systems in a realistic and useful way, one has to consider the heterogeneity of the medium (different phases) and the fact that the flow fluctuates with time.

Another important application is the analysis of flow and heat transfer in advanced energy systems such as fuel cells and porous combustors. In addition, many transport processes in chemical engineering equipment also occur at a high velocity within a porous bed. The two independent characteristics mentioned previously (namely, the medium heterogeneity and flow turbulence) have never been covered in a book and, as mentioned, this new knowledge could be useful to scientists, engineers, and environmentalists dealing with natural disasters and working on the development of advanced energy systems.

As such, for practical, simple, and tractable analyses, engineers, scientists, and environmentalists tend to look at these systems as if the medium were made by a unique material (*after application of a volumetric average*) and did not present high-frequency variation in its fluid-phase velocity (*by using a model for handling turbulence effects*). The advantages of having an easy-to-use tool based on this macroscopic treatment are many, such as unveiling important overall flow

characteristics without having to resort to sole experimental analysis, which, in turn, can be time consuming and expensive.

Turbulence models proposed for such flows depend on the order of application of time- and volume-averaged operators. Two developed methodologies, following the two orders of integration, lead to different governing equations for the statistical quantities. This book will review recently published methodologies to characterize turbulent transport in porous media mathematically.

The concept of *double decomposition* is discussed in detail, and models are classified in terms of the order of application of time- and volume-averaged operators, among other peculiarities. A total of four major classes of models have been identified, and a general discussion on their main characteristics takes place.

For hybrid media, involving both a porous structure and a clear flow region, difficulties arise due to the proper mathematical treatment given at the interface. This book also presents and discusses numerical solutions for such hybrid media, considering a channel partially filled with a porous layer through which fluid flows in the turbulent regime. In addition, macroscopic forms of buoyancy terms were considered in both the mean and the turbulent fields. Cases reviewed include heat transfer in porous square enclosures, as well as cavities partially and totally filled with porous material.

In summary, this book presents an overview of both porous media modeling and turbulence modeling, with the aim of positioning this work in relation to these two classical analyses. It begins with a review of governing equations for clear flow before the averaging operations are applied to them. Then the *double-decomposition* concept is presented and thoroughly discussed prior to the derivation of macroscopic governing equations. Equations for turbulent momentum transport follow, showing detailed derivation for the mean and turbulent field quantities. The statistical  $k-\varepsilon$  model for clear domains, used to model turbulence effects, also serves as the basis for modeling. Turbulent heat transport in porous matrices is then reviewed in light of the double-decomposition concept. Models for treating buoyancy effects, double diffusion, reactive flows, and systems having a moving solid phase are also considered in the book. A chapter on numerical modeling and algorithms details major methodologies and techniques used to solve the flow governing equations numerically. A final chapter on applications in hybrid media covers forced flows in composite channels, channels with porous and solid baffles, turbulent impinging jet onto a porous layer, buoyant flows, heat transfer in a back-step and heat transport in porous burners and in moving beds.

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