PROBABILITY AND STATISTICS

FOR ENGINEERING AND THE SCIENCES

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JAY L. DEVORE

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Probability and Statistics for Engineering and the Sciences

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Preface

Purpose

The use of probability models and statistical methods for analyzing data has become common practice in virtually all scientific disciplines. This book attempts to provide a comprehensive introduction to those models and methods most likely to be encountered and used by students in their careers in engineering and the natural sciences. Although the examples and exercises have been designed with scientists and engineers in mind, most of the methods covered are basic to statistical analyses in many other disciplines, so that students of business and the social sciences will also profit from reading the book.

Approach

Students in a statistics course designed to serve other majors may be initially skeptical of the value and relevance of the subject matter, but my experience is that students *can* be turned on to statistics by the use of good examples and exercises that blend their everyday experiences with their scientific interests. Consequently, I have worked hard to find examples of real, rather than artificial, data—data that someone thought was worth collecting and analyzing. Many of the methods presented, especially in the later chapters on statistical inference, are illustrated by analyzing data taken from a published source, and many of the exercises also involve working with such data. Sometimes the reader may be unfamiliar with the context of a particular problem (as indeed I often was), but I have found that students are more attracted by real problems with a somewhat strange context than by patently artificial problems in a familiar setting.

Mathematical Level

The exposition is relatively modest in terms of mathematical development. Substantial use of the calculus is made only in Chapter 4 and parts of Chapters 5 and 6. In particular, with the exception of an occasional remark or aside, calculus appears in the inference part of the book only in the second section of Chapter 6. Matrix algebra is not used at all. Thus almost all the exposition should be accessible to those whose mathematical background includes one semester or two quarters of differential and integral calculus.

Content

Chapter 1 begins with some basic concepts and terminology—population, sample, descriptive and inferential statistics, enumerative versus analytic studies, and so on—and continues with a survey of important graphical and numerical descriptive methods. A rather traditional development of probability is given in Chapter 2, followed by probability distributions of discrete and continuous random variables in Chapters 3 and 4, respectively. Joint distributions and their properties are discussed in the first part of Chapter 5. The latter part of this chapter introduces statistics and their sampling distributions, which form the bridge between probability and inference. The next three chapters cover point estimation, statistical intervals, and hypothesis testing based on a single sample. Methods of inference involving two independent samples and paired data are presented in Chapter 9. The analysis of variance is the subject of Chapters 10 and 11 (single-factor and multifactor, respectively). Regression makes its initial appearance in Chapter 12 (the simple linear regression model and correlation) and returns for an extensive encore in Chapter 13. The last three chapters develop chi-squared methods, distribution-free (nonparametric) procedures, and techniques from statistical quality control.

Helping Students Learn

Although the book's mathematical level should give most science and engineering students little difficulty, working toward an understanding of the concepts and gaining an appreciation for the logical development of the methodology may sometimes require substantial effort. To help students gain such an understanding and appreciation, I have provided numerous exercises ranging in difficulty from many that involve routine application of text material to some that ask the reader to extend concepts discussed in the text to somewhat new situations. There are many more exercises than most instructors would want to assign during any particular course, but I recommend that students be required to work a substantial number of them; in a problem-solving discipline, active involvement of this sort is the surest way to identify and close the gaps in understanding that inevitably arise. Answers to odd-numbered exercises appear in the answer section at the back of the text. In addition, a Student Solutions Manual, consisting of worked-out solutions to virtually all the odd-numbered exercises, is available.

New for This Edition

- The first section of Chapter 1 has been rewritten to emphasize from the outset that variation is the source from which all statistical methodology flows. The techniques from exploratory and descriptive statistics introduced in this chapter are utilized to a greater extent than before in the inferential part of the book.
- The material on sampling distributions in Chapter 5 has been reorganized to convey more clearly the central idea on which inferential methods are based: The value of any statistic (quantity calculated from sample data) will in general vary when sample after sample is selected from the same population.
- One-sided confidence and prediction intervals are now featured in Chapter 7 along with their two-sided counterparts. A new confidence interval for a population propor-

tion (the Agresti–Coull "score" interval) is included. Normal tolerance intervals, previously relegated to an exercise, are now discussed in more detail in the text itself, and a table of tolerance critical values for one- and two-sided intervals is included.

- There is increased emphasis on P-values for testing hypotheses. The appendix now contains a table of t curve tail areas, so that a statement such as $P\text{-value} \approx .017$, rather than just $.01 < P\text{-value} < .025$, can be made. A new chi-squared table also allows for more precise P-value information for chi-squared tests, and a more detailed F table does the same thing for F tests.
- Notation in the regression chapters has been streamlined, allowing for the use of more concise formulas. There is now a short subsection in Chapter 13 on logistic regression.
- Finally, numerous examples have been updated, and many new exercises have supplemented or replaced those from previous editions.

Recommended Coverage

There is enough material in this book for a year-long course. Anyone teaching a course of shorter duration will have to be selective in the choice of topics to be included. At Cal Poly, we teach a two-quarter sequence, meeting four hours per week. During the first ten weeks we cover much of the material in Chapters 1–7 (going lightly over joint distributions and the details of estimation by maximum likelihood and the method of moments). The second quarter begins with hypothesis testing and moves on to two-sample inferences, ANOVA, regression, and selections from the chi-squared, distribution-free, and quality control chapters. Coverage of material in a one-semester course would obviously have to be somewhat more restrictive. There is, of course, never enough time to teach students all that we would like them to know!

Acknowledgments

My colleagues here at Cal Poly have provided me with invaluable support and encouragement over the years. I am also grateful to the many users of previous editions who have made suggestions for improvement (and pointed out occasional errors). A note of thanks goes to Julie Seely and Beth Eltinge for their work on the Student Solutions Manual.

I gratefully acknowledge the plentiful feedback provided by the following reviewers of this and previous editions: Robert L. Armacost, University of Central Florida; Douglas M. Bates, University of Wisconsin–Madison; David M. Cresap, University of Portland; Don E. Deal, University of Houston; Charles E. Donaghey, University of Houston; Mark Duva, University of Virginia; Nasser S. Fard, Northeastern University; Celso Grebogi, University of Maryland; James J. Halavin, Rochester Institute of Technology; Wei-Min Huang, Lehigh University; Stephen Kokoska, Colgate University; Arnold R. Miller, University of Denver; Don Ridgeway, North Carolina State University; Larry J. Ringer, Texas A&M University; Richard M. Soland, The George Washington University; Clifford Spiegelman, Texas A&M University; Jery Stedinger, Cornell University; David Steinberg, Tel Aviv University; G. Geoffrey Vining, University of Florida; Bhutan Wadhwa, Cleveland State University; and Michael G. Zabetakis, University of Pittsburgh.

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Jay L. Devore

Contents

1

Overview and Descriptive Statistics

Introduction

Statistical concepts and methods are not only useful but indeed often indispensable in understanding the world around us. They provide ways of gaining new insights into the behavior of many phenomena that you will encounter in your chosen field of specialization in engineering or science.

The discipline of statistics teaches us how to make intelligent judgments and informed decisions in the presence of uncertainty and variation. Without uncertainty or variation, there would be little need for statistical methods or statisticians. If every component of a particular type had exactly the same lifetime, if all resistors produced by a certain manufacturer had the same resistance value, if pH determinations for soil specimens from a particular locale gave identical results, and so on, then a single observation would reveal all desired information.

An interesting manifestation of variation arises in the course of performing emissions testing on motor vehicles. The expense and time requirements of the Federal Test Procedure (FTP) preclude its widespread use in vehicle inspection programs. As a result, many agencies have developed less costly and quicker tests, which it is hoped replicate FTP results. According to the article "Motor Vehicle

Emissions Variability" (*J. of the Air and Waste Mgmt. Assoc.,* 1996: 667–675), the acceptance of the FTP as a gold standard has led to the widespread belief that repeated measurements on the same vehicle would yield identical (or nearly identical) results. The authors of the article applied the FTP to seven vehicles characterized as "high emitters." Here are the results for one such vehicle:

HC (gm/mile)	13.8	18.3	32.2	32.5
CO (gm/mile)	118	149	232	236

The substantial variation in both the HC and CO measurements casts considerable doubt on conventional wisdom and makes it much more difficult to make precise assessments about emissions levels.

How can statistical techniques be used to gather information and draw conclusions? Suppose, for example, that a materials engineer has developed a coating for retarding corrosion in metal pipe under specified circumstances. If this coating is applied to different segments of pipe, variation in environmental conditions and in the segments themselves will result in more substantial corrosion on some segments than on others. Methods of statistical analysis could be used on data from such an experiment to decide whether the *average* amount of corrosion exceeds an upper specification limit of some sort or to predict how much corrosion will occur on a single piece of pipe.

Alternatively, suppose the engineer has developed the coating in the belief that it will be superior to the currently used coating. A comparative experiment could be carried out to investigate this issue by applying the current coating to some segments of pipe and the new coating to other segments. This must be done with care lest the wrong conclusion emerge. For example, perhaps the average amount of corrosion is identical for the two coatings. However, the new coating may be applied to segments that have superior ability to resist corrosion and under less stressful environmental conditions compared to the segments and conditions for the current coating. The investigator would then likely observe a difference between the two coatings attributable not to the coatings themselves but just to extraneous variation. Statistics offers not only methods for analyzing the results of experiments once they have been carried out, but also suggestions for how experiments can be performed in an efficient manner to mitigate the effects of variation and have a better chance of producing correct conclusions.

1.1 Populations, Samples, and Processes

Engineers and scientists are constantly exposed to collections of facts, or **data,** both in their professional capacities and in everyday activities. The discipline of statistics provides methods for organizing and summarizing data, and for drawing conclusions based on information contained in the data.

An investigation will typically focus on a well-defined collection of objects constituting a **population** of interest. In one study, the population might consist of all gelatin capsules of a particular type produced during a specified period. Another investigation might involve the population consisting of all individuals who received a B.S. in engineering during the most recent academic year. When desired information is available for all objects in the population, we have what is called a **census.** Constraints on time, money, and other scarce resources usually make a census impractical or infeasible. Instead, a subset of the population—a **sample**—is selected in some prescribed manner. Thus we might obtain a sample of bearings from a particular production run as a basis for investigating whether bearings are conforming to manufacturing specifications, or we might select a sample of last year's engineering graduates to obtain feedback about the quality of the engineering curricula.

We are usually interested only in certain characteristics of the objects in a population: the number of flaws on the surface of each casing, the thickness of each capsule wall, the gender of an engineering graduate, the age at which the individual graduated, and so on. A characteristic may be categorical, such as gender or type of malfunction, or it may be numerical in nature. In the former case, the *value* of the characteristic is a category (e.g., female or insufficient solder), whereas in the latter case, the value is a number (e.g., age = 23 years or diameter = .502 cm). A **variable** is any characteristic whose value may change from one object to another in the population. We shall initially denote variables by lowercase letters from the end of our alphabet. Examples include

x = gender of a graduating engineer

y = number of major defects on a newly manufactured automobile

z = braking distance of an automobile under specified conditions

Data results from making observations either on a single variable or simultaneously on two or more variables. A **univariate** data set consists of observations on a single variable. For example, we might determine the type of transmission, automatic (A) or manual (M), on each of ten automobiles recently purchased at a certain dealership, resulting in the categorical data set

M A A A M A A M A A

The following sample of lifetimes (hours) of brand D batteries put to a certain use is a numerical univariate data set:

5.6 5.1 6.2 6.0 5.8 6.5 5.8 5.5

We have **bivariate** data when observations are made on each of two variables. Our data set might consist of a (height, weight) pair for each basketball player on a team, with the first observation as (72, 168), the second as (75, 212), and so on. If an engineer determines the value of both x = component lifetime and y = reason for component failure, the resulting data set is bivariate with one variable numerical and the other categorical. **Multivariate** data arises when observations are made on more than two variables. For example, a research physician might determine the systolic blood pressure, diastolic blood pressure, and serum cholesterol level for each patient participating in a study. Each observation would be a triple of numbers, such as (120, 80, 146). In many multivariate data sets, some variables are numerical and others are categorical. Thus the annual automobile issue of *Consumer Reports* gives values of such variables as type of vehicle (small, sporty, compact, mid-size, large), city fuel efficiency (mpg), highway fuel efficiency (mpg), drive train type (rear wheel, front wheel, four wheel), and so on.

Branches of Statistics

An investigator who has collected data may wish simply to summarize and describe important features of the data. This entails using methods from **descriptive statistics.** Some of these methods are graphical in nature; the construction of histograms, boxplots, and scatter plots are primary examples. Other descriptive methods involve calculation of numerical summary measures, such as means, standard deviations, and correlation coefficients. The wide availability of statistical computer software packages has made these tasks much easier to carry out than they used to be. Computers are much more efficient than human beings at calculation and the creation of pictures (once they have received appropriate instructions from the user!). This means that the investigator doesn't have to expend much effort on "grunt work" and will have more time to study the data and extract important messages. Throughout this book, we will present output from various packages such as MINITAB, SAS, and S-Plus.

Example 1.1 The tragedy that befell the space shuttle *Challenger* and its astronauts in 1986 led to a number of studies to investigate the reasons for mission failure. Attention quickly focused on the behavior of the rocket engine's O-rings. Here is data consisting of observations on x = O-ring temperature (°F) for each test firing or actual launch of the shuttle rocket engine (*Presidential Commission on the Space Shuttle Challenger Accident,* Vol. 1, 1986: 129–131).

84	49	61	40	83	67	45	66	70	69	80	58
68	60	67	72	73	70	57	63	70	78	52	67
53	67	75	61	70	81	76	79	75	76	58	31

Without any organization, it is very difficult to get a sense of what a typical or representative temperature might be, whether the values are highly concentrated about a typical value or quite spread out, whether there are any gaps in the data, what percentage of the values are in the 60's, and so on. Figure 1.1 shows what is called a *stem-and-leaf display* of the data, as well as a *histogram.* Shortly, we will discuss construction and interpretation of these pictorial summaries; for the moment, we hope you see how they begin to tell us how the values of temperature are distributed along the measurement scale. Some of these launches/firings were successful and others resulted in failure. In Chapter 13, we will consider whether temperature had a bearing on the likelihood of a successful launch.

```
Stem-and-leaf of temp N = 36
Leaf Unit = 1.0
 1    3   1
 1    3
 2    4   0
 4    4   59
 6    5   23
 9    5   788
13    6   0113
(7)   6   6777789
16    7   000023
10    7   556689
 4    8   0134
```

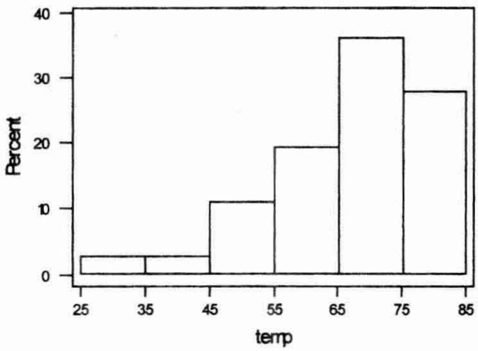

Figure 1.1 A MINITAB stem-and-leaf display and histogram of the O-ring temperature data ■

Having obtained a sample from a population, an investigator would frequently like to use sample information to draw some type of conclusion (make an inference of some sort) about the population. That is, the sample is a means to an end rather than an end in itself. Techniques for generalizing from a sample to a population are gathered within the branch of our discipline called **inferential statistics.**

Example 1.2 Material strength investigations provide a rich area of application for statistical methods. The article "Effects of Aggregates and Microfillers on the Flexural Properties of Concrete" (*Magazine of Concrete Research,* 1997: 81–98) reported on a study of strength properties of high-performance concrete obtained by using superplasticizers and certain binders. The compressive strength of such concrete had previously been investigated, but not much was known about flexural strength (a measure of ability to resist failure in bending). The accompanying data on flexural strength (in MegaPascal, MPa, where 1 Pa (Pascal) $= 1.45 \times 10^{-4}$ psi) appeared in the article cited:

5.9	7.2	7.3	6.3	8.1	6.8	7.0	7.6	6.8	6.5	7.0	6.3	7.9	9.0
8.2	8.7	7.8	9.7	7.4	7.7	9.7	7.8	7.7	11.6	11.3	11.8	10.7	

Suppose we want an *estimate* of the average value of flexural strength for all beams that could be made in this way (if we conceptualize a population of all such beams, we are trying to estimate the population mean). It can be shown that, with a high degree of confidence, the population mean strength is between 7.48 MPa and 8.80 MPa; we call this a *confidence interval* or *interval estimate.* Alternatively, this data could be used to predict the flexural strength of a *single* beam of this type. With a high degree of confidence, the strength of a single such beam will exceed 7.35 MPa; the number 7.35 is called a *lower prediction bound.* ■

The main focus of this book is on presenting and illustrating methods of inferential statistics that are useful in scientific work. The most important types of inferential procedures—point estimation, hypothesis testing, and estimation by confidence intervals—are introduced in Chapters 6–8 and then used in more complicated settings in Chapters 9–16. The remainder of this chapter presents methods from descriptive statistics that are most used in the development of inference.

Chapters 2–5 present material from the discipline of probability. This material ultimately forms a bridge between the descriptive and inferential techniques and leads to a better understanding of how inferential procedures are developed and used, how statistical conclusions can be translated into everyday language and interpreted, and when and where pitfalls can occur in applying the methods. Probability and statistics both deal with questions involving populations and samples, but do so in an "inverse manner" to one another.

In a probability problem, properties of the population under study are assumed known (e.g., in a numerical population, some specified distribution of the population values may be assumed), and questions regarding a sample taken from the population are posed and answered. In a statistics problem, characteristics of a sample are available to the experimenter, and this information enables the experimenter to draw conclusions about the population. The relationship between the two disciplines can be summarized by saying that probability reasons from the population to the sample (deductive reasoning), whereas inferential statistics reasons from the sample to the population (inductive reasoning). This is illustrated in Figure 1.2.

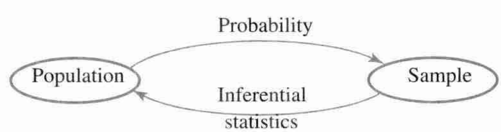

Figure 1.2 The relationship between probability and inferential statistics

Before we can understand what a particular sample can tell us about the population, we should first understand the uncertainty associated with taking a sample from a given population. This is why we study probability before statistics.

As an example of the contrasting focus of probability and inferential statistics, consider drivers' use of manual lap belts in cars equipped with automatic shoulder belt systems. (The article "Automobile Seat Belts: Usage Patterns in Automatic Belt Systems," *Human Factors,* 1998: 126–135, summarizes usage data.) In probability, we might assume that 50% of all drivers of cars equipped in this way in a certain metropolitan area regularly use their lap belt (an assumption about the population), so we might ask, "How likely is it that a sample of 100 such drivers will include at least 70 who regularly use their lap belt?" or "How many of the drivers in a sample of size 100 can we expect to regularly use their lap belt?" On the other hand, in inferential statis-