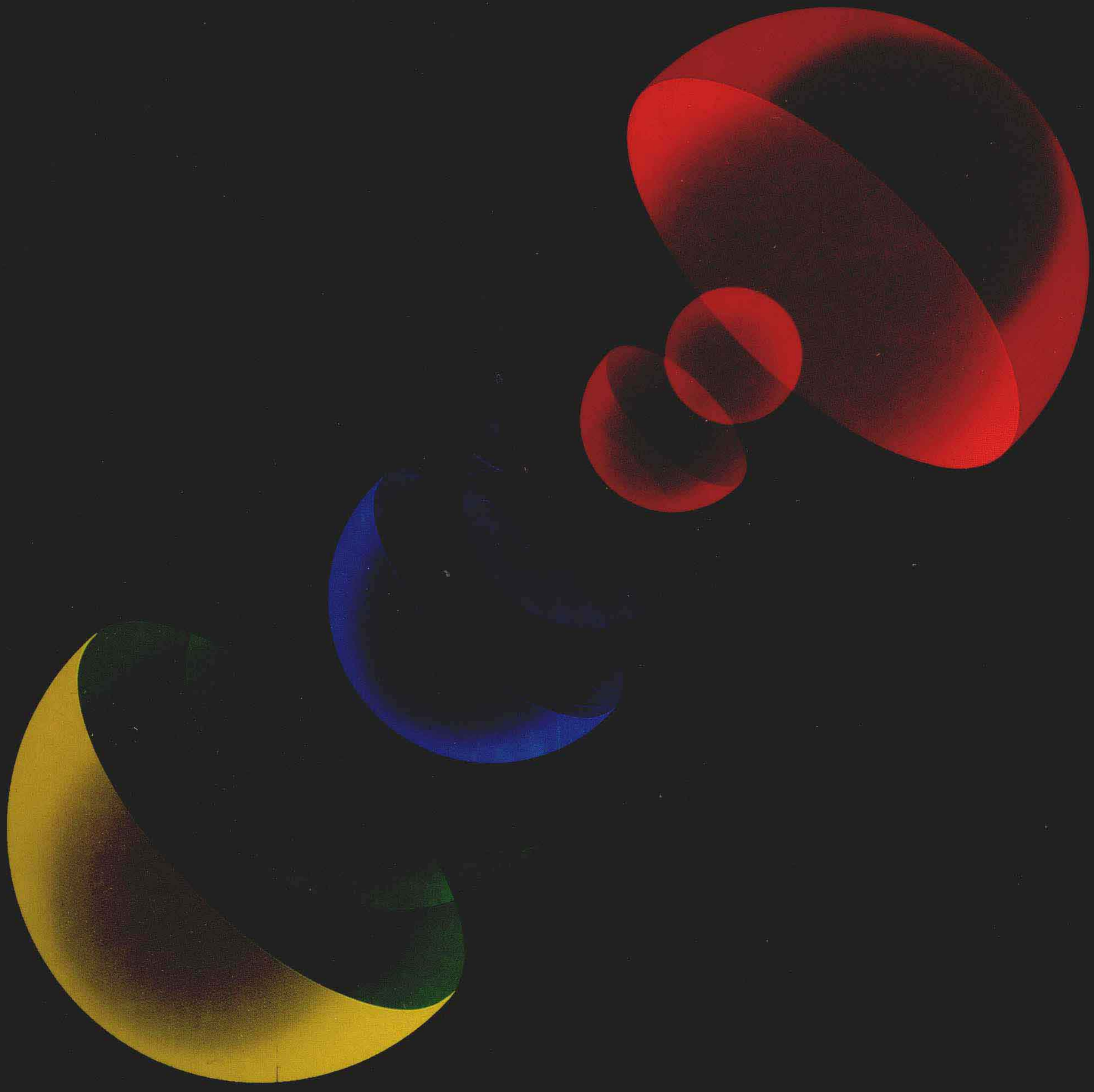
College Algebra

Larson / Hostetler



COLLEGE ALGEBRA

Roland E. Larson
Robert P. Hostetler
THE PENNSYLVANIA STATE UNIVERSITY
THE BEHREND COLLEGE

With the assistance of
David E. Heyd
THE PENNSYLVANIA STATE UNIVERSITY
THE BEHREND COLLEGE

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PREFACE

Success in college level mathematics courses for those interested in any one of a variety of disciplines such as computer science, engineering, management, statistics, or one of the natural sciences begins with a firm understanding of algebraic concepts. The goal of our textbook is to further the preparation of students, who have completed two years of high school algebra, in such important areas as graphical techniques, functions, and analytic geometry. These are some of the fundamental elements used in the calculus and other mathematical endeavors that many students pursue.

The features of our book have been designed to create a comprehensive teaching instrument that employs effective pedagogical techniques.

• *Order of Topics.* Chapter 1 provides a thorough review of the concepts of algebra, including complex numbers. With this early coverage of complex numbers, the algebra of polynomial and rational functions (Chapters 1 through 5) can be brought to a logical conclusion with a discussion of the Fundamental Theorem of Algebra. Then, in Chapters 6 through 9 coverage is given to additional topics in algebra: exponential and logarithmic functions, systems of linear equations, matrices, sequences, series, and probability.

• *Algebra of Calculus.* Special emphasis has been given to the *algebra of calculus*. Many examples and exercises consist of algebra problems that arise in the study of calculus. These examples are clearly identified.

• *Examples.* The text contains over 600 examples, each carefully chosen to illustrate a particular concept or problem-solving technique. Each example is titled for quick reference and many examples include side comments (set in color) to justify or explain the steps in the solution.

• *Exercises.* Over 3000 exercises are included that are designed to build competence, skill, and understanding. Each exercise set is graded in difficulty to allow students to gain confidence and understanding in the use of algebra. To help prepare students for calculus, we stress a graphical approach in many sections and have included numerous graphs in the exercises.

• *Graphics.* The ability to visualize a problem is a critical part of a student's ability to solve a problem. This text includes over 600 figures.

• *Applications.* Throughout the textual material we have included numerous word problems that give students concrete ideas about the usefulness of the topics included.

• *Calculators.* Although we do not require the use of calculators in any section, techniques for calculator use are provided at appropriate places throughout the text. In addition, calculators have allowed us to include many realistic applications that are often excluded because of lengthy or tedious computations. Exercises meant to be solved with the help of a calculator are clearly indicated.

• *Supplements.* For the student, the *Study and Solutions Guide* by Dianna L. Zook is available. This guide includes detailed steps of solutions to nearly half of the odd-numbered exercises. The guide also includes a review of important concepts for each chapter as well as practice chapter tests. For the instructor, the *Instructor's Guide* by Meredythe M. Burrows is available and it includes answers to the even-numbered exercises as well as sample tests for each chapter.

We would like to thank the many people who have helped us at various stages of this project. Their encouragement, criticisms, and suggestions have been invaluable to us. The following reviewers offered many excellent ideas: Ben P. Bockstage, Broward Community College; Daniel D. Bonar, Denison University; H. Eugene Hall, DeKalb Community College; William B. Jones, University of Colorado; Jimmie D. Lawson, Louisiana State University; Jerome L. Paul, University of Cincinnati; and Shirley C. Sorensen, University of Maryland.

The mathematicians listed below completed a survey conducted by D. C. Heath and Company in 1983 which helped outline our topical coverage: Stan Adamski, University of Toledo; Daniel D. Anderson, University of Iowa; James E. Arnold, University of Wisconsin; Prem N. Bajaj, Wichita State University; Imogene C. Beckemeyer, Southern Illinois University; Bruce Blake, Clemson University; Dale E. Boye, Schoolcraft College; Sarah W. Bradsher, Danville Community College; John H. Brevit, Western Kentucky University; Milo F. Bryn, South Dakota State University; Gary G. Carlson, Brigham Young University; Louis J. Chatterley, Brigham Young University; Mary Clarke, Cerritos College; Lee G. Corbin, College of the Canyons; Milton D. Cox, Miami University; Robert G. Cromie, St. Lawrence University; Bettyann Daley, University of Delaware; Clinton O. Davis, Brevard Community College; Karen R. Dougan, University of Florida; Richard B. Duncan, Tidewater Community College; Don Duttenhoeffer, Brevard Community College; Bruce R. Ebanks, Texas Technological University; Susan L. Ehlers, St. Louis Community College; Delvis A. Fernandez, Chabot College; Leslee Francis, Brigham Young University; August J. Garver, University of Missouri; Douglas W. Hall, Michigan State University; James E. Hall, University of Wisconsin; Nancy Harbour, Brevard Community College; Ferdinand Haring, North Dakota State University; Cecilia Holt, Calhoun Community College; James Howard, Ferris State College; Don Jefferies, Orange

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Several of our colleagues also worked on this project with us. David E. Heyd assisted us with the text; Dianna Zook wrote the *Study and Solutions Guide;* and Meredythe Burrows wrote the *Instructor's Guide*. Three students helped with the computer graphics and accuracy checks: Wendy Hafenmaier, Timothy Larson, and A. David Salvia. Linda Matta spent many hours carefully typing the instructor's manual and proofreading the galleys and page-proofs. Deanna Larson had the enormous job of typing the entire manuscript.

On a personal level, we are grateful to our children for their interest and support during the three years the book was being written and produced, and to our wives, Deanna Larson and Eloise Hostetler, for their love, patience, and understanding.

If you have suggestions for improving this text, please feel free to write us. Over the past several years we have received many useful comments from both instructors and students and we value this very much.

Roland E. Larson
Robert P. Hostetler

THE LARSON AND HOSTETLER PRECALCULUS SERIES

To accommodate the different methods of teaching college algebra, trigonometry, and analytic geometry, we have prepared four volumes. These separate titles are described below.

COLLEGE ALGEBRA

A text designed for a one-term course covering standard topics such as algebraic functions, exponential and logarithmic functions, matrices, determinants, probability, sequences, and series.

TRIGONOMETRY

This text is used in a one-term course covering the trigonometric functions, exponential and logarithmic functions, and analytical geometry.

ALGEBRA AND TRIGONOMETRY

This title combines the content of the two texts mentioned above (with the exception of analytic geometry). It is comprehensive enough for two terms of courses or may be covered, with careful selection, in one term.

PRECALCULUS

With this book, students cover the algebraic and trigonometric functions, and analytic geometry in preparation for a course in calculus. This may be used in a one- or two-term course.

INTRODUCTION TO CALCULATORS

This text includes some examples and exercises that make use of a scientific calculator. A calculator can assist you in both learning and applying mathematics. Moreover, a calculator can significantly extend the range of practical applications. Instructions in the use of a calculator will be given as we encounter new functions and applications. Of necessity, the instructions that we provide are somewhat general and may not agree precisely with the steps required by your calculator.

One of the basic differences in calculators is their internal hierarchy (priority) of operations. For use with this text, we recommend a calculator with the following features.

1. (At least) 8-digit display with scientific notation
2. Four arithmetic operations: [+], [−], [×], [÷]
3. Exponential keys: [y^x] or [a^x], [e^x] or [INV] [ln x]
4. Natural logarithm: [ln x]
5. Pi: [π]
6. Inverse, reciprocal, square root: [INV], [1/x], [√‾]
7. Trigonometric functions: [sin], [cos], [tan]
8. Memory: [M] or [STO]
9. Parentheses: [(], [)]
10. Change sign key: [+/−] (Note that this is not the subtraction key. It is used to enter negative numbers into the calculator.)

In this text, all calculator steps will be given with *algebraic logic,* that is, the calculator logic using the normal algebraic order of operations. For example, the calculation $4.69[5 + 2(6.87 - 3.042)]$ can be performed with the following sequence of steps:

4.69 [×] [(] 5 [+] 2 [×] [(] 6.87 [−] 3.042 [)] [)] [=]

which should yield the value 59.35664. Without parentheses, we would work from the inside out with the following sequence to obtain the same result:

6.87 [−] 3.042 [=] [×] 2 [+] 5 [=] [×] 4.69 [=]

When rounding off decimals, we use the following rules:

1. Determine the number of positions you wish to keep. The digit in the last position you keep is called the rounding digit, and the digit in the first position you discard is called the decision digit.
2. If the decision digit is 5 or greater (≥ 5), round up by adding 1 to the rounding digit.
3. If the decision digit is 4 or less (≤ 4), round down by leaving the rounding digit unchanged.
4. Keep the decimal point in the same place.

We cannot control the internal round-off that occurs in calculators. What does your calculator display when you compute $2 \div 3$? Some calculators simply truncate (drop) the digits that exceed their display range (of eight digits) and display .66666666. Others have an internal round-off subroutine and display .66666667. Although the second display is more accurate, *both* of these decimal representations of $\frac{2}{3}$ contain a round-off error. One of the best ways to minimize error due to round-off is to *leave numbers in your calculator* until your calculations are complete. If you want to save a number for future use, store it in your calculator memory.

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Sequences, Series and Probability

Chapter 9

CHAPTER 1 REVIEW OF FUNDAMENTAL CONCEPTS OF ALGEBRA

Algebra: Its Nature and Use 1.1

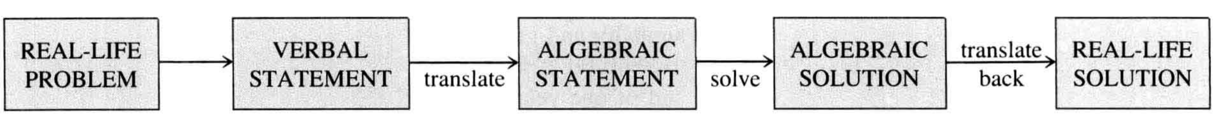
What Is Algebra?

Whatever your algebraic background, you already have some concept of what algebra is. First and foremost, algebra is a *useful language for solving practical* (real-life) *problems*. Second, algebra is a *convenient tool for writing generalizations* of specific statements involving the operations of arithmetic. For instance, the equation

$$a + b = b + a$$

is used to indicate that the operation of addition is commutative on the set of real numbers. Algebra, in this sense, provides a kind of short-hand version of the rules of arithmetic. Finally, knowledge of algebra is a *prerequisite for the study of advanced courses in mathematics*.

Since real-life problems do not come to us expressed in algebraic form, one truly challenging part of algebra is the translation of such word problems into algebraic problems. The algebraic solution is then translated back as the solution to the word problem. This process has the following scheme:



For Those Planning to Take Calculus

To prepare for calculus, you will need to spend time learning to manipulate or rewrite algebraic expressions. For example, consider the following expression:

$$\frac{9 - 2x}{x^2 + x - 6}$$

which has the equivalent partial fractions form

$$\frac{9 - 2x}{x^2 + x - 6} = \frac{1}{x - 2} - \frac{3}{x + 3}$$

Integration, a basic operation in calculus, is difficult to carry out on the left-hand expression, but is rather simple to do on the right-hand expression. Throughout the text, we will refer to the algebra used in calculus as the **algebra of calculus.** In many instances, this algebra of calculus will seem backwards—the reverse of our regular algebra. For instance, adding and subtracting fractions would be a regular use of algebra—but in the partial fractions example just cited, we used algebra to *rewrite* a given fraction as the difference of two simpler fractions. Study the following chart for a preview of what algebra can do for us.

HOW DO WE USE ALGEBRA?

A. TO SYMBOLIZE REAL-LIFE PROBLEMS

Verbal Statement	Algebraic Statement
• Sixteen is 45% of what number?	• $x =$ Number $16 = (0.45)x$
• Ed is now twice as old as Cindy. Eight years ago he was 18 years older than Cindy was. How old is each now?	• *Now* / *8 Years Ago* Cindy's Age: x / $x - 8$ Ed's Age: $2x$ / $2x - 8$ $(2x - 8) = 18 + (x - 8)$
• A certain amount is invested at 12% compounded annually and grows to \$840.00 at the end of one year. How much was the original investment?	• $x =$ Original Investment $x + (0.12)x = 840$

B. TO WRITE GENERAL STATEMENTS OF ARITHMETIC PROPERTIES

Arithmetic Equation	Algebraic Rule
• $3 + 2 = 2 + 3$ $\frac{1}{2} + \frac{3}{4} = \frac{3}{4} + \frac{1}{2}$	• $a + b = b + a$
• $\sqrt[3]{8} = \sqrt[3]{2^3} = 2$ $\sqrt{4^3} = (\sqrt{4})^3 = 2^3 = 8$	• $\sqrt[n]{a^m} = (\sqrt[n]{a})^m = a^{m/n}$
• $\frac{1}{2} + \frac{3}{5} = \frac{5 + 6}{10}$	• $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$
• $13(49) = 13(50 - 1)$ $= 650 - 13 = 637$	• $a(b - c) = ab - ac$

C. TO SIMPLIFY THE OPERATIONS OF CALCULUS

Algebraic Expression	Rewritten for Calculus
• $\sqrt[3]{(2x - x^2)^5}$	• $(2x - x^2)^{5/3}$
• $\frac{5}{(2x + 5)^4}$	• $5(2x + 5)^{-4}$
• $\ln \frac{\sqrt{x}}{(x + 2)^2}$	• $\frac{1}{2} \ln x - 2 \ln(x + 2)$
• $\frac{x^3 - 3x^2 + 5x}{x^3}$	• $1 - \frac{3}{x} + 5x^{-2}$
• $\frac{1}{1 + e^{-x}}$	• $\frac{e^x}{e^x + 1}$
• $\frac{3x^2 - x - 2}{x^3 + 2x^2}$	• $\frac{3}{x + 2} - \frac{1}{x^2}$

1.2 The Real Number System

We begin our study of algebra with a look at the **real number system.** We use real numbers every day to describe quantities like age, miles per gallon, container size, population, and so on. To represent real numbers we use symbols such as

$$9, \quad -5, \quad \sqrt{2}, \quad \pi, \quad \tfrac{4}{3}, \quad 0.6666\ldots,$$
$$28.21, \quad 0, \quad \text{and} \quad \sqrt[3]{-32}$$

There are four fundamental operations on the real numbers: **addition, subtraction, multiplication,** and **division,** denoted by the symbols $+$, $-$, $\times$ (or $\cdot$), and $\div$. The set of real numbers is **closed** relative to these four operations (with the exception that division by zero is undefined). This means that the *sum, difference, product, and quotient of two real numbers are also real numbers*.

The set of real numbers is made up of the following five subsets:

Natural Numbers	$\{1, 2, 3, 4, \ldots\}$
Whole Numbers	$\{0, 1, 2, 3, \ldots\}$
Integers	$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
Rational Numbers	{all numbers of the form p/q}* or {all terminating or repeating decimals}
Irrational Numbers	{all nonrepeating, nonterminating decimals}

Note in Figure 1.1 that if we begin with the natural numbers we have closure for addition and multiplication. However, we do not obtain closure for subtraction until we get to the integers and we do not obtain closure for division until we get to the rational numbers.

For the purpose of this text, we consider addition and multiplication as the two basic operations of arithmetic and we use these operations without formally defining them. Later, we will define subtraction and division in terms of addition and multiplication, respectively. The following list summarizes the properties of real numbers under the two basic operations.

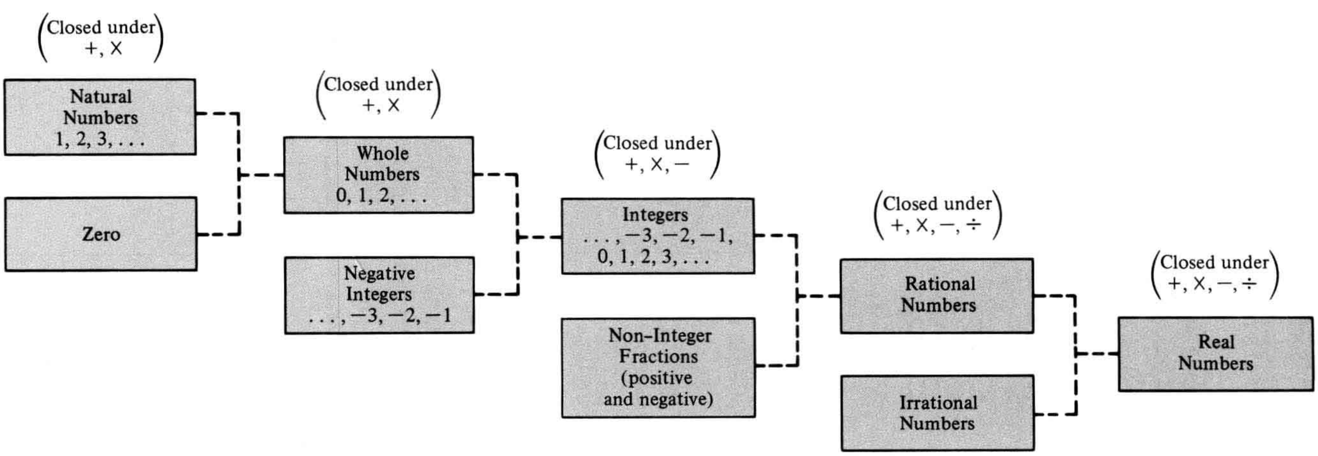


FIGURE 1.1 *Subsets of the Real Numbers*

*Rational numbers can be expressed as the ratio of two integers; that is, they can be written in the form p/q, where p and q are integers with $q \neq 0$.

PROPERTIES OF REAL NUMBERS

For all real numbers a, b, and c:

Property	Addition	Multiplication
1. **Closure:**	$a + b$ is a real number.	$a \cdot b$ is a real number.
2. **Commutative:**	$a + b = b + a$	$a \cdot b = b \cdot a$
3. **Associative:**	$(a + b) + c = a + (b + c)$	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
4. **Identity:**	0 is the identity. $a + 0 = a = 0 + a$	1 is the identity. $a \cdot 1 = a = 1 \cdot a$
5. **Inverse:**	$-a$ is the inverse of a. $a + (-a) = 0 = (-a) + a$	$\frac{1}{a}$ is the inverse of a. $a\left(\frac{1}{a}\right) = 1 = \left(\frac{1}{a}\right)a, \quad a \neq 0$
6. **Distributive:**	$a(b + c) = a \cdot b + a \cdot c$	*(left)*
	$(a + b)c = a \cdot c + b \cdot c$	*(right)*

EXAMPLE 1
Properties of Real Numbers

Identify the property illustrated in each of the given equations.

(a) $5 + 4 = 4 + 5$ (b) $(3 + 7)2 = 3 \cdot 2 + 7 \cdot 2$

(c) $4x\left(\frac{1}{4x}\right) = 1, \quad x \neq 0$ (d) $(x + 6) + 8 = x + (6 + 8)$

Solution:

(a) $5 + 4 = 4 + 5$ *Commutative*

(b) $(3 + 7)2 = 3 \cdot 2 + 7 \cdot 2$ *Distributive*

(c) $4x\left(\frac{1}{4x}\right) = 1, \quad x \neq 0$ *Inverse*

(d) $(x + 6) + 8 = x + (6 + 8)$ *Associative*

Remark: When working with the additive inverse (Property 5), don't confuse the *negative of a number* with a *negative number*. If a number b is already negative, then its additive inverse, $-b$, is positive. For instance, if $b = -5$, then $-b = -(-5) = 5$.

PROPERTIES OF NEGATIVES

For all real numbers a and b:

Properties	Examples
1. $(-1)a = -a$	$(-1)7 = -7$
2. $-(-a) = a$	$-(-6) = 6$

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