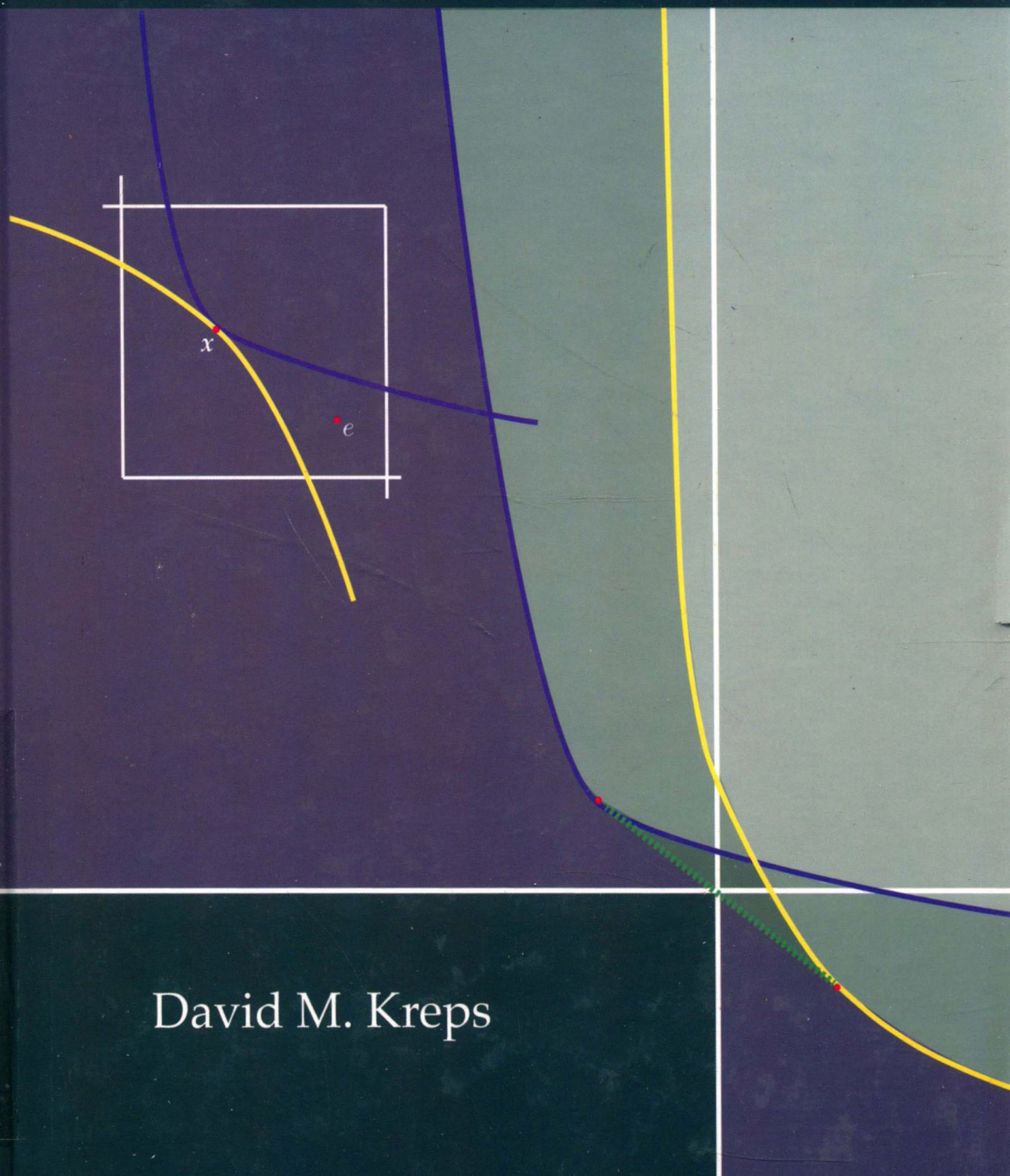


# Microeconomic Foundations I

---

## Choice and Competitive Markets



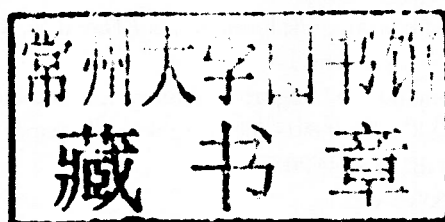
David M. Kreps

# Microeconomic Foundations I

---

## Choice and Competitive Markets

David M. Kreps



PRINCETON UNIVERSITY PRESS | PRINCETON AND OXFORD

Copyright © 2013 by Princeton University Press  
Published by Princeton University Press, 41 William Street,  
Princeton, New Jersey 08540

In the United Kingdom: Princeton University Press, 6 Oxford Street,  
Woodstock, Oxfordshire OX20 1TW

pup.princeton.edu

All rights reserved

**Library of Congress Cataloging-in-Publication Data**

Kreps, David M.

Microeconomic foundations / David M. Kreps

p. cm.

Includes bibliographical references and index.

ISBN 978-0-691-15583-8 (hbk. : alk. paper) 1. Microeconomics. I. Title.

HB172.K744 2013

338.5—dc23

2012011926

British Library Cataloging-in-Publication Data is available

This book has been composed in Helvetica, Palatino, and Computer Modern typefaces.

The publisher would like to acknowledge the author of this volume for providing the camera-ready copy from which this book was published.

Printed on acid-free paper ∞

Printed in the United States of America

1 3 5 7 9 10 8 6 4 2

---

## **Microeconomic Foundations I**

---

To Kenneth J. Arrow

# Preface

---

This book is for aspiring academic economists and those in related fields. It provides a rigorous treatment of some of the basic tools of economic modeling and reasoning, bundled together with enough commentary and reflection so that the reader can appreciate both the strengths and weaknesses of these tools. The target audience (to whom this preface is directly addressed) consists of first-year graduate students who are taking the standard “theory sequence” and would like to go more deeply into a selection of foundational issues, as well as students who, having taken a first-year graduate course out of one of the standard textbooks, would like a deeper dive. At the Stanford Graduate School of Business, this book (more or less) has been the basis of the first-quarter, first-year theory course for Ph.D. students, many of whom had taken a course out of the admirable textbook by Mas-Colell, Whinston, and Green,<sup>1</sup> and so for whom this is an opportunity to review and extend their command of that material.

The objective of the book is captured by the word “command.” In my experience, most students emerge from the standard first-year graduate theory course with an understanding of the material that is good but not great. There is little doubt that almost any student would benefit from a structured review of this material using her original text. But, in my opinion, the standard textbooks are not written with command or mastery of the material as their primary objective. Because they are written to serve very broad audiences, breadth of coverage is stressed over depth, and the authors sometimes omit technical details, to avoid panicking less well-prepared readers. This book sacrifices breadth for depth, avoids compromises about details (with a few exceptions), and tries to explain to the reader both why economic foundations are done the way they are done and what are some of the limitations in how things are done.

Clearly, words like “command” and “mastery” must be taken with many grains of salt. If your career objectives are to do research in any topic covered by this book, the coverage here is inadequate to bring you to the level of understanding you will require. Every chapter in this book could be expanded to a book-length treatment on its own and, even then, important work on the topic would be left out. In some cases, the book comes closer to the research frontier than in others; perhaps not surprisingly, this is true on topics on which I myself have made contributions. But in no case will you finish a chapter and be prepared to tackle frontier research on the topic of that chapter.

Instead, when I use the terms “command” and “mastery,” I have in mind something less ambitious. The foundations of economics are abstract and mathematical (more about this momentarily), and as with any abstract, mathematics-based dis-

---

<sup>1</sup> *Microeconomic Theory*, Oxford: Oxford University Press, 1995.



cipline, the more comfortable you are with the foundations, the more likely it is that you will use those foundations well. Errors in thought are much more likely the closer you are working to the frontiers of your understanding. If you ever find yourself leaning on formal mathematics that you don't fully understand—if you find yourself thinking, “I'm not sure why my model generates this result, but that's what emerges”—you are in grave danger. You should understand the tools you use deeply enough so that you aren't fooled by them.

So that's the objective here: to bring you (closer) to command level on a relatively limited set of results, rather than to a nodding-acquaintance level with a broader set. If you understand a few things deeply, you will understand what it means to acquire deep understanding, and then you can strive for a similar depth of understanding on whatever (other) subject is of interest to you. My objective is to turn that “if” into a “when,” while covering a selection of important microeconomic foundations.

Given this objective, can this book be used as a primary text in the first-graduate-theory course? It is used that way for some of the students at the Stanford GSB, so of course I think the answer is yes. But bear in mind the book's trade-off of breadth for depth. You should complement this book with one that provides broader coverage. Indeed, since this material is part of the foundation of what (I expect) you hope to be your career, you should in any case invest in multiple perspectives. And, having given you that advice in general, let me be a bit more specific: One of the many virtues of Mas-Colell, Whinston, and Green (*ibid.*) is its enormous breadth. You ought to have a copy on your shelf, if not your desk.<sup>2</sup>

## Volume I?

The title is *Microeconomic Foundations I* with subtitle *Choice and Competitive Markets*, suggesting that further volumes are in preparation. “In preparation” is an overstatement, as I write these words; “planned” is more accurate, and I plan not only *II: Strategic Interaction, Information, and Imperfect Competition*, but also *III: Institutions and Behavior*. The volume you are holding deals with economic foundations that existed in (nearly) finished form in the mid 1970s: various models of individual choice; consumer and producer theory (for price-taking or competitive consumers and firms); and (some) general equilibrium theory. The intended second volume will cover material that entered the mainstream of economic thought and practice from the mid 1970s to, say, 1990: information economics and noncooperative game theory, in particular. The third piece is the most speculative: I have in mind a volume that will wrap together developments in behavioral and institutional economics, with (perhaps) transaction cost economics playing a central (but not *the* central) role. I am trying to write this so that each volume would correspond to one ten-week course, fitting the academic calendar of Stanford University. But that's an ambitious agenda; only time will tell if the second and third parts ever appear.

---

<sup>2</sup> Of course, many other excellent treatments of these topics can be found; I make no attempt to list them all. But one resource that may be harder to find is a sequence of excellent notes on a variety of topics in microeconomics and related mathematics, prepared by Kim Border. Go to the URL <http://www.hss.caltech.edu/~kcb/Notes.shtml> for a list of these notes.

## Mathematics in this book and in economics

The approach of this book is resolutely mathematical, because the foundations of economics are resolutely mathematical. The level of mathematics required is not *extremely* high; nearly everything takes place within finite-dimensional Euclidean space. This is a deliberate choice: I have tried to hold the mathematics employed to a level that most graduate students in economics will have. With exceptions limited to a few topics, to navigate this book you must know the sort of mathematics covered in an undergraduate course on real analysis, plus the first few weeks of an undergraduate course in abstract algebra (concerning binary relations).<sup>3</sup> You will need to know more about some specific mathematics, notably some convex analysis, some theory of correspondences, and basics of constrained optimization. But most of the prerequisite mathematics and all of these specific topics are reviewed in a series of appendices at the end of the book.<sup>4</sup>

However, while high-level mathematics is not required, what is commonly called “mathematical sophistication” is needed from start to finish. To make it through the book, you need to be comfortable with mathematical abstraction and with a definition–proposition–proof style of presentation. For students with a strong background in mathematics, this will not be problematic and may even be comforting; but for many students, this will be the real barrier to using this book. I make no apologies for imposing this hurdle, because this, in my opinion, is essential to command-level understanding of the mathematical tools economists employ. I take proofs seriously, providing in most cases details or at least an outline of the proof. (I will sometimes skip steps or “leave the proof to the reader.” In every case where this happens, if you aren’t sure you see how to fill in the gaps, then you really should take the time to figure out how to do so.)

Each chapter comes with some problems, often including requests that you provide proofs that I leave to the reader. You won’t achieve mastery of this material if you don’t do them. So do them. Answers to problems with asterisks—as in, \*2.3, meaning Problem 3 in Chapter 2—are provided in a *Student’s Guide*, which also gives summaries of each chapter. (This includes roughly half the problems and, in most cases, problems where I ask you to fill in gaps left in the text.) You can freely download chapter-by-chapter pieces of the *Student’s Guide* at the URL <http://www.microfoundations1.stanford.edu/student>.<sup>5</sup>

Concerning mathematics and its role in economics: Some first-year graduate

<sup>3</sup> So my earlier claim that I have tried to avoid compromises is, at best, a relative statement. And sometimes the lure of going beyond finite-dimensional Euclidean spaces is irresistible: in a very few places, I employ some measure-theoretic probability theory; to do some of the problems, you must know some theory of stochastic processes; toward the end of the book, I informally discuss economies with a continuum of agents. But none of this material is essential for the main expositional flow of the text. I also expect all readers to be reasonably facile with spreadsheets; I employ MSExcel.

<sup>4</sup> I also provide a very detailed appendix on the methods of dynamic programming, which I expect few readers will have seen before. This material is not used in this book except in the problems connected to Chapter 7, but these are useful tools in modern macroeconomics and in topics to be discussed in the second volume, and it seemed appropriate to cover these methods in connection with Chapter 7, which concerns dynamic choice.

<sup>5</sup> Solutions to the other problems are provided in an *Instructor’s Manual*, which also provides my



students are utterly turned off by their first-year theory courses. They have come to the study of economics to understand real-world phenomena and, perhaps, to make a difference in the real world, not to study mathematics. To those students, my response is that if you plan to use economic techniques to understand the real world and to see how to make a difference, your effectiveness will depend in part on how well you understand those techniques; doing all this math builds your understanding of the techniques. Aspiring novelists or essayists may not see the value in learning to diagram sentences in fifth grade, but if diagramming sentences in fifth grade improves the clarity of their sentences—and I think it does—it is an important drill along the way to becoming a novelist or essayist.

A different objection is that economics is a poorer discipline *because of its reliance on mathematical models*. To be tractable—a word you are likely to come to dislike—mathematical models must be relatively simple. So mathematics forces all sorts of simplifications on economic models that make the models less realistic. Because of this, some critics decry the study of mathematical models in economics; they say it is indoctrination of the young into a false and limiting faith.

Mathematical modeling is a mixed blessing for economics. Mathematical modeling provides real advantages in terms of precision of thought, in seeing how assumptions are linked to conclusions, in generating and communicating insights, in generalizing propositions, and in exporting knowledge from one context to another. In my opinion, these advantages are monumental, far outweighing the costs. But the costs are not zero. Mathematical modeling limits what can be tackled and what is considered legitimate inquiry. You may decide, with experience, that the sorts of models in this book do not help you understand the economic phenomena that you want to understand. Since, as I write these lines, I don't know what phenomena you want to understand, I can't say that you are surely wrong. And the position is defensible. But, based on my own experiences, you are probably wrong. In any case, you are more likely to succeed in convincing others and changing the way economists as a whole do business if you have mastered the sort of mathematical models presented here, which continue to be the foundation of modern economics.

## Conventions

Within each chapter, propositions, definitions, lemmas, and so forth are numbered sequentially. That is, if the first such item in Chapter 6 is a definition, it is Definition 6.1; if the second such item in Chapter 6 is a proposition, it is Proposition 6.2. Figures in a chapter are also numbered sequentially, but in a different list. So the first figure in Chapter 6 is Figure 6.1. Problems are numbered sequentially in still another list, and equations in still another list.

The use of third-person singular pronouns in books such as this has become an exercise in political correctness. I use *she*, *her*, and *hers* when only one actor is

involved; the second actor is *he*, *him*, and *his*. Keeping with PC requirements, when there are two actors and a logical status ordering, *she* has higher status, as in: *she is the employer, he is the employee*. With a tip of the hat to Robert Aumann, in some places she is Alice and he is Bob.

Having paid my dues to PC as outlined in the previous paragraph, the dollar is the standard currency in this book.

## Acknowledgments

Many generations of Ph.D. students at the Stanford Graduate School of Business have suffered through typo-laden drafts of pieces of this volume, and they have done quite a lot to reduce (not to zero, I'm sorry to say) the number of typos. I thank them.

As I was wrapping up the final version of the manuscript, Alejandro Francetich took on the task of reading for internal consistency. He did much more, finding a host of both typos and think-os, including some that are best described as howlers. I don't know that he got them all, but he improved the final product immensely.

In a world of email, it is very easy to "reach out" to colleagues with a specific question. When writing a book of this sort, which encompasses a lot of material about which I am *not* an expert, the urge to ask colleagues who are experts has been too much for me to resist, and the equally good grace and advice of many colleagues have made this a far better book. I am bound to have forgotten some individuals in this category (to whom I apologize), but among those who have been generous with their time and expertise are Kenneth Arrow, Susan Athey, Robert Aumann, Kim Border, Eddie Dekel, Erwin Diewert, Yossi Feinberg, Faruk Gul, Matt Jackson, Jim Jordan, Vijay Krishna, Sunil Kumar, Mark Machina, Michael Ostrovsky, Phil Reny, John Roberts, Tom Sargent, José Scheinkman, Andy Skrzypacz, Hugo Sonnenschein, and Peter Wakker.

The production of this book required the efforts of a number of folks at Princeton University Press, who were very patient with a crazy, opinionated, and stubborn author. I'm particularly grateful to acquisition editor Seth Ditchik, production editors Ben Holmes and Kathleen Cioffi, copy editor Richard Isomaki, indexer Sheila Bodell, and senior book designer Lorraine Doneker.

I produced this book using TeXtures, an implementation of TeX by Blue Sky TeX Systems. The people at Blue Sky have always been there for me when I have had technical issues. Figures were produced using Adobe Illustrator.

This volume records the contributions of many economists, some of whom it has been my privilege to know as role models and friends. I am grateful to them all, and I take this opportunity to recognize one in particular:

Given the nature of this book and my unbounded admiration for both the individual and his work, it is a pleasure and honor to dedicate this volume to Kenneth Arrow.

---

# Microeconomic Foundations I

# Contents

---

Preface	xiii
Chapter One. Choice, Preference, and Utility	1
1.1. Consumer Choice: The Basics	1
1.2. Proving Most of Proposition 1.2, and More	5
1.3. The No-Better-Than Sets and Utility Representations	7
1.4. Strict Preference and Indifference	9
1.5. Infinite Sets and Utility Representations	10
1.6. Choice from Infinite Sets	15
1.7. Equivalent Utility Representations	17
1.8. Commentary	18
Bibliographic Notes	23
Problems	23
Chapter Two. Structural Properties of Preferences and Utility Functions	30
2.1. Monotonicity	31
2.2. Convexity	32
2.3. Continuity	35
2.4. Indifference Curve Diagrams	38
2.5. Weak and Additive Separability	39
2.6. Quasi-linearity	43
2.7. Homotheticity	44
Bibliographic Notes	45
Problems	45
Chapter Three. Basics of Consumer Demand	50
3.1. The Consumer's Problem	50
3.2. Basic Facts about the CP	52
3.3. The Marshallian Demand Correspondence and Indirect Utility Function	54
3.4. Solving the CP with Calculus	56
Bibliographic Notes	63
Problems	64

<b>Chapter Four. Revealed Preference and Afriat's Theorem</b>	<b>67</b>
4.1. An Example and Basic Ideas	67
4.2. GARP and Afriat's Theorem	70
4.3. Comparative Statics and the Own-Price Effect	74
Bibliographic Notes	77
Problems	78
 <b>Chapter Five. Choice under Uncertainty</b>	 <b>79</b>
5.1. Two Models and Three Representations	79
5.2. The Mixture-Space Theorem	89
5.3. States of Nature and Subjective Expected Utility	101
5.4. Subjective and Objective Probability and the Harsanyi Doctrine	108
5.5. Empirical and Theoretical Critiques	110
Bibliographic Notes	116
Problems	116
 <b>Chapter Six. Utility for Money</b>	 <b>123</b>
6.1. Properties of Utility Functions for Money	123
6.2. Induced Preferences for Income	134
6.3. Demand for Insurance and Risky Assets	138
Bibliographic Notes	140
Problems	140
 <b>Chapter Seven. Dynamic Choice</b>	 <b>148</b>
7.1. The Standard Strategic Approach	149
7.2. Dynamic Programming	152
7.3. Testable Restrictions of the Standard Model	153
7.4. Three Alternatives to the Standard Model	156
Bibliographic Notes	161
Problems	161
 <b>Chapter Eight. Social Choice and Efficiency</b>	 <b>166</b>
8.1. Arrow's Theorem	166
8.2. What Do We Give Up?	172
8.3. Efficiency	175
8.4. Identifying the Pareto Frontier: Utility Imputations and Bergsonian Social Utility Functionals	176
8.5. Syndicate Theory and Efficient Risk Sharing: Applying Proposition 8.10	184
8.6. Efficiency?	192
Bibliographic Notes	194
Problems	194

<b>Chapter Nine. Competitive and Profit-Maximizing Firms</b>	<b>197</b>
9.1. The Production-Possibility Set	198
9.2. Profit Maximization	199
9.3. Basics of the Firm's Profit-Maximization Problem	201
9.4. Afriat's Theorem for Firms	207
9.5. From Profit Functions to Production-Possibility Sets	211
9.6. How Many Production-Possibility Sets Give the Same Profit Function?	213
9.7. What Is Going On Here, Mathematically?	216
9.8. Differentiability of the Profit Function	219
9.9. Cost Minimization and Input-Requirement Sets	222
9.10. Why Do We Care?	228
Bibliographic Notes	229
Problems	229
 <b>Chapter Ten. The Expenditure-Minimization Problem</b>	 <b>233</b>
10.1. Defining the EMP	233
10.2. Basic Analysis of the EMP	235
10.3. Hicksian Demand and the Expenditure Function	236
10.4. Properties of the Expenditure Function	238
10.5. How Many Continuous Utility Functions Give the Same Expenditure Function?	240
10.6. Recovering Continuous Utility Functions from Expenditure Functions	247
10.7. Is an Alleged Expenditure Function Really an Expenditure Function?	248
10.8. Connecting the CP and the EMP	254
Bibliographic Notes	255
Problems	255
 <b>Chapter Eleven. Classic Demand Theory</b>	 <b>258</b>
11.1. Roy's Identity and the Slutsky Equation	258
11.2. Differentiability of Indirect Utility	262
11.3. Duality of Utility and Indirect Utility	269
11.4. Differentiability of Marshallian Demand	274
11.5. Integrability	279
11.6. Complements and Substitutes	283
11.7. Integrability and Revealed Preference	284
Bibliographic Notes	286
Problems	287
 <b>Chapter Twelve. Producer and Consumer Surplus</b>	 <b>289</b>
12.1. Producer Surplus	289



12.2. Consumer Surplus	296
Bibliographic Notes	304
Problems	304
<b>Chapter Thirteen. Aggregating Firms and Consumers</b>	<b>306</b>
13.1. Aggregating Firms	307
13.2. Aggregating Consumers	310
13.3. Convexification through Aggregation	318
Bibliographic Notes	326
Problems	326
<b>Chapter Fourteen. General Equilibrium</b>	<b>329</b>
14.1. Definitions	329
14.2. Basic Properties of Walrasian Equilibrium	333
14.3. The Edgeworth Box	335
14.4. Existence of Walrasian Equilibria	338
14.5. The Set of Equilibria for a Fixed Economy	351
14.6. The Equilibrium Correspondence	354
Bibliographic Notes	354
Problems	355
<b>Chapter Fifteen. General Equilibrium, Efficiency, and the Core</b>	<b>358</b>
15.1. The First Theorem of Welfare Economics	359
15.2. The Second Theorem of Welfare Economics	362
15.3. Walrasian Equilibria Are in the Core	366
15.4. In a Large Enough Economy, Every Core Allocation Is a Walrasian-Equilibrium Allocation	370
15.5. Externalities and Lindahl Equilibrium	380
Bibliographic Notes	383
Problems	383
<b>Chapter Sixteen. General Equilibrium, Time, and Uncertainty</b>	<b>386</b>
16.1. A Framework for Time and Uncertainty	386
16.2. General Equilibrium with Time and Uncertainty	389
16.3. Equilibria of Plans, Prices, and Price Expectations: I. Pure Exchange with Contingent Claims	392
16.4. EPPPE: II. Complex Financial Securities and Complete Markets	402
16.5. EPPPE: III. Complex Securities with Real Dividends and Complete Markets	418
16.6. Incomplete Markets	419
16.7. Firms	424
Bibliographic Notes	431
Problems	432

Contents	xi
About the Appendices	437
Appendix One: Mathematical Induction	439
Appendix Two: Some Simple Real Analysis	441
A2.1. The Setting	441
A2.2. Distance, Neighborhoods, and Open and Closed Sets	441
A2.3. Sequences and Limits	445
A2.4. Boundedness, (Completeness), and Compactness	446
A2.5. Continuous Functions	447
A2.6. Simply Connected Sets and the Intermediate-Value Theorem	448
A2.7. Suprema and Infima; Maxes and Mins	448
A2.8. The Maximum of a Continuous Function on a Compact Set	449
A2.9. Lims Sup and Inf	450
A2.10. Upper and Lower Semi-continuous Functions	451
Appendix Three: Convexity	452
A3.1. Convex Sets	452
A3.2. The Separating- and Supporting-Hyperplane Theorems	457
A3.3. The Support-Function Theorem	459
A3.4. Concave and Convex Functions	461
A3.5. Quasi-concavity and Quasi-convexity	463
A3.6. Supergradients and Subgradients	466
A3.7. Concave and Convex Functions and Calculus	468
Appendix Four: Correspondences	469
A4.1. Functions and Correspondences	470
A4.2. Continuity of Correspondences	471
A4.3. Singleton-Valued Correspondences and Continuity	474
A4.4. Parametric Constrained Optimization Problems and Berge's Theorem	475
A4.5. Why this Terminology?	477
Appendix Five: Constrained Optimization	479
Appendix Six: Dynamic Programming	485
A6.1. Several Examples	485
A6.2. A General Formulation	489
A6.3. Bellman's Equation	494
A6.4. Conserving and Unimprovable Strategies	496
A6.5. Additive Rewards	501

A6.6. States of the System	504
A6.7. Solving Finite-Horizon Problems	506
A6.8. Infinite-Horizon Problems and Stationarity	509
A6.9. Solving Infinite-Horizon (Stationary) Problems with Unimprovability	512
A6.10. Policy Iteration (and Transience)	516
A6.11. Value Iteration	518
A6.12. Examples	521
A6.13. Things Not Covered Here: Other Optimality Criteria; Continuous Time and Control Theory	527
A6.14. Multi-armed Bandits and Complexity	528
A6.15. Four More Problems You Can Solve	530
 Appendix Seven: The Implicit Function Theorem	 534
 Appendix Eight: Fixed-Point Theory	 535
 References	 543
 Index	 551