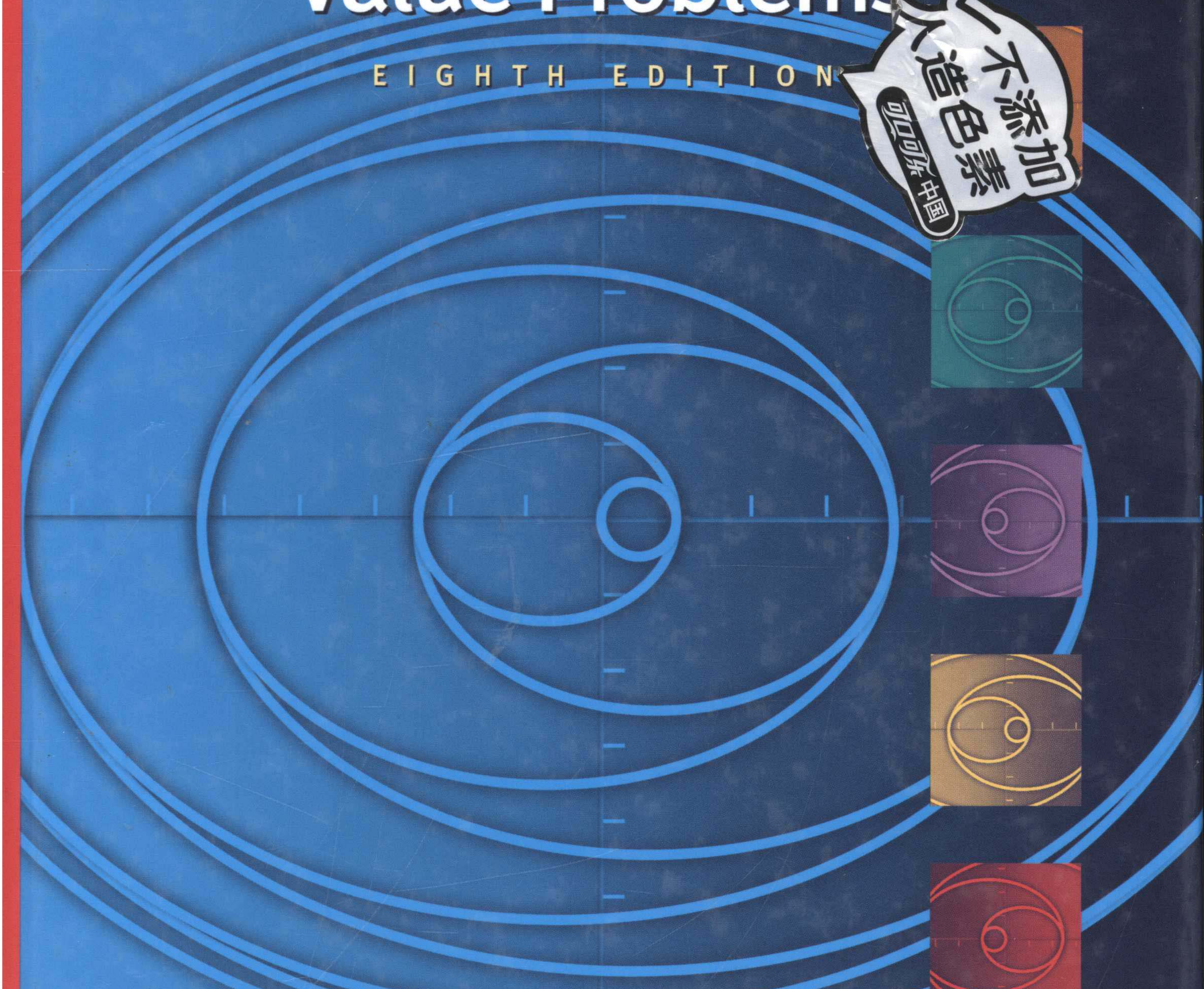
Elementary Differential Equations and Boundary Value Problems

EIGHTH EDITION



William E. Boyce | Richard C. DiPrima

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Elementary Differential Equations and Boundary Value Problems

To Elsa and Maureen

To Siobhan, James, Richard, Jr., Carolyn, and Ann

And to the next generation:
Charles, Aidan, Stephanie, Veronica, and Deirdre

The Authors

William E. Boyce received his B.A. degree in Mathematics from Rhodes College, and his M.S. and Ph.D. degrees in Mathematics from Carnegie-Mellon University. He is a member of the American Mathematical Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics. He is currently the Edward P. Hamilton Distinguished Professor Emeritus of Science Education (Department of Mathematical Sciences) at Rensselaer. He is the author of numerous technical papers in boundary value problems and random differential equations and their applications. He is the author of several textbooks including two differential equations texts, and is the coauthor (with M.H. Holmes, J.G. Ecker, and W.L. Siegmann) of a text on using Maple to explore Calculus. He is also coauthor (with R.L. Borrelli and C.S. Coleman) of *Differential Equations Laboratory Workbook* (Wiley 1992), which received the EDUCOM Best Mathematics Curricular Innovation Award in 1993. Professor Boyce was a member of the NSF-sponsored CODEE (Consortium for Ordinary Differential Equations Experiments) that led to the widely-acclaimed *ODE Architect*. He has also been active in curriculum innovation and reform. Among other things, he was the initiator of the "Computers in Calculus" project at Rensselaer, partially supported by the NSF. In 1991 he received the William H. Wiley Distinguished Faculty Award given by Rensselaer.

Richard C. DiPrima (deceased) received his B.S., M.S., and Ph.D. degrees in Mathematics from Carnegie-Mellon University. He joined the faculty of Rensselaer Polytechnic Institute after holding research positions at MIT, Harvard, and Hughes Aircraft. He held the Eliza Ricketts Foundation Professorship of Mathematics at Rensselaer, was a fellow of the American Society of Mechanical Engineers, the American Academy of Mechanics, and the American Physical Society. He was also a member of the American Mathematical Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics. He served as the Chairman of the Department of Mathematical Sciences at Rensselaer, as President of the Society for Industrial and Applied Mathematics, and as Chairman of the Executive Committee of the Applied Mechanics Division of ASME. In 1980, he was the recipient of the William H. Wiley Distinguished Faculty Award given by Rensselaer. He received Fulbright fellowships in 1964–65 and 1983 and a Guggenheim fellowship in 1982–83. He was the author of numerous technical papers in hydrodynamic stability and lubrication theory and two texts on differential equations and boundary value problems. Professor DiPrima died on September 10, 1984.

PREFACE

Audience and Prerequisites

This edition, like its predecessors, is written from the viewpoint of the applied mathematician, whose interest in differential equations may sometimes be quite theoretical, sometimes intensely practical, and often somewhere in between. We have sought to combine a sound and accurate (but not abstract) exposition of the elementary theory of differential equations with considerable material on methods of solution, analysis, and approximation that have proved useful in a wide variety of applications.

The book is written primarily for undergraduate students of mathematics, science, or engineering, who typically take a course on differential equations during their first or second year of study. The main prerequisite for reading the book is a working knowledge of calculus, gained from a normal two- or three-semester course sequence or its equivalent. Some familiarity with matrices will also be helpful in the chapters on systems of differential equations

A Changing Learning Environment

The environment in which instructors teach, and students learn, differential equations has changed enormously in the past several years and continues to evolve at a rapid pace. Computing equipment of some kind, whether a graphing calculator, a notebook computer, or a desktop workstation is available to most students of differential equations. This equipment makes it relatively easy to execute extended numerical calculations, to generate graphical displays of a very high quality, and, in many cases, to carry out complex symbolic manipulations. A high-speed internet connection offers an enormous range of further possibilities.

The fact that so many students now have these capabilities enables instructors, if they wish, to modify very substantially their presentation of the subject and their expectations of student performance. Not surprisingly, instructors have widely varying opinions as to how a course on differential equations should be taught under these circumstances. Nevertheless, at many colleges and universities courses on differential equations are becoming more visual, more quantitative, more project-oriented, and less formula-centered than in the past.

A Flexible Approach

To be widely useful a textbook must be adaptable to a variety of instructional strategies. This implies at least two things. First, instructors should have maximum flexibility to choose both the particular topics that they wish to cover and also the order in which they want to cover them. Second, the book should be useful to students having access to a wide range of technological capability.

Modular Chapters

With respect to content, we provide this flexibility by making sure that, so far as possible, individual chapters are independent of each other. Thus, after the basic parts of the first three chapters are completed (roughly Sections 1.1 through 1.3, 2.1 through 2.5, and 3.1 through 3.6) the selection of additional topics, and the order and depth in which they are covered, is at the discretion of the instructor. For example, an instructor who wishes to emphasize a systems approach to differential equations can take up Chapter 7 (Linear Systems) and perhaps even Chapter 9 (Nonlinear Autonomous Systems) immediately after Chapter 2. Or, while we present the basic theory of linear equations first in the context of a single second order equation (Chapter 3), many instructors have combined this material with the corresponding treatment of higher order equations (Chapter 4) or of linear systems (Chapter 7). Or, while the main discussion of numerical methods is in Chapter 8, an instructor who wishes to emphasize this approach can introduce some of this material in conjunction with Chapter 2. Many other choices and combinations are also possible and have been used effectively with earlier editions of this book.

Technology

With respect to technology, we note repeatedly in the text that computers are extremely useful for investigating differential equations and their solutions, and many of the problems are best approached with computational assistance. Nevertheless, the book is adaptable to courses having various levels of computer involvement, ranging from little or none to intensive. The text is independent of any particular hardware platform or software package. More than 450 problems are marked with the symbol to indicate that we consider them to be technologically intensive. These problems may call for a plot, or for substantial numerical computation, or for extensive symbolic manipulation, or for some combination of these requirements. Naturally, the designation of a problem as technologically intensive is a somewhat subjective judgment, and the is intended only as a guide. Many of the marked problems can be solved, at least in part, without computational help, and a computer can be used effectively on many of the unmarked problems.

Homework Problems

From a student's point of view, the problems that are assigned as homework and that appear on examinations drive the course. We believe that the most outstanding feature of this book is the number, and above all the variety and range, of the problems that it contains. Many problems are entirely straightforward, but many others are more challenging, and some are fairly open-ended, and can serve as the basis for independent student projects. There are far more problems than any instructor can use in any given course, and this provides instructors with a multitude of possible choices in tailoring their course to meet their own goals and the needs of their students.

One of the choices that an instructor now has to make concerns the role of computing in the course. For instance, many more or less routine problems, such as those requesting the solution of a first or second order initial value problem, are now easy to solve by a computer algebra system. This edition includes quite a few such problems, just as its predecessors did. We do not state in these problems how they should be solved, because we believe that it is up to each instructor to specify whether their students should solve such problems by hand, with computer assistance, or perhaps both ways. Also, there are many problems that call for a graph of the solution. Instructors have the option of specifying whether they want an accurate computer-generated plot or a hand-drawn sketch, or perhaps both.

Mathematical Modeling

Building from Basic Models

The main reason for solving many differential equations is to try to learn something about an underlying physical process that the equation is believed to model. It is basic to the importance of differential equations that even the simplest equations correspond to useful physical models, such as exponential growth and decay, spring-mass systems, or electrical circuits. Gaining an understanding of a complex natural process is usually accomplished by combining or building upon simpler and more basic models. Thus a thorough knowledge of these basic models, the equations that describe them, and their solutions, is the first and indispensable step toward the solution of more complex and realistic problems. We describe the modeling process in detail in Sections 1.1, 1.2, and 2.3. Careful constructions of models appear also in Sections 2.5, 3.8, and in the appendices to Chapter 10. Differential equations resulting from the modeling process appear frequently throughout the book, especially in the problem sets.

A Combination of Tools–Analytical and Numerical

Nonroutine problems often require the use of a variety of tools, both analytical and numerical. Paper and pencil methods must often be combined with effective use of a computer. Quantitative results and graphs, often produced by a computer, serve to illustrate and clarify conclusions that may be obscured by complicated analytical expressions. On the other hand, the implementation of an efficient numerical procedure typically rests on a good deal of preliminary analysis—to determine the qualitative features of the solution as a guide to computation, to investigate limiting or special cases, or to discover which ranges of the variables or parameters may require or merit special attention. Thus, a student should come to realize that investigating a difficult problem may well require both analysis and computation; that good judgment may be required to determine which tool is best-suited for a particular task; and that results can often be presented in a variety of forms.

Gaining Insight into the Behavior of a Process

We believe that it is important for students to understand that (except perhaps in courses on differential equations) the goal of solving a differential equation is seldom simply to obtain the solution. Rather, one is interested in the solution in order to obtain insight into the behavior of the process that the equation purports to model. In other words, the solution is not an end in itself. Thus, we have included a great many problems, as well as some examples in the text, that call for conclusions to be drawn about the solution. Sometimes this takes the form of asking for the value of the independent variable at which the solution has a certain property, or to determine the long term behavior of the solution. Other problems ask for the effect of variations in a parameter, or for the determination of a critical value of a parameter at which the solution experiences a substantial change. Such problems are typical of those that arise in the applications of differential equations, and, depending on the goals of the course, an instructor has the option of assigning few or many of these problems.

Notable Changes in the Eighth Edition

Readers familiar with the preceding edition will observe that the general structure of the book is unchanged. The revisions that we have made in this edition have several goals: to enlarge the range of applications that are considered, to make the presentation more visual by adding some new figures, and to improve the exposition by including several new or improved examples. More specifically, the most important changes are the following:

1. There are approximately 65 new problems scattered throughout the book. There are also about 15 new figures and 8 new or modified examples.
2. Section 2.1, "Linear Equations; Method of Integrating Factors," has been substantially rewritten, with two new examples, to reduce repetition.
3. Section 2.5, "Autonomous Equations and Population Dynamics," has been modified to give more prominence to the phase line as an aid to sketching solutions.
4. In Section 3.9 the general case of damped vibrations is considered before the special case of undamped vibrations, reversing the order of previous editions. The presentation is more detailed and there are three new figures.
5. The proof of the convolution theorem in Section 6.6 has been rewritten and six new problems on integral and integro-differential equations have been added.
6. To illustrate the occurrence of systems of higher than second order a new example on coupled oscillators has been added to Section 7.6, with three accompanying figures and several related problems.
7. An example has been added to Section 7.9 to demonstrate the use of Laplace transforms for nonhomogeneous linear systems.

8. There are several new problems in Sections 2.5, 9.4, and 9.7 to illustrate the occurrence of bifurcations in one- and two-dimensional nonlinear systems.
9. There are new problems in Section 10.6 on heat conduction in the presence of external heat sources, in Section 10.7 on dispersive waves, and in Section 10.8 on the flow through an aquifer.

As the subject matter of differential equations continues to grow, as new technologies become commonplace, as old areas of application are expanded, and as new ones appear on the horizon, the content and viewpoint of courses and their textbooks must also evolve. This is the spirit we have sought to express in this book.

William E. Boyce
Grafton, New York
February 23, 2004

Supplemental Resources for Instructors and Students

The *ODE Architect* CD is included with every copy of the text. *ODE Architect* is a prize-winning, state-of-the-art NSF-sponsored learning software package, which is Windows-compatible. A solver tool allows you to build your own models with ODEs and study them in a truly interactive point-and-click environment. The *Architect* includes an interactive library of more than one hundred model differential equation systems with graphs of solutions. The *Architect* also has 14 interactive multimedia modules, which provide a range of models and phenomena, from a golf game to chaos.

An Instructor's Solutions Manual, ISBN 0-471-67972-0, includes solutions for all problems in the text.

A Student Solutions Manual, ISBN 0-471-43340-3, includes solutions for selected problems in the text.

A Companion Web site, www.wiley.com/college/boyce, provides a wealth of resources for students and instructors, including:

- PowerPoint slides of important ideas and graphics for study and note taking.
- Review and Study Outlines to help students prepare for quizzes and exams.
- Online Review Quizzes to enable students to test their knowledge of key concepts. For further review diagnostic feedback is provided that refers to pertinent sections in the text.
- Getting Started with *ODE Architect*. This guide introduces students and professors to *ODE Architect*'s simulations and multimedia.
- Additional problems for use with Mathematica, Maple, and MATLAB, allowing opportunities for further exploration of important ideas in the course utilizing these computer algebra and numerical analysis packages.

eGrade Plus

eGrade Plus is a powerful online tool that provides instructors with an integrated suite of teaching and learning resources in one easy-to-use Web site.

eGrade Plus is organized around the essential activities you perform in class:

- **Prepare & Present:** Create class presentations using a wealth of Wiley-provided resources–such as an online version of the textbook, PowerPoint slides, and interactive simulations–making your class preparation more efficient. You may easily adapt, customize, and add to this content to meet the needs of your course.
- **Create Assignments:** Automate the assigning and grading of homework or quizzes by using Wiley-provided question banks, or by writing your own. Student responses will be graded automatically and the results recorded in your gradebook. eGrade Plus can link homework problems to the relevant section of the online text, providing students with context-sensitive help.
- **Track Student Progress:** Keep track of your students' progress via an instructor's gradebook, which allows you to analyze individual and overall class results to determine their progress and level of understanding.

- **Administer Your Course:** eGrade Plus can easily be integrated with another course management system, gradebook, or other resources you are using in your class, providing you with the flexibility to build your course, your way.

For more information about the features and benefits of eGrade Plus, please view our online demo at www.wiley.com/college/egradeplus.

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William E. Boyce

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