

结构力学

Structural Mechanics

何丽艳◆主编 陈思远 刘宇刚◆副主编



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陈思远

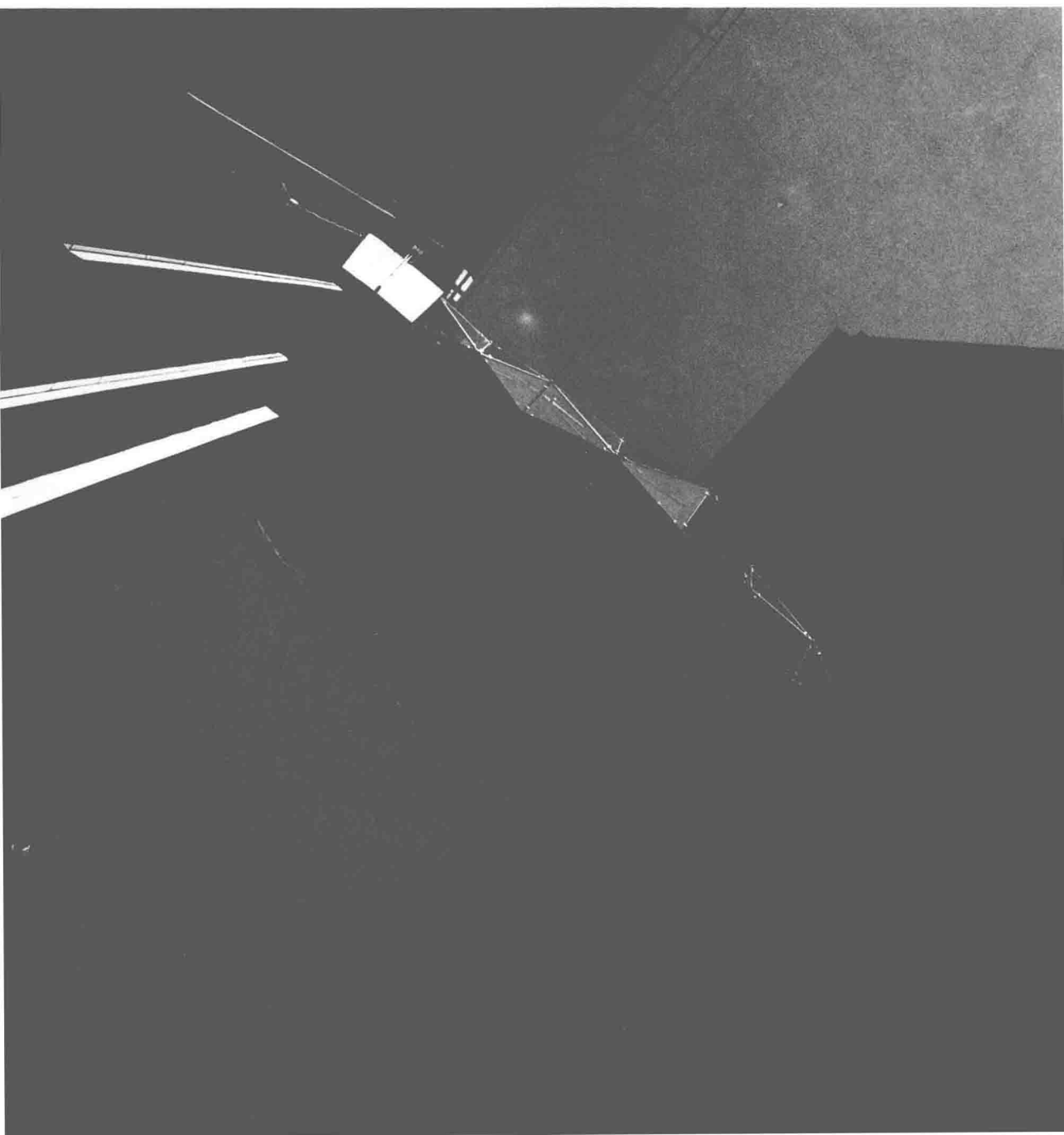
刘宇刚◆副主编

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Chapter 1 Statics of Structures

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1.1 Introduction

With few exceptions, structures must be stable under all conditions of load; that is, they must be able to support applied loads (their own weight, anticipated live loads, wind, and so forth) without changing shape, undergoing large displacements, or collapsing. Since structures that are stable do not move perceptibly when loaded, their analysis — the determination of both internal and external forces (reactions) — is based in large part on the principles and techniques contained in the branch of engineering mechanics called statics. The subject of statics, which you have studied previously, covers force systems acting on rigid bodies at rest (the most common case) or moving at constant velocity; that is, in either case the acceleration of the body is zero.

Although the structures we will study in this book are not absolutely rigid because they undergo small elastic deformations when loaded, in most situations the deflections are so small that we can (1) treat the structure or its components as rigid bodies and (2) base the analysis on the initial dimensions of the structure.

We begin this chapter with a brief review of statics. In this review we consider the characteristics of forces, discuss the equations of static equilibrium for two-dimensional (planar) structures, and use the equations of static equilibrium to determine the reactions and internal forces in a variety of simple determinate structures such as beams, trusses, and simple frames.

We conclude this chapter with a discussion of determinacy and stability. By determinacy, we mean procedures to establish if the equations of statics alone are sufficient to permit a complete analysis of a structure. If the structure cannot be analyzed by the equations of statics, the structure is termed indeterminate. To analyze an indeterminate structure, we must supply additional equations by considering the geometry of the deflected shape. Indeterminate structures will be discussed in later chapters.

beam 梁
truss 桁架
frame 框架

deflected 挠度

determinate 决定性的
indeterminate 不确定的

稳定性 稳定性

By stability, we mean the geometric arrangement of members and supports required to produce a stable structure, that is, a structure that can resist load from any direction without undergoing either a radical change in shape or large rigid-body displacements. In this chapter we consider the stability and determinacy of structures that can be treated as either a single rigid body or as several interconnected rigid bodies. The principles that we establish for these simple structures will be extended to more complex structures in later chapters.

1.2 Forces

To solve typical structural problems, we use equations involving forces or their components. Forces may consist of either a linear force that tends to produce translation or a couple that tends to produce rotation of the body on which it acts. Since a force has magnitude and direction, it can be represented by a vector. For example, Figure 1.1a shows a force F lying in the xy plane and passing through point A .

A couple consists of a pair of equal and oppositely directed forces lying in the same plane (see Fig. 1.1b). The moment M associated with the couple equals the product of the force F and the perpendicular distance (or arm) d between forces. Since a moment is a vector, it has magnitude as well as direction. Although we often represent a moment by a curved arrow to show that it acts in the clockwise or counter-clockwise direction (see Fig. 1.1c), we can also represent a moment by a vector — usually a double-headed arrow — using the right-hand rule. In the right-hand rule we curl the fingers of the right hand in the direction of the moment, and the direction in which the thumb points indicates the direction of the vector.

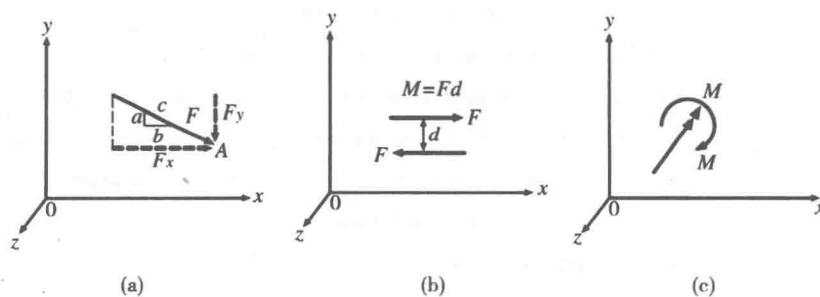


Figure 1.1: Force and moment vectors: (a) linear force vector resolved into x and y components; (b) couple of magnitude Fd ; (c) alternative representation of moment M , by a vector using the right-hand rule.

We must frequently carry out computations that require either resolving a force into its components or combining several forces to produce a single resultant force.

To facilitate these calculations, it is convenient to select arbitrarily horizontal and vertical axes—an x - y coordinate system—as the basic reference directions.

A force can be resolved into components by using the geometric relationship—similar triangles—that exists between the vector components and the slope of the vector. For example, to express the vertical component F_y of the vector F in Figure 1.1a in terms of the slope of the vector, we write, using similar triangles,

$$\frac{F_y}{a} = \frac{F}{c} \quad \text{and} \quad F_y = \frac{a}{c}F$$

Similarly, if we set up a proportion between the horizontal component F_x and F and the sides of the slope triangle noted on the vector, we can write

$$F_x = \frac{b}{c}F$$

If a force is to be resolved into components that are not parallel to an x - y coordinate system, the law of sines provides a simple relationship between length of sides and interior angles opposite the respective sides. For the triangle shown in Figure 1.2, we can state the law of sines as

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

where A is the angle opposite side a , B is the angle opposite side b , and C is the angle opposite side c .

Example 1.1 illustrates the use of the law of sines to compute the orthogonal components of a vertical force in arbitrary directions.

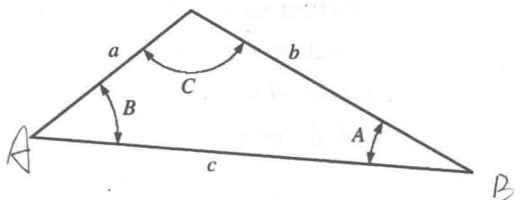


Figure 1.2: Diagram to illustrate law of sines.

[Example 1.1]

Using the law of sines, resolve the 75-lb vertical force F_{AB} in Figure 1.3a into components directed along lines a and b .

Solution

Through point B draw a line parallel to line b , forming triangle ABC . The interior angles of the triangle are easily computed from the information given. Vectors AC and CB (Fig. 1.3b) represent the required components of force F_{AB} . From the law of sines we can write

pound 44482 N

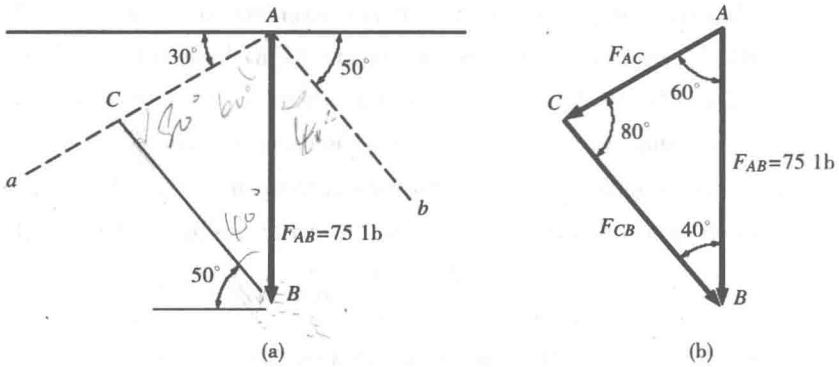


Figure 1.3: Resolution of a vertical force into components.

$$\frac{\sin 80^\circ}{75} = \frac{\sin 40^\circ}{F_{AC}} = \frac{\sin 60^\circ}{F_{CB}}$$

where $\sin 80^\circ = 0.985$, $\sin 60^\circ = 0.866$, and $\sin 40^\circ = 0.643$. Solving for F_{AC} and F_{CB} yields

$$F_{AC} = \frac{\sin 40^\circ}{\sin 80^\circ}(75) = 48.961 \text{ lb}$$

$$F_{CB} = \frac{\sin 60^\circ}{\sin 80^\circ}(75) = 65.941 \text{ lb}$$

Resultant of a Planar Force System

In certain structural problems we will need to determine the magnitude and location of the resultant of a force system. Since the resultant is a single force that produces the same external effect on a body as the original force system, the resultant R must satisfy the following three conditions:

1. The horizontal component of the resultant R_x must equal the algebraic sum of the horizontal components of all forces:

$$R_x = \sum F_x \quad (1.1a)$$

2. The vertical component of the resultant R_y must equal the algebraic sum of the vertical components of all forces:

$$R_y = \sum F_y \quad (1.1b)$$

3. The moment M_o produced by the resultant about a reference axis through point O must equal the moment about point O produced by all forces and couples that make up the original force system.

$$M_o = R_d = \sum F_i d_i + \sum M_i \quad (1.1c)$$

where

$$R = \text{resultant force} = \sqrt{R_x^2 + R_y^2}$$

d = perpendicular distance from line of action of resultant

to axis about which moments are computed

(1.1d)

$\sum F_i d_i$ = moment of all forces about reference axis

$\sum M_i$ = moment of all couples about reference axis

[Example 1.2]

Computation of a Resultant

Determine the magnitude and location of the resultant R of the three wheel loads shown in Figure 1.4.

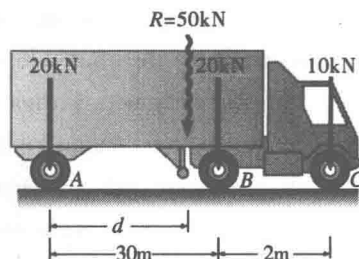


Figure 1.4

Solution

Since none of the forces act in the horizontal direction or have components in the horizontal direction,

$$R_x = 0$$

Using Equation 1.1b gives

$$R = R_y = \sum F_y = 20 + 20 + 10 = 50 \text{ kN}$$

Locate the position of the resultant, using Equation 1.1c; that is, equate the moment produced by the original force system to the moment produced by the resultant R . Select a reference axis through point A (choice of A arbitrary).

$$50d = 20(0) + 20(3) + 10(5), \quad d = 2.2 \text{ m}$$

Resultant of a Distributed Load

In addition to concentrated loads and couples, many structures carry distributed loads. The external effect of a distributed load (the computation of reactions it produces, for example) is most easily handled by replacing the distributed loads by an equivalent resultant force. As you have learned previously in statics and mechanics of materials courses, the magnitude of the resultant of a distributed load equals the area under the load curve and acts at its centroid (see Table A.1 for values of area and location of the centroid for several common geometric shapes). Example 1.3 illustrates the use of integration to compute the magnitude and location of the resultant of a distributed load with a parabolic variation.

If the shape of a distributed load is complex, the designer can often simplify the computation of the magnitude and position of the resultant by subdividing the area into several smaller geometric areas whose properties are known. In most cases distributed loads are uniform or vary linearly. For the latter case, you can divide the area

centroid
重心
centroid
axis
重心轴

into triangular and rectangular areas (see Example 1.7).

As an alternative procedure, the designer may replace a distributed load that varies in a complex manner by a statically equivalent set of concentrated loads using the equations in Figure 1.5. To use these equations, we divide the distributed loads into an arbitrary number of segments of length h . The ends of the segments are termed the nodes. Figure 1.5 shows two typical segments. The nodes are labeled 1, 2, and 3. The number of segments into which the load is divided depends on the length and shape of the distributed load and the quantity we will compute. If the distributed load varies linearly between nodes, the equivalent concentrated force at each node is given by the equations in Figure 1.5a. The equations for forces labeled P_1 and P_3 apply at an exterior node—a segment is located on only one side of the node, and P_2 applies to an interior node—segments are located on both sides of a node.

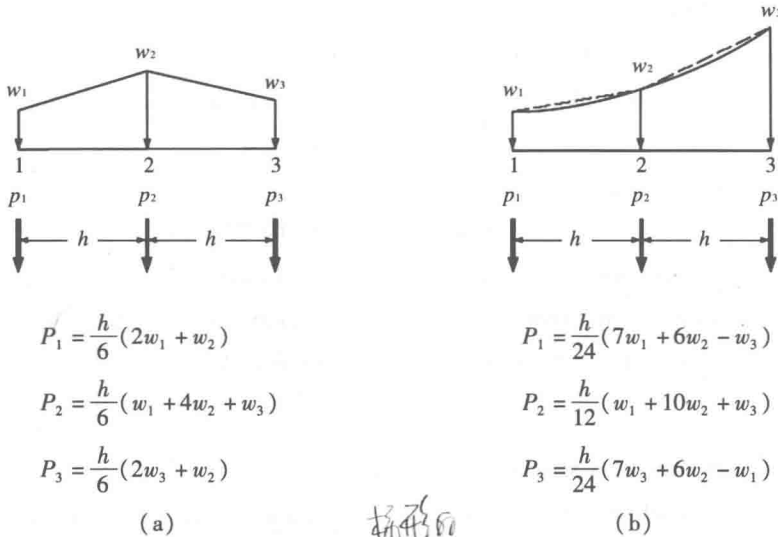


Figure 1.5: (a) Expressions to convert a trapezoidal variation of load to a set of statically equivalent, equally spaced, concentrated loads; (b) equations to convert a parabolic variation of load to a statically equivalent set of concentrated loads. Equations are valid for concave downward parabolas also, and will give a close approximation for higher-order curves.

For a distributed load with a parabolic variation (either concave up or concave down), the equations in Figure 1.5b should be used. These equations will also give good results (within 1 or 2 percent of the exact values) for distributed loads whose shape is represented by a higher-order curve. If the length of the segments is not too large, the simpler equations in Figure 1.5a can also be applied to a distributed load whose ordinates lie on a curve such as shown in Figure 1.5b. When they are applied in this fashion, we are in effect replacing the actual loading curve by a series of trapezoidal elements, as shown by the dashed line in Figure 1.5b. As we reduce the dis-

tance h between nodes (or equivalently increase the number of segments), the trapezoidal approximation approaches the actual curve. Example 1.4 illustrates the use of the equations of Figure 1.5.

Although the resultant of a distributed load produces the same external effect on a body as the original loading, the internal stresses produced by the resultant are not the same as those produced by the distributed load. For example, the resultant force can be used to compute the reactions of a beam, but the computations for internal forces—for example, shear and moment—must be based on the actual loading.

[Example 1.3]

Compute the magnitude and location of the resultant of the parabolic loading shown in Figure 1.6. The slope of the parabola is zero at the origin.

Solution

Compute R by integrating the area under the parabola $y = (w/L^2)x^2$.

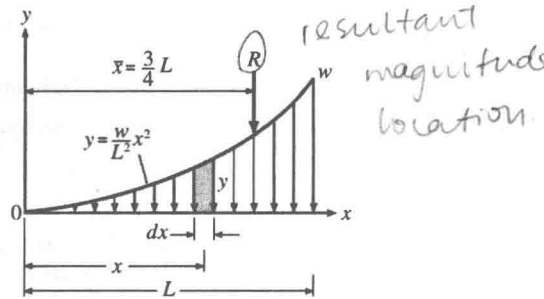


Figure 1.6

centroid method. 重心法.
$$R = \int_0^L y dx = \int_0^L \frac{wx^2}{L^2} dx = \left[\frac{wx^3}{3L^2} \right]_0^L = \frac{wL}{3}$$
 $y = kx^2$

Locate the position of the centroid. Using Equation 1.1c and summing moments about the origin o gives

$$R\bar{x} = \int_0^L y dx(x) = \int_0^L \frac{w}{L^2} x^3 dx = \left[\frac{wx^4}{4L^2} \right]_0^L = \frac{wL^2}{4}$$

$w = kx^2$
 $k = \frac{w}{L^2}$

Substituting $R = wL/3$ and solving the equation above for $\bar{x}$ yield

$$\bar{x} = \frac{3}{4}L$$

[Example 1.4]

The beam in Figure 1.7a supports a (distributed load) whose ordinate lie on a parabolic curve. Replace the distributed load by a statically equivalent set of concentrated loads.

Solution

Divide the load into three segments where $h = 5$ ft. Evaluate the equivalent loads, using the equations in Figure 1.5b.

$$P_1 = \frac{h}{24}(7w_1 + 6w_2 - w_3) = \frac{5}{24}[7(4) + 6(6.25) - 9] = 11.77 \text{ kips}$$

$$P_2 = \frac{h}{12}(w_1 + 10w_2 + w_3) = \frac{5}{12}[4 + 10(6.25) + 9] = 31.46 \text{ kips}$$

$$P_3 = \frac{h}{12}(w_2 + 10w_3 + w_4) = \frac{5}{12}[6.25 + 10(9) + 12.25] = 45.2 \text{ kips}$$

$$P_4 = \frac{h}{24}(7w_1 + 6w_2 - w_3) = \frac{5}{24}[7(12.25) + 6(9) - 6.25] = 27.81 \text{ kips}$$

Also compute the approximate values of loads P_1 and P_2 , using the equations in Figure 1.5a for a trapezoidal distribution of load.

$$P_1 = \frac{h}{6}(2w_1 + w_2) = \frac{5}{6}[2(4) + 6.25] = 11.88 \text{ kips}$$

$$P_2 = \frac{h}{6}(w_1 + 4w_2 + w_3) = \frac{5}{6}[4 + 4(6.25) + 9] = 31.67 \text{ kips}$$

The analysis above indicates that for this case the approximate values of P_1 and P_2 deviate less than 1 percent from the exact values.

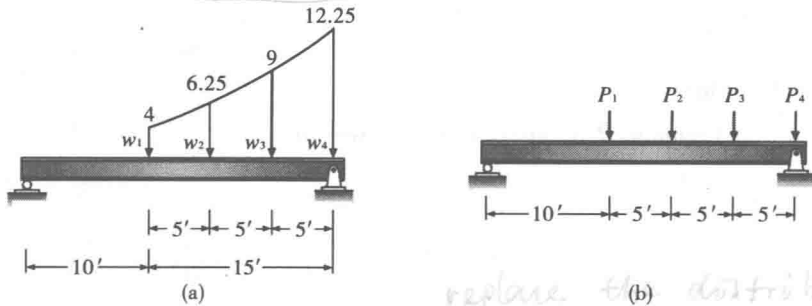


Figure 1.7: (a) Beam with a distributed load (units of load in kips per foot); (b) beam with equivalent concentrated loads.

Principle of Transmissibility 传递率

The principle of transmissibility states that a force may be moved along its line of action without changing the external effect that it produces on a body. For example, in Figure 1.8a we can see from a consideration of equilibrium in the x direction that the horizontal force P applied to the beam at point A creates a horizontal reaction at support C equal to P . If the force at point A is moved along its line of action to point D at the right end of the beam (see Fig. 1.8b), the same horizontal reaction P develops at C . Although the effect of moving the force along its line of action produces no change in the reactions, we can see that the internal force in the member is affected by the position of the load. For example, in Figure 1.8a compression stresses develop between points A and C . On the other hand, if the load acts at D , the stress between points A and C is zero and tensile stresses are created between C and D (see Fig. 1.8b).

The ability of the engineer to move vectors along their line of action is used frequently in structural analysis to simplify computations, to solve problems involving vectors graphically, and to develop a better understanding of behavior. For example,