

GEOFFREY A. JEHLE
PHILIP J. RENY

ADVANCED MICROECONOMIC THEORY

THIRD EDITION

FT Prentice Hall
FINANCIAL TIMES

Advanced Microeconomic Theory

THIRD EDITION

GEOFFREY A. JEHLE

Vassar College

PHILIP J. RAY

University of Chicago



Financial Times
Prentice Hall
is an imprint of

PEARSON

Harlow, England • London • New York • Boston • San Francisco • Toronto • Sydney • Singapore • Hong Kong
Tokyo • Seoul • Taipei • New Delhi • Cape Town • Madrid • Mexico City • Amsterdam • Munich • Paris • Milan

Pearson Education Limited

Edinburgh Gate
Harlow
Essex CM20 2JE
England

and Associated Companies throughout the world

Visit us on the World Wide Web at:
www.pearsoned.co.uk

First published 2011

© Geoffrey A. Jehle and Philip J. Reny 2011

The rights of Geoffrey A. Jehle and Philip J. Reny to be identified as author of this work have been asserted by them in accordance with the Copyright, Designs and Patents Act 1988.

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without either the prior written permission of the publisher or a licence permitting restricted copying in the United Kingdom issued by the Copyright Licensing Agency Ltd, Saffron House, 6–10 Kirby Street, London EC1N 8TS.

ISBN: 978-0-273-73191-7

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library

Library of Congress Cataloging-in-Publication Data

A catalog record for this book is available from the Library of Congress

10 9 8 7 6 5
14 13 12

Typeset in 10/12 pt and Times-Roman by 75
Printed and bound in Great Britain by Ashford Colour Press Ltd, Gosport, Hampshire

PEARSON

We work with leading authors to develop the strongest educational materials in economics, bringing cutting-edge thinking and best learning practice to a global market.

Under a range of well-known imprints, including Financial Times Prentice Hall, we craft high quality print and electronic publications that help readers to understand and apply their content, whether studying or at work.

To find out more about the complete range of our publishing, please visit us on the World Wide Web at:
www.pearsoned.co.uk

To Rana and Kamran

G.A.J.

To Dianne, Lisa, and Elizabeth

P.J.R.

PREFACE

In preparing this third edition of our text, we wanted to provide long-time readers with new and updated material in a familiar format, while offering first-time readers an accessible, self-contained treatment of the essential core of modern microeconomic theory.

To those ends, every chapter has been revised and updated. The more significant changes include a new introduction to general equilibrium with contingent commodities in Chapter 5, along with a simplified proof of Arrow's theorem and a new, careful development of the Gibbard-Satterthwaite theorem in Chapter 6. Chapter 7 includes many refinements and extensions, especially in our presentation on Bayesian games. The biggest change – one we hope readers find interesting and useful – is an extensive, integrated presentation in Chapter 9 of many of the central results of mechanism design in the quasi-linear utility, private-values environment.

We continue to believe that working through exercises is the surest way to master the material in this text. New exercises have been added to virtually every chapter, and others have been updated and revised. Many of the new exercises guide readers in developing for themselves extensions, refinements or alternative approaches to important material covered in the text. Hints and answers for selected exercises are provided at the end of the book, along with lists of theorems and definitions appearing in the text. We will continue to maintain a readers' forum on the web, where readers can exchange solutions to exercises in the text. It can be reached at <http://alfred.objects.net>.

The two full chapters of the Mathematical Appendix still provide students with a lengthy and largely self-contained development of the set theory, real analysis, topology, calculus, and modern optimisation theory

which are indispensable in modern microeconomics. Readers of this edition will now find a fuller, self-contained development of Lagrangian and Kuhn-Tucker methods, along with new material on the Theorem of the Maximum and two separation theorems. The exposition is formal but presumes nothing more than a good grounding in single-variable calculus and simple linear algebra as a starting point. We suggest that even students who are very well-prepared in mathematics browse both chapters of the appendix early on. That way, if and when some review or reference is needed, the reader will have a sense of how that material is organised.

Before we begin to develop the theory itself, we ought to say a word to new readers about the role mathematics will play in this text. Often, you will notice we make certain assumptions purely for the sake of mathematical expediency. The justification for proceeding this way is simple, and it is the same in every other branch of science. These abstractions from 'reality' allow us to bring to bear powerful mathematical methods that, by the rigour of the logical discipline they impose, help extend our insights into areas beyond the reach of our intuition and experience. In the physical world, there is 'no such thing' as a frictionless plane or a perfect vacuum. In economics, as in physics, allowing ourselves to accept assumptions like these frees us to focus on more important aspects of the problem and thereby helps to establish benchmarks in theory against which to gauge experience and observation in the real world. This does not mean that you must wholeheartedly embrace every 'unrealistic' or purely formal aspect of the theory. Far from it. It is *always* worthwhile to cast a critical eye on these matters as they arise and to ask yourself what is gained, and what is sacrificed, by the abstraction at hand. Thought and insight on these points are the stuff of which advances in theory and knowledge are made. From here on, however, we will take the theory as it is and seek to understand it on its own terms, leaving much of its critical appraisal to your moments away from this book.

Finally, we wish to acknowledge the many readers and colleagues who have provided helpful comments and pointed out errors in previous editions. Your keen eyes and good judgements have helped us make this third edition better and more complete than it otherwise would be. While we cannot thank all of you personally, we must thank Eddie Dekel, Roger Myerson, Derek Neal, Motty Perry, Arthur Robson, Steve Williams, and Jörgen Weibull for their thoughtful comments.

CONTENTS

PREFACE	xv
---------	----

PART I	
ECONOMIC AGENTS	1

CHAPTER 1	CONSUMER THEORY	3
------------------	------------------------	----------

1.1	Primitive Notions	3
-----	--------------------------	----------

1.2	Preferences and Utility	4
-----	--------------------------------	----------

1.2.1	Preference Relations	5
-------	----------------------	---

1.2.2	The Utility Function	13
-------	----------------------	----

1.3	The Consumer's Problem	19
-----	-------------------------------	-----------

1.4	Indirect Utility and Expenditure	28
-----	---	-----------

1.4.1	The Indirect Utility Function	28
-------	-------------------------------	----

1.4.2	The Expenditure Function	33
-------	--------------------------	----

1.4.3	Relations Between the Two	41
-------	---------------------------	----

1.5	Properties of Consumer Demand	48
-----	--------------------------------------	-----------

1.5.1	Relative Prices and Real Income	48
-------	---------------------------------	----

1.5.2	Income and Substitution Effects	50
-------	---------------------------------	----

1.5.3	Some Elasticity Relations	59
-------	---------------------------	----

1.6	Exercises	63
-----	------------------	-----------

CHAPTER 2	TOPICS IN CONSUMER THEORY	73
	2.1 Duality: A Closer Look	73
	2.1.1 Expenditure and Consumer Preferences	73
	2.1.2 Convexity and Monotonicity	78
	2.1.3 Indirect Utility and Consumer Preferences	81
	2.2 Integrability	85
	2.3 Revealed Preference	91
	2.4 Uncertainty	97
	2.4.1 Preferences	98
	2.4.2 Von Neumann-Morgenstern Utility	102
	2.4.3 Risk Aversion	110
	2.5 Exercises	118
CHAPTER 3	THEORY OF THE FIRM	125
	3.1 Primitive Notions	125
	3.2 Production	126
	3.2.1 Returns to Scale and Varying Proportions	132
	3.3 Cost	135
	3.4 Duality in Production	143
	3.5 The Competitive Firm	145
	3.5.1 Profit Maximisation	145
	3.5.2 The Profit Function	147
	3.6 Exercises	154
PART II	MARKETS AND WELFARE	163
CHAPTER 4	PARTIAL EQUILIBRIUM	165
	4.1 Perfect Competition	165
	4.2 Imperfect Competition	170
	4.2.1 Cournot Oligopoly	174

4.2.2	Bertrand Oligopoly	175
4.2.3	Monopolistic Competition	177
4.3	Equilibrium and Welfare	179
4.3.1	Price and Individual Welfare	179
4.3.2	Efficiency of the Competitive Outcome	183
4.3.3	Efficiency and Total Surplus Maximisation	186
4.4	Exercises	188
CHAPTER 5 GENERAL EQUILIBRIUM		195
5.1	Equilibrium in Exchange	196
5.2	Equilibrium in Competitive Market Systems	201
5.2.1	Existence of Equilibrium	203
5.2.2	Efficiency	212
5.3	Equilibrium in Production	220
5.3.1	Producers	220
5.3.2	Consumers	223
5.3.3	Equilibrium	225
5.3.4	Welfare	232
5.4	Contingent Plans	236
5.4.1	Time	236
5.4.2	Uncertainty	236
5.4.3	Walrasian Equilibrium with Contingent Commodities	237
5.5	Core and Equilibria	239
5.5.1	Replica Economies	240
5.6	Exercises	251
CHAPTER 6 SOCIAL CHOICE AND WELFARE		267
6.1	The Nature of the Problem	267
6.2	Social Choice and Arrow's Theorem	269
6.2.1	A Diagrammatic Proof	274

6.3	Measurability, Comparability, and Some Possibilities	279
6.3.1	The Rawlsian Form	282
6.3.2	The Utilitarian Form	284
6.3.3	Flexible Forms	285
6.4	Justice	288
6.5	Social Choice and the Gibbard-Satterthwaite Theorem	290
6.6	Exercises	296
<hr/>		
	PART III STRATEGIC BEHAVIOUR	303
	CHAPTER 7 GAME THEORY	305
<hr/>		
7.1	Strategic Decision Making	305
7.2	Strategic Form Games	307
7.2.1	Dominant Strategies	308
7.2.2	Nash Equilibrium	311
7.2.3	Incomplete Information	319
7.3	Extensive Form Games	325
7.3.1	Game Trees: A Diagrammatic Representation	328
7.3.2	An Informal Analysis of Take-Away	330
7.3.3	Extensive Form Game Strategies	331
7.3.4	Strategies and Payoffs	332
7.3.5	Games of Perfect Information and Backward Induction Strategies	333
7.3.6	Games of Imperfect Information and Subgame Perfect Equilibrium	337
7.3.7	Sequential Equilibrium	347
7.4	Exercises	364
<hr/>		
	CHAPTER 8 INFORMATION ECONOMICS	379
<hr/>		
8.1	Adverse Selection	380
8.1.1	Information and the Efficiency of Market Outcomes	380

8.1.2	Signalling	385
8.1.3	Screening	404
8.2	Moral Hazard and the Principal-Agent Problem	413
8.2.1	Symmetric Information	414
8.2.2	Asymmetric Information	416
8.3	Information and Market Performance	420
8.4	Exercises	421
<hr/>		
CHAPTER 9	AUCTIONS AND MECHANISM DESIGN	427
<hr/>		
9.1	The Four Standard Auctions	427
9.2	The Independent Private Values Model	428
9.2.1	Bidding Behaviour in a First-Price, Sealed-Bid Auction	429
9.2.2	Bidding Behaviour in a Dutch Auction	432
9.2.3	Bidding Behaviour in a Second-Price, Sealed-Bid Auction	433
9.2.4	Bidding Behaviour in an English Auction	434
9.2.5	Revenue Comparisons	435
9.3	The Revenue Equivalence Theorem	437
9.3.1	Incentive-Compatible Direct Selling Mechanisms: A Characterisation	441
9.3.2	Efficiency	444
9.4	Designing a Revenue Maximising Mechanism	444
9.4.1	The Revelation Principle	444
9.4.2	Individual Rationality	445
9.4.3	An Optimal Selling Mechanism	446
9.4.4	A Closer Look at the Optimal Selling Mechanism	451
9.4.5	Efficiency, Symmetry, and Comparison to the Four Standard Auctions	453
9.5	Designing Allocatively Efficient Mechanisms	455
9.5.1	Quasi-Linear Utility and Private Values	456
9.5.2	Ex Post Pareto Efficiency	458

9.5.3	Direct Mechanisms, Incentive Compatibility and the Revelation Principle	458
9.5.4	The Vickrey-Clarke-Groves Mechanism	461
9.5.5	Achieving a Balanced Budget: Expected Externality Mechanisms	466
9.5.6	Property Rights, Outside Options, and Individual Rationality Constraints	469
9.5.7	The IR-VCG Mechanism: Sufficiency of Expected Surplus	472
9.5.8	The Necessity of IR-VCG Expected Surplus	478
9.6	Exercises	484

MATHEMATICAL APPENDICES **493**

CHAPTER A1 **SETS AND MAPPINGS** **495**

A1.1 **Elements of Logic** **495**

A1.1.1	Necessity and Sufficiency	495
A1.1.2	Theorems and Proofs	496

A1.2 **Elements of Set Theory** **497**

A1.2.1	Notation and Basic Concepts	497
A1.2.2	Convex Sets	499
A1.2.3	Relations and Functions	503

A1.3 **A Little Topology** **505**

A1.3.1	Continuity	515
A1.3.2	Some Existence Theorems	521

A1.4 **Real-Valued Functions** **529**

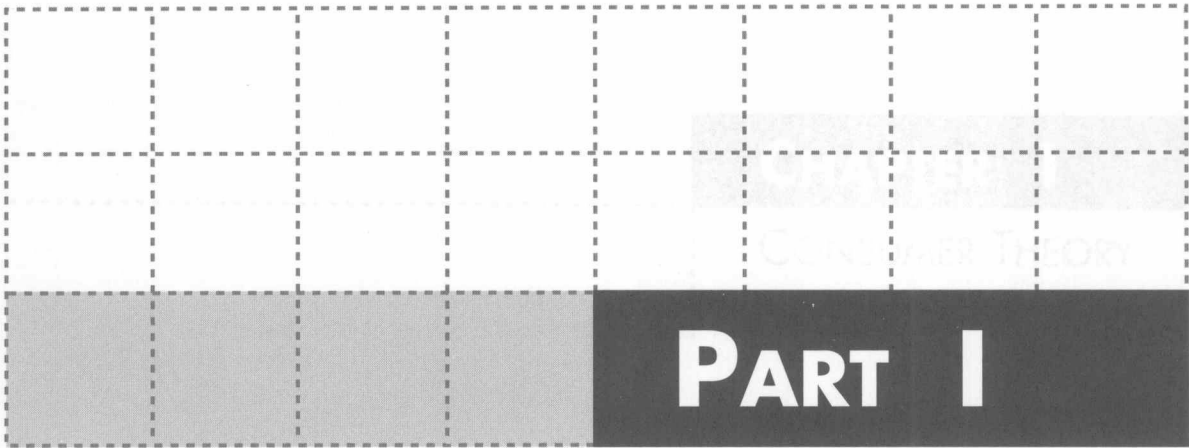
A1.4.1	Related Sets	530
A1.4.2	Concave Functions	533
A1.4.3	Quasiconcave Functions	538
A1.4.4	Convex and Quasiconvex Functions	542

A1.5 **Exercises** **546**

CHAPTER A2 **CALCULUS AND OPTIMISATION** **551**

A2.1 **Calculus** **551**

A2.1.1	Functions of a Single Variable	551
A2.1.2	Functions of Several Variables	553
A2.1.3	Homogeneous Functions	561
A2.2	Optimisation	566
A2.2.1	Real-Valued Functions of Several Variables	567
A2.2.2	Second-Order Conditions	570
A2.3	Constrained Optimisation	577
A2.3.1	Equality Constraints	577
A2.3.2	Lagrange's Method	579
A2.3.3	Geometric Interpretation	584
A2.3.4	Second-Order Conditions	588
A2.3.5	Inequality Constraints	591
A2.3.6	Kuhn-Tucker Conditions	595
A2.4	Optimality Theorems	601
A2.5	Separation Theorems	607
A2.6	Exercises	611
	LIST OF THEOREMS	619
	LIST OF DEFINITIONS	625
	HINTS AND ANSWERS	631
	REFERENCES	641
	INDEX	645



In the first two chapters of this book, we will explore the basic principles of consumer theory. In chapter 1, we will introduce the basic concepts of consumer theory, and in chapter 2, we will explore the theory of consumer choice. You will learn the basics of consumer theory, and you will see how it is applied in the real world. You will also see how it is used to explain consumer behavior.

ECONOMIC AGENTS

1.1 POSITIVE NOTIONS

There are four basic positive notions in consumer theory: utility, income, price, and choice. Utility is the satisfaction that a consumer derives from consuming a good or service. Income is the amount of money that a consumer has available to spend on goods and services. Price is the amount of money that a consumer must pay for a good or service. Choice is the decision that a consumer makes about which goods and services to consume.

The notion of a consumer's utility function is central to consumer theory. A utility function represents the set of all combinations of goods and services that a consumer can consume, given their income and the prices of the goods and services. The utility function is a curve that shows the relationship between the quantity of a good or service consumed and the utility derived from it.

Let us now consider the notion of a consumer's budget constraint. A budget constraint is a line that shows the combinations of goods and services that a consumer can afford to purchase, given their income and the prices of the goods and services. The budget constraint is a straight line that is downward sloping, and it is tangent to the utility function at the point of consumer equilibrium.

CHAPTER 1

CONSUMER THEORY

In the first two chapters of this volume, we will explore the essential features of modern consumer theory – a bedrock foundation on which so many theoretical structures in economics are built. Some time later in your study of economics, you will begin to notice just how central this theory is to the economist's way of thinking. Time and time again you will hear the echoes of consumer theory in virtually every branch of the discipline – how it is conceived, how it is constructed, and how it is applied.

1.1 PRIMITIVE NOTIONS

There are four building blocks in any model of consumer choice. They are the consumption set, the feasible set, the preference relation, and the behavioural assumption. Each is conceptually distinct from the others, though it is quite common sometimes to lose sight of that fact. This basic structure is extremely general, and so, very flexible. By specifying the form each of these takes in a given problem, many different situations involving choice can be formally described and analysed. Although we will tend to concentrate here on specific formalisations that have come to dominate economists' view of an individual consumer's behaviour, it is well to keep in mind that 'consumer theory' *per se* is in fact a very rich and flexible *theory of choice*.

The notion of a **consumption set** is straightforward. We let the consumption set, X , represent the set of all alternatives, or complete consumption plans, that the consumer can conceive – whether some of them will be achievable in practice or not. What we intend to capture here is the universe of alternative choices over which the consumer's mind is capable of wandering, unfettered by consideration of the realities of his present situation. The consumption set is sometimes also called the **choice set**.

Let each commodity be measured in some infinitely divisible units. Let $x_i \in \mathbb{R}$ represent the number of units of good i . We assume that only non-negative units of each good are meaningful and that it is always possible to conceive of having *no* units of any particular commodity. Further, we assume there is a finite, fixed, but arbitrary number n of different goods. We let $\mathbf{x} = (x_1, \dots, x_n)$ be a vector containing different quantities of each of the n commodities and call \mathbf{x} a **consumption bundle** or a **consumption plan**. A consumption