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Mathematics for Engineering Technicians

K. A. Stroud

Book 2A
Practical Applications

S.T.(P) Technology Today Series

Mathematics for Engineering Technicians

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BOOK 2A
Practical Applications

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BOOK 2A
Practical Applications

MATHEMATICS FOR ENGINEERING TECHNICIANS

The series is planned to include

Book 1	*(Level I)*	*full unit*	*Introductory course*
Book 2	*(Level II)*	*half unit*	
Book 2A	*(Level II)*	*half unit*	*Practical applications*
Book 2B	*(Level II)*	*half unit*	*Analytical approach*
Book 3	*(Level III)*	*full unit*	

Also (in preparation) MATHEMATICS FOR ELECTRICAL ENGINEERING TECHNICIANS (Level II and Level III)

PREFACE

The present volume continues the series of texts designed to provide a sound coverage of the Mathematics requirements of the T.E.C. Certificate courses as detailed in the programme issued by the Technical Education Council.

The structure of this book follows closely the pattern established in the author's previous books in the same 'Mathematics for Engineering Technicians' series. It is designed as a second half-unit course at Level II stage and is intended to follow the first half-unit course at Level II presented in Book 2. It deals largely with practical applications at the appropriate standard and is especially intended for students who may not be pursuing Mathematics as a formal topic at Level III.

The complete Level II course for such students consists therefore of

Book 2 + Book 2A (Practical applications).

Those students who anticipate proceeding eventually to take Mathematics at the Level III stage are advised to follow Book 2 with Book 2B which presents a second half-unit course with a more analytical approach to the development of the work and which, therefore, provides a sounder foundation in preparation for the more advanced Level III course.

The present series of texts has been developed to provide considerable flexibility to meet the range of ability, previous experience and future requirements of students embarked on the technicians' courses and results in a system which has proved highly successful, both in the classroom and for individual learning.

Each chapter is built on a common structure, details of which are outlined below. The underlying element throughout is that, at this level, mathematics is essentially a practical subject and that progress can be maintained only when the student is continually and actively involved. Numerous exercises are, therefore, provided at each stage of development and a complete set of answers is appended.

CHAPTER STRUCTURE

Each chapter opens with an optional *Pre-Test* designed to assess the student's previous knowledge of the topics covered in the chapter. This is particularly useful when individualised learning or private study is being pursued, for, if the student can demonstrate mastery of the subject by a high score in the pre-test, he may well be advised not to work through that particular chapter, but to move on to the next topic. By this means, each student can make most efficient use of his time and effort, and sustain interest in his progress.

The material is developed as a logical sequence of topic sections with sufficient mathematical proof to substantiate the methods used. Each section of work is accompanied by *Worked Examples* and numerous short *Exercises* based on specific learning points are included for continued student involvement. *Revision Exercises* are provided at suitable intervals, and where, appropriate, more extensive worked examples and further exercises founded on typical examination questions are included.

A *Revision Summary* of the material covered is provided towards the end of each chapter and prepares the student for the *Final Test A* based specifically on the work of the whole chapter. Following the Final Test A, a short remedial *Guided Revision* section in programmed form is included, primarily to assist those students who have been less successful in the Test exercise. In this section, the slower student is guided through a number of revision examples in detail, with decreasing support as mastery is attained. After the completion of the Guided Revision, a further *Final Test B* is provided for final assessment of such students.

With each chapter, the Pre-Test also demonstrates the objectives of the ensuing area of work and, furthermore, shows clearly the standard of achievement to be expected. The complete set of Pre-Tests and Final Tests spread throughout the book, provides valuable exercises for revision at the completion of the course. The level of difficulty of each of these tests reflects the standard to which the subject matter has been developed within the appropriate chapter and is specifically designed to provide evidence of a sound, but basic, understanding of the subject matter covered.

I wish to record my sincere appreciation of all those who willingly discussed the original preparation of the manuscript and also of the publishers for their continued co-operation, valuable suggestions and advice.

K.A. Stroud 1981

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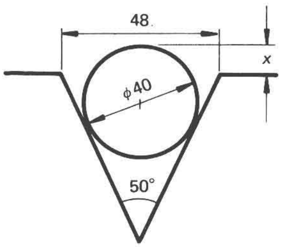
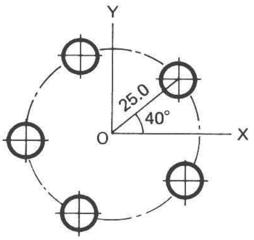
Chapter 1 Trigonometry — Applications I

PRE-TEST 1

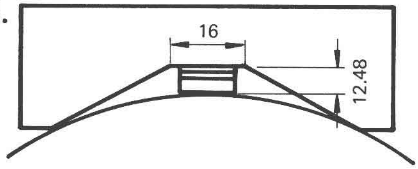
1. 

A test plug, 40 mm in diameter, rests in a 50° vee-groove. Calculate x, the distance the top of the plug protrudes above the top of the groove.

2. 

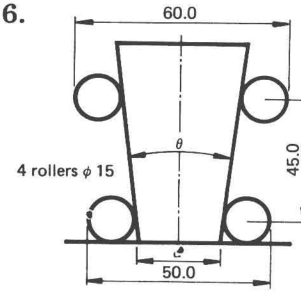
Five holes are equally spaced on a pitch circle of 50.0 mm diameter. Compile a table showing the five co-ordinate hole centres.

3. A 200 mm sine bar is set up to check a taper angle of 8°42′. The lower roller is set 12.25 mm above the horizontal bed. Calculate the height setting of the upper roller.

4. 

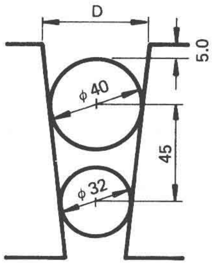
The 120° vee-gauge shown has a flat 16.00 mm wide. Slip gauges totalling 12.48 mm are required between the flat and the circumference of the job. Calculate the diameter of the job.

5. When inserted into a bore, a 150 mm point gauge rocks through a total distance of 24.00 mm. Calculate the diameter of the bore.

6. 

Dimensions over two sets of identical rollers at different heights are shown. Calculate

(a) the angle of taper θ,
(b) the diameter d at the smaller end of the taper.

7. 

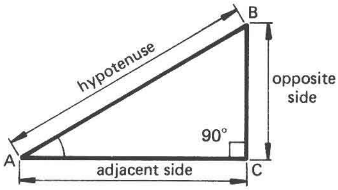
For the taper hole shown, calculate
(a) the angle of taper,
(b) the top diameter D.

TRIGONOMETRY — APPLICATIONS I

1.1 REVISION OF PREVIOUS WORK

1.1.1 Trigonometrical ratios

The six trigonometrical ratios of an angle are defined in terms of the ratios of the sides of a right-angled triangle containing that angle.



Main ratios

$$\sin A = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{BC}{AB}$$

$$\cos A = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{AC}{AB}$$

$$\tan A = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{BC}{AC}$$

Reciprocal ratios

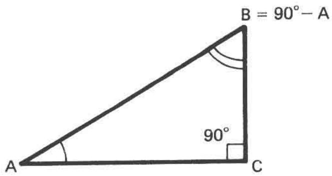
$$\cot A = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{AC}{BC} \qquad \therefore \cot A = \frac{1}{\tan A}$$

$$\sec A = \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{AB}{AC} \qquad \therefore \sec A = \frac{1}{\cos A}$$

$$\operatorname{cosec} A = \frac{\text{hypotenuse}}{\text{opposite side}} = \frac{AB}{BC} \qquad \therefore \operatorname{cosec} A = \frac{1}{\sin A}$$

1.1.2 Complementary angles

Two angles are complementary when the sum of the angles is 90°.



In $\triangle ABC$, angles A and B are complementary, since the three angles of any triangle add up to 180° and, in this case, 90° is taken up by the right-angle ($\angle C$), leaving the remaining 90° between $\angle A$ and $\angle B$.

$$\therefore \angle B = 90° - \angle A.$$

$$\sin A = \frac{BC}{AB} = \cos B \qquad \therefore \sin A = \cos(90° - A)$$

$$\cos A = \frac{AC}{AB} = \sin B \qquad \therefore \cos A = \sin(90° - A)$$

$$\tan A = \frac{BC}{AC} = \cot B \qquad \therefore \tan A = \cot(90° - A)$$

$$\cot A = \frac{AC}{BC} = \tan B \qquad \therefore \cot A = \tan(90° - A)$$

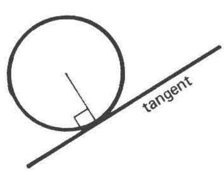
$$\sec A = \frac{AB}{AC} = \mathrm{cosec}\, B \qquad \therefore \sec A = \mathrm{cosec}(90° - A)$$

$$\mathrm{cosec}\, A = \frac{AB}{BC} = \sec B \qquad \therefore \mathrm{cosec}\, A = \sec(90° - A).$$

1.1.3 Applications

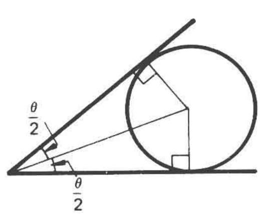
Applications of trigonometry to practical problems often depend on recognising right-angled triangles and applying the trigonometrical ratios, the values of which can be obtained from tables or calculators. Worked examples are shown below, but two useful points are worth remembering:

(a)



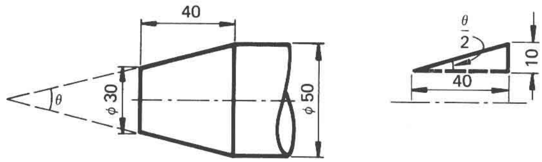
When a circle touches a line, i.e. a tangent, the radius at the point of contact is at right-angles to the tangent.

(b)



When a circle rests in a vee-groove, the line joining the apex of the vee and the centre of the circle, bisects the angle of the groove.

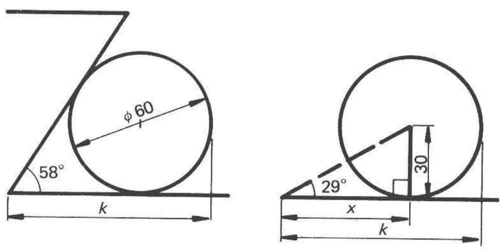
EXAMPLE 1 A bar of diameter 50 mm is tapered as shown. Calculate θ, the angle of taper.



$$\tan\frac{\theta}{2} = \frac{10}{40} = 0.25 \qquad \therefore \frac{\theta}{2} = 14°2'$$

$$\therefore \underline{\theta = 28°4'}$$

EXAMPLE 2 A 60 mm plug is placed in an angular groove. Calculate the dimension k.

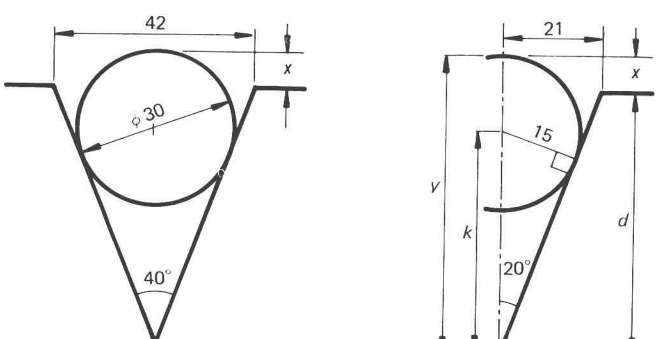


$$\frac{x}{30} = \cot 29° \qquad \therefore x = 30 \times \cot 29°$$

$$= 30 \times 1.804 = 54.12\,\text{mm}$$

$$k = x + 30 \qquad \therefore k = 54.12 + 30 = 84.12\,\text{mm} \qquad \underline{k = 84.1\,\text{mm}}$$

EXAMPLE 3 A test bar, 30 mm diameter, rests in a 40° vee-groove, 42 mm wide at the top. Calculate x, the distance the top of the bar protrudes above the top of the groove.



(a) To find k: $\frac{k}{15} = \text{cosec } 20° \quad \therefore k = 15 \times 2.924 = 43.86\,\text{mm}$

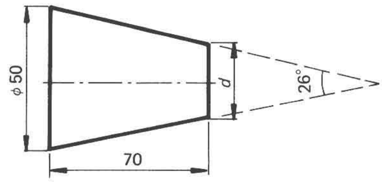
(b) To find y: $y = k + 15 = 43.86 + 15 \quad \therefore y = 58.86\,\text{mm}$

(c) To find d: $\frac{d}{21} = \cot 20° \quad \therefore d = 21 \times 2.747 = 57.70\,\text{mm}$

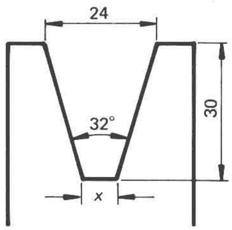
(d) To find x: $x = y - d = 58.86 - 57.70 = 1.16\,\text{mm}$

$$\underline{x = 1.16\,\text{mm}}$$

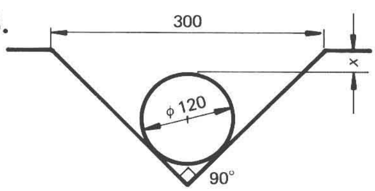
Exercise 1

1. 

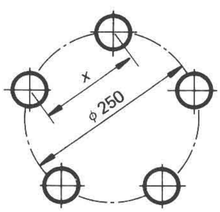
The angle of taper on the plug shown is 26°. Calculate the smaller diameter d.

2. 

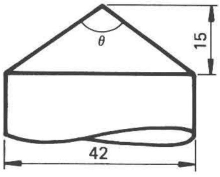
The groove in a pulley for a vee-belt drive has the dimensions shown. Calculate the dimension x.

3. 

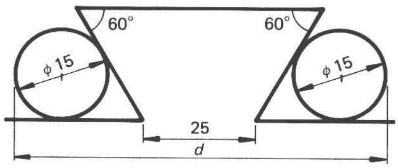
A plug, 120 mm diameter, is placed in a 90° vee-block, 300 mm wide at the top. Calculate x, the distance between the top of the plug and the top of the block.

4. 

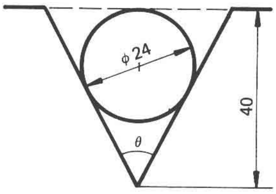
Five holes are equally spaced on a pitch circle of diameter 250 mm. Calculate x, the distance between centres of two adjacent holes measured along a chord.

5. 

The end of a bar, 42 mm diameter, is coned to the dimensions shown. Calculate the angle θ.

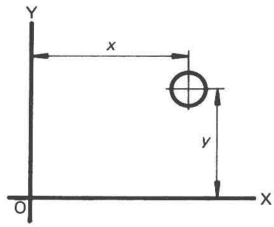
6. 

Two 15 mm test plugs are used to check the dovetail slide shown. Calculate the dimension d.

7. 

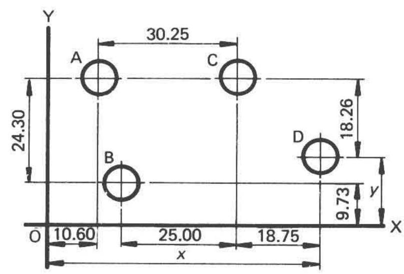
A test plug, 24 mm diameter, rests in a vee-groove of depth 40 mm. The top of the plug is level with the top of the groove. Calculate the groove angle θ.

1.2 CO-ORDINATE HOLE CENTRES



The position of the centre of a hole to be drilled can be located by knowing its distances from two lines of reference, or datum lines, at right-angles to each other. These distances are its co-ordinates, normally denoted by x and y.

EXAMPLE 1 Four holes are drilled with centres at A, B, C and D. Find their co-ordinate dimensions relative to the axes OX and OY.



For the centre D:

$$x = 10.60 + 30.25 + 18.75 = 59.60\,\text{mm}$$

$$y = 9.73 + 24.30 - 18.26 = 15.77\,\text{mm}$$

The co-ordinates of each hole centre can be found in the same way.

A: $x = 10.60\,\text{mm}$; $y = 9.73 + 24.30 = 34.03\,\text{mm}$

B: $x = 10.60 + 30.25 - 25.00 = 15.85\,\text{mm}$; $y = 9.73\,\text{mm}$

C: $x = 15.85\,(x \text{ for B}) + 25.00 = 40.85\,\text{mm}$;

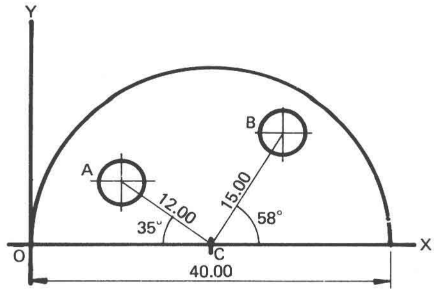
$y = 9.73 + 24.30 = 34.03\,\text{mm}$

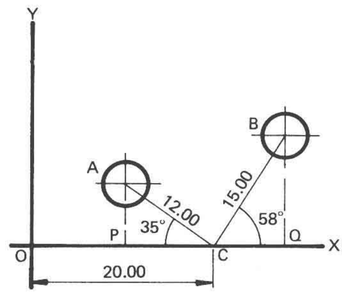
D: $x = 59.60\,\text{mm}$; $y = 15.77\,\text{mm}$ as found before.

For clarity, we can put these results into tabular form, as follows:

Hole	Centre co-ordinates	
	x	y
A	10.60	34.03
B	15.85	9.73
C	40.85	34.03
D	59.60	15.77

EXAMPLE 2 Two holes are drilled in a semi-circular plate at centres A and B as shown. Find the hole centre co-ordinates relative to OX and OY.





Hole A: $\dfrac{AP}{12.00} = \sin 35°$

$\therefore \; AP = 12.00 \times \sin 35° = 6.883$

$\dfrac{PC}{12.00} = \cos 35°$

$\therefore \; PC = 12.00 \times \cos 35° = 9.830$

$x = 20 - PC = 20.00 - 9.83 = 10.17$ $\qquad \underline{x = 10.17}$

$y = AP = 6.88$ $\qquad \underline{y = 6.88}$

Hole B: $\dfrac{BQ}{15.00} = \sin 58° \quad \therefore \; BQ = 15.00 \times \sin 58° = 12.72$

$\dfrac{CQ}{15.00} = \cos 58° \quad \therefore \; CQ = 15.00 \times \cos 58° = 7.95$

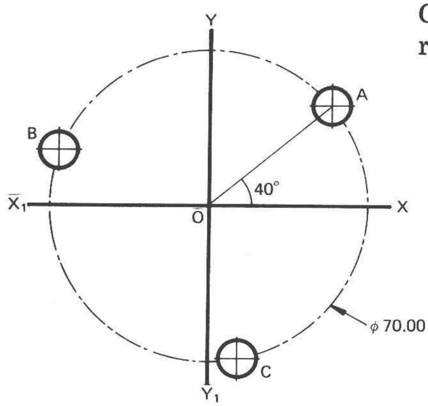
$x = 20 + CQ = 20 + 7.95 = 27.95$ $\qquad \underline{x = 27.95}$

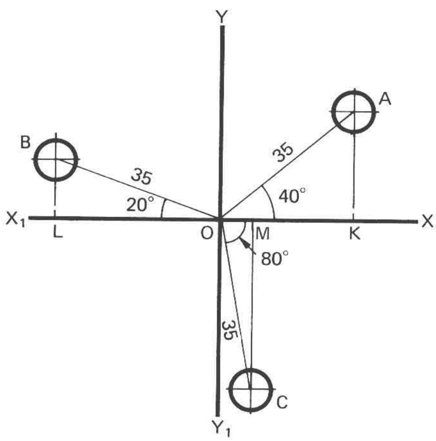
$y = BQ = 12.72$ $\qquad \underline{y = 12.72}$

So we have:

Hole	Centre co-ordinates	
	x	y
A	10.17	6.88
B	27.95	12.72

EXAMPLE 3 A, B and C are three equally spaced hole centres on a pitch circle of diameter 70.00 mm. Calculate the co-ordinate hole centres relative to the x and y axes indicated.





Holes A, B and C are separated by

$$\frac{360°}{3} = 120°.$$

$$\therefore \angle BOX_1 = 20°$$

$$\angle COX = 80°$$

For A:	OK	$= 35.00 \times \cos 40° = 26.81$	$x =$	26.81
	AK	$= 35.00 \times \sin 40° = 22.50$	$y =$	22.50
For B:	OL	$= 35.00 \times \cos 20° = 32.89$	$x =$	−32.89
	BL	$= 35.00 \times \sin 20° = 11.97$	$y =$	11.97
For C:	OM	$= 35.00 \times \cos 80° = 6.08$	$x =$	6.08
	CM	$= 35.00 \times \sin 80° = 34.47$	$y =$	−34.47

So we have:

Hole	**Centre co-ordinates**	
	x	y
A	26.81	22.50
B	−32.89	11.97
C	6.08	−34.47