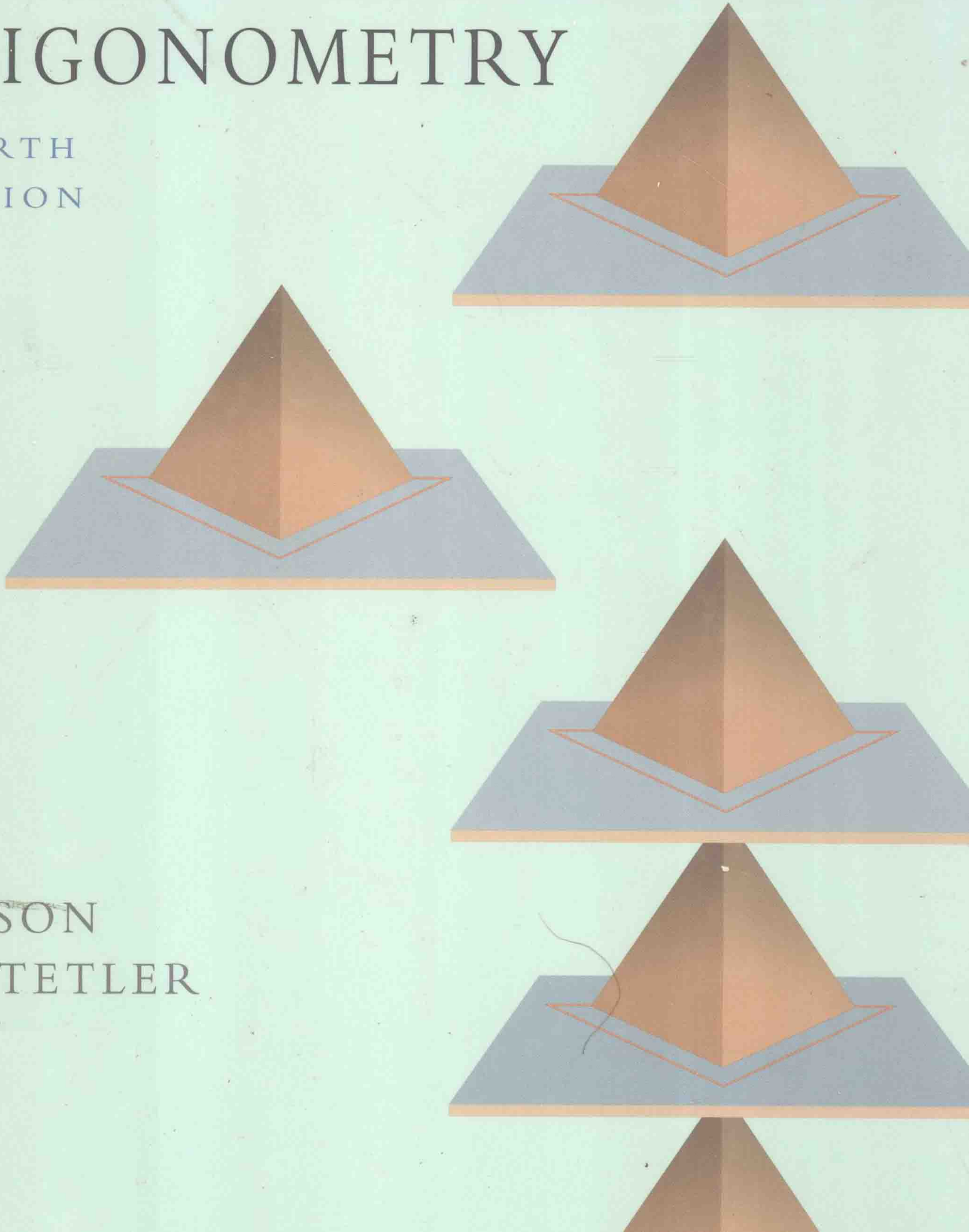
TRIGONOMETRY

FOURTH EDITION



LARSON
HOSTETLER

Trigonometry

FOURTH EDITION

Roland E. Larson **Robert P. Hostetler**

The Pennsylvania State University
The Behrend College

With the assistance of

David E. Heyd

The Pennsylvania State University
The Behrend College

HOUGHTON MIFFLIN COMPANY Boston New York

Sponsoring Editor: Christine B. Hoag
Senior Associate Editor: Maureen Brooks
Managing Editor: Catherine B. Cantin
Assistant Editor: Carolyn Johnson
Supervising Editor: Karen Carter
Associate Project Editor: Rachel D'Angelo Wimberly
Editorial Assistant: Caroline Lipscomb
Production Supervisor: Lisa Merrill
Art Supervisor: Gary Crespo
Marketing Manager: Charles Cavaliere
Marketing Associate: Ros Kane
Marketing Assistant: Kate Burden Thomas

Cover design by Harold Burch Design, NYC

Composition: Meridian Creative Group

Printed in the U.S.A.

Library of Congress Catalog Card Number: 96-076658

ISBN: 0-669-41737-8

789–DC–00

Preface

A firm foundation in algebra and trigonometry is necessary for success in college-level mathematics courses. *Trigonometry,* Fourth Edition, is designed to help students develop their proficiency in trigonometry, and so strengthen their understanding of the underlying concepts. Although the basic concepts of algebra are reviewed in the text, it is assumed that most students taking this course have completed two years of high school algebra.

The text takes every opportunity to show how algebra with trigonometry is a modern modeling language for real-life problems. Examples, exercises, and group activities—many using real data—provide a real-life context to help students grasp mathematical concepts. As appropriate, graphing technology is utilized throughout the text to enhance student understanding of mathematical concepts.

New to the Fourth Edition

All text elements in the previous edition were considered for revision, and many new examples, exercises, and applications were added to the text. Following are the major changes in the Fourth Edition.

Improved Coverage Chapter P, "Prerequisites," contains material that the student should have studied in earlier courses. All or part of this review material may be covered or omitted, offering greater flexibility in designing the course syllabus.

As a result of user requests, Chapter 6, "Topics in Analytic Geometry," has been reordered so that the topic of parametric equations is discussed before polar equations.

CD-ROM To accommodate a variety of teaching and learning styles, *Trigonometry,* Fourth Edition, is also available in a multimedia, CD-ROM format. *Interactive Trigonometry* offers students a variety of additional tutorial assistance, including examples and exercises with detailed solutions; pre-, post-, and self-tests with answers; and *TI-82* and *TI-83* graphing calculator emulators. (See pages xvi–xviii for more detailed information.)

Technology The new Fourth Edition acknowledges the increasing availability of graphing technology by offering the opportunity to use graphing utilities throughout, without requiring their use. This is achieved through a combination of features, including—at point of use—many opportunities for exploration using technology (see page 84); graphing utility instructions in the text, (see page 31); and clearly labeled exercises that require the use of a graphing utility (see page 365). In addition, *Interactive Trigonometry* offers the text in a CD-ROM format, as well as additional tutorial and technology enhanced features.

Data Analysis and Modeling Throughout the Fourth Edition, students are offered many more opportunities to collect and interpret data, to make conjectures, and to construct mathematical models. Students are encouraged to use mathematical models to make predictions or draw conclusions from real data (see page 392); invited to compare models (see page 403); and asked to use curve-fitting techniques to write models from data (see page 50). This edition encourages greater use of charts, tables, scatter plots, and graphs to summarize, analyze, and interpret data.

Applications To emphasize for students the connection between mathematical concepts and real-world situations, up-to-date, real-life applications are integrated throughout the text. Appearing as chapter introductions with related exercises (see pages 213 and 229), examples (see page 164), exercises (see page 169), Group Activities (see page 50), and Chapter Projects (see pages 210–211), these applications offer students frequent opportunities to use and review their problem-solving skills. The applications cover a wide range of disciplines including areas such as physics, chemistry, the social sciences, biology, and business.

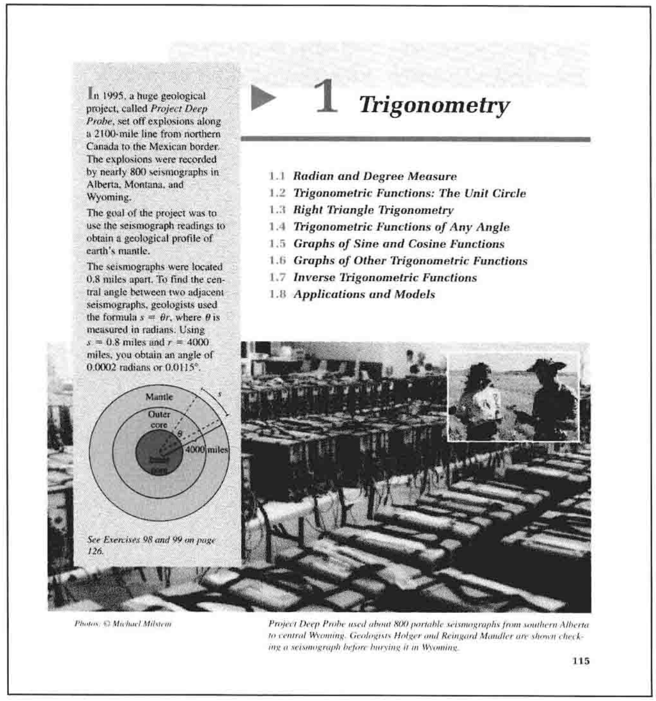
Group Activities Each section ends with a Group Activity. These exercises reinforce students' understanding by exploring mathematical concepts in a variety of ways, including interpretation of mathematical concepts and results (see page 199); problem posing and error analysis (see page 132); and constructing mathematical models, tables, and graphs (see page 392). Designed to be completed in class or as homework assignments, the Group Activities give the students the opportunity to work cooperatively as they think, talk, and write about mathematics.

Connections In addition to highlighting the connections between algebra and areas outside mathematics through real-world applications, this text emphasizes the connections between algebra and other branches of mathematics, such as geometry (see page 203) and statistics (see Appendix A). Many examples and exercises throughout the text also reinforce the connections through graphical, numerical, and analytical representations of important algebraic concepts (see page 164).

There are many other new features of the Fourth Edition as well, including Exploration, Study Tips, Historical Notes, Focus on Concepts, and Chapter Projects. These and other features of the Fourth Edition are described in greater detail on the following pages.

Features of the Fourth Edition

Chapter Opener Each chapter opens with a look at a real-life application. Real data is presented using graphical, numerical, and algebraic techniques. In addition, a list of the section titles shows students how the topics fit into the overall development of algebra and trigonometry.


In 1995, a huge geological project, called *Project Deep Probe*, set off explosions along a 2100-mile line from northern Canada to the Mexican border. The explosions were recorded by nearly 800 seismographs in Alberta, Montana, and Wyoming.

The goal of the project was to use the seismograph readings to obtain a geological profile of earth's mantle.

The seismographs were located 0.8 miles apart. To find the central angle between two adjacent seismographs, geologists used the formula $s = \theta r$, where θ is measured in radians. Using $s = 0.8$ miles and $r = 4000$ miles, you obtain an angle of 0.0002 radians or 0.0115°.

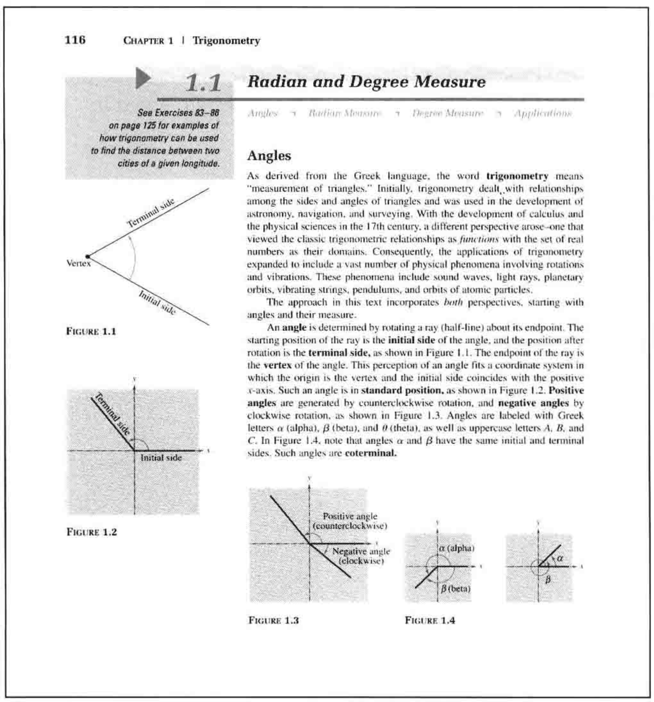
See Exercises 98 and 99 on page 126.

▶ 1 *Trigonometry*

Photos: © Michael Milstein

Project Deep Probe used about 800 portable seismographs from southern Alberta to central Wyoming. Geologists Holger and Reingard Mandler are shown checking a seismograph before burying it in Wyoming.

115


116 CHAPTER 1 | Trigonometry

▶ 1.1 **Radian and Degree Measure**

Angles → Radian Measure → Degree Measure → Applications

See Exercises 83–88 on page 125 for examples of how trigonometry can be used to find the distance between two cities of a given longitude.

Angles

As derived from the Greek language, the word **trigonometry** means "measurement of triangles." Initially, trigonometry dealt with relationships among the sides and angles of triangles and was used in the development of astronomy, navigation, and surveying. With the development of calculus and the physical sciences in the 17th century, a different perspective arose–one that viewed the classic trigonometric relationships as *functions* with the set of real numbers as their domains. Consequently, the applications of trigonometry expanded to include a vast number of physical phenomena involving rotations and vibrations. These phenomena include sound waves, light rays, planetary orbits, vibrating strings, pendulums, and orbits of atomic particles.

The approach in this text incorporates *both* perspectives, starting with angles and their measure.

An **angle** is determined by rotating a ray (half-line) about its endpoint. The starting position of the ray is the **initial side** of the angle, and the position after rotation is the **terminal side,** as shown in Figure 1.1. The endpoint of the ray is the **vertex** of the angle. This perception of an angle fits a coordinate system in which the origin is the vertex and the initial side coincides with the positive x-axis. Such an angle is in **standard position,** as shown in Figure 1.2. **Positive angles** are generated by counterclockwise rotation, and **negative angles** by clockwise rotation, as shown in Figure 1.3. Angles are labeled with Greek letters α (alpha), β (beta), and θ (theta), as well as uppercase letters A, B, and C. In Figure 1.4, note that angles α and β have the same initial and terminal sides. Such angles are **coterminal.**

FIGURE 1.1

FIGURE 1.2

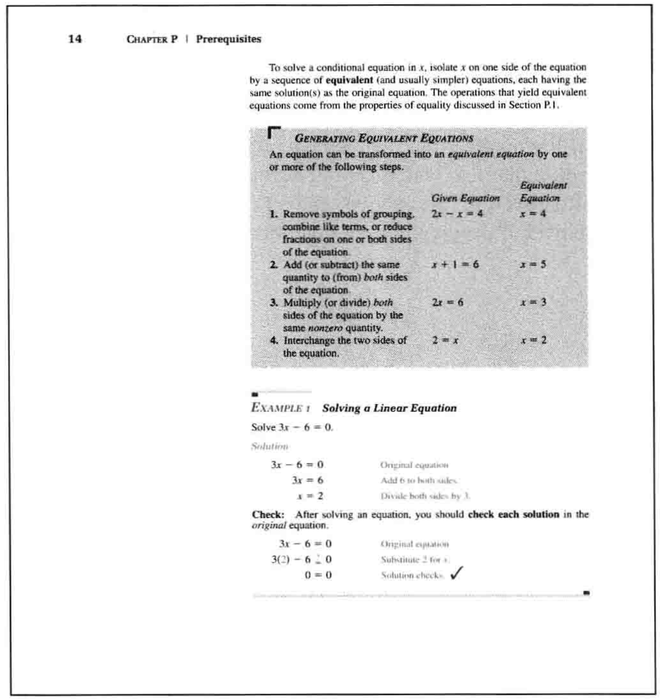
FIGURE 1.3

FIGURE 1.4

Section Outline Each section begins with a list of the major topics covered in the section. These topics are also the subsection titles and can be used for easy reference and review by students. In addition, an exercise application that uses a skill or illustrates a concept covered in the section is highlighted to emphasize the connection between mathematical concepts and real-life situations.

Graphics Visualization is a critical problem-solving skill. To encourage the development of this ability, the text has nearly 2300 figures in examples, exercises, and answers to exercises. Included are graphs of equations and functions, geometric figures, displays of statistical information, scatter plots, and numerous screen outputs from graphing technology. All graphs of equations and functions are computer- or calculator-generated for accuracy, and they are designed to resemble students' actual screen outputs as closely as possible. Graphics are also used to emphasize graphical interpretation, comparison, and estimation.

Theorems, Definitions, and Guidelines All of the important rules, formulas, theorems, guidelines, properties, definitions, and summaries are highlighted for emphasis. Each is also titled for easy reference.



14 CHAPTER P | Prerequisites

To solve a conditional equation in x, isolate x on one side of the equation by a sequence of **equivalent** (and usually simpler) equations, each having the same solution(s) as the original equation. The operations that yield equivalent equations come from the properties of equality discussed in Section P.1.

GENERATING EQUIVALENT EQUATIONS

An equation can be transformed into an *equivalent equation* by one or more of the following steps.

	Given Equation	Equivalent Equation
1. Remove symbols of grouping, combine like terms, or reduce fractions on one or both sides of the equation.	$2x - x = 4$	$x = 4$
2. Add (or subtract) the same quantity to (from) *both* sides of the equation.	$x + 1 = 6$	$x = 5$
3. Multiply (or divide) *both* sides of the equation by the same *nonzero* quantity.	$2x = 6$	$x = 3$
4. Interchange the two sides of the equation.	$2 = x$	$x = 2$

EXAMPLE 1 ***Solving a Linear Equation***

Solve $3x - 6 = 0$.

Solution

$3x - 6 = 0$ Original equation

$3x = 6$ Add 6 to both sides.

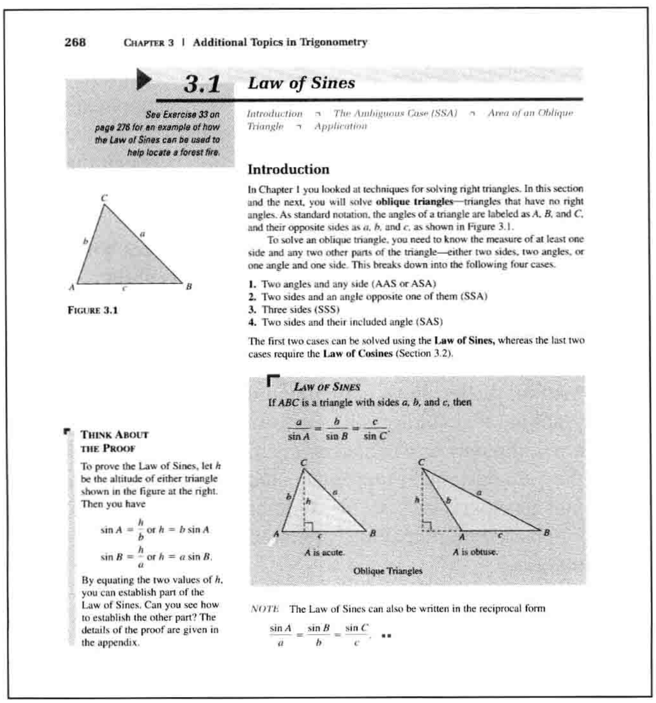
$x = 2$ Divide both sides by 3.

Check: After solving an equation, you should **check each solution** in the *original* equation.

$3x - 6 = 0$ Original equation

$3(2) - 6 \stackrel{?}{=} 0$ Substitute 2 for x.

$0 = 0$ Solution checks. ✓



268 CHAPTER 3 | Additional Topics in Trigonometry

3.1 Law of Sines

See Exercise 33 on page 276 for an example of how the Law of Sines can be used to help locate a forest fire.

Introduction ▫ *The Ambiguous Case (SSA)* ▫ *Area of an Oblique Triangle* ▫ *Application*

Introduction

In Chapter 1 you looked at techniques for solving right triangles. In this section and the next, you will solve **oblique triangles**—triangles that have no right angles. As standard notation, the angles of a triangle are labeled as A, B, and C, and their opposite sides as a, b, and c, as shown in Figure 3.1.

To solve an oblique triangle, you need to know the measure of at least one side and any two other parts of the triangle—either two sides, two angles, or one angle and one side. This breaks down into the following four cases.

1. Two angles and any side (AAS or ASA)
2. Two sides and an angle opposite one of them (SSA)
3. Three sides (SSS)
4. Two sides and their included angle (SAS)

The first two cases can be solved using the **Law of Sines,** whereas the last two cases require the **Law of Cosines** (Section 3.2).

FIGURE 3.1

LAW OF SINES

If ABC is a triangle with sides a, b, and c, then

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

A is acute. A is obtuse.

Oblique Triangles

THINK ABOUT THE PROOF

To prove the Law of Sines, let h be the altitude of either triangle shown in the figure at the right. Then you have

$\sin A = \frac{h}{b}$ or $h = b \sin A$

$\sin B = \frac{h}{a}$ or $h = a \sin B$.

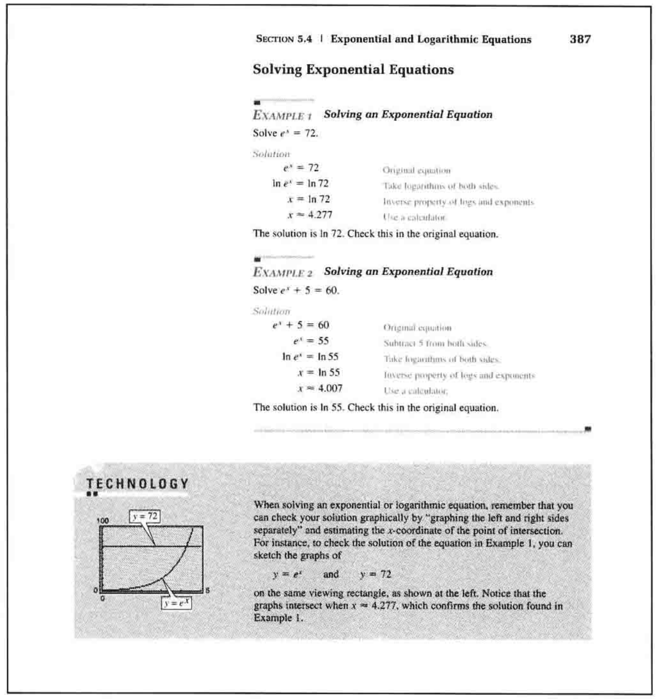
By equating the two values of h, you can establish part of the Law of Sines. Can you see how to establish the other part? The details of the proof are given in the appendix.

NOTE The Law of Sines can also be written in the reciprocal form

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

Think About the Proof Located in the margin adjacent to the corresponding theorem, each Think About the Proof feature offers strategies for proving the theorem. Detailed proofs for selected theorems are given in Appendix B.

Technology Instructions for using graphing utilities appear in the text at point of use. They offer convenient reference for students using graphing technology, and they can easily be omitted if desired. Additionally, problems in the Exercise Sets that require a graphing utility have been identified with the icon ▦.



SECTION 5.4 | Exponential and Logarithmic Equations 387

Solving Exponential Equations

EXAMPLE 1 ***Solving an Exponential Equation***

Solve $e^x = 72$.

Solution

$e^x = 72$ Original equation

$\ln e^x = \ln 72$ Take logarithms of both sides.

$x = \ln 72$ Inverse property of logs and exponents

$x \approx 4.277$ Use a calculator.

The solution is ln 72. Check this in the original equation.

EXAMPLE 2 ***Solving an Exponential Equation***

Solve $e^x + 5 = 60$.

Solution

$e^x + 5 = 60$ Original equation

$e^x = 55$ Subtract 5 from both sides.

$\ln e^x = \ln 55$ Take logarithms of both sides.

$x = \ln 55$ Inverse property of logs and exponents

$x \approx 4.007$ Use a calculator.

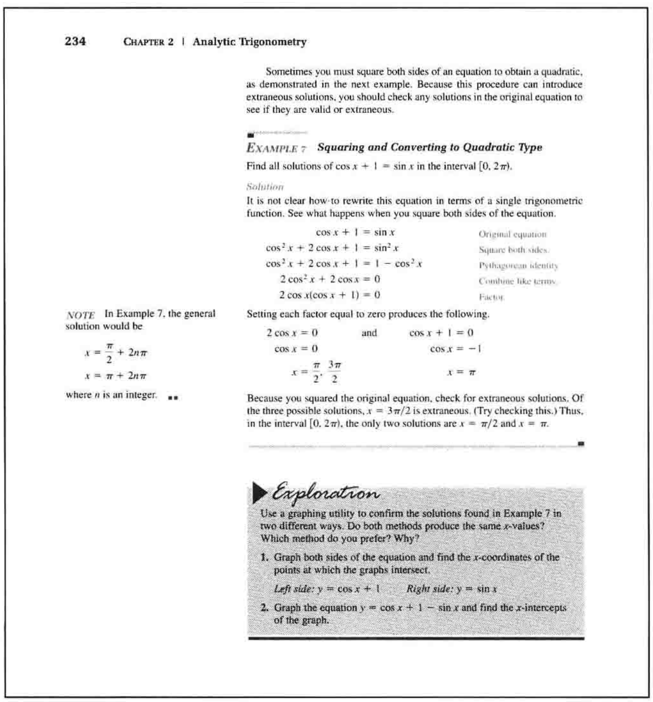
The solution is ln 55. Check this in the original equation.

TECHNOLOGY

When solving an exponential or logarithmic equation, remember that you can check your solution graphically by "graphing the left and right sides separately" and estimating the x-coordinate of the point of intersection. For instance, to check the solution of the equation in Example 1, you can sketch the graphs of

$y = e^x$ and $y = 72$

on the same viewing rectangle, as shown at the left. Notice that the graphs intersect when $x \approx 4.277$, which confirms the solution found in Example 1.



234 CHAPTER 2 | Analytic Trigonometry

Sometimes you must square both sides of an equation to obtain a quadratic, as demonstrated in the next example. Because this procedure can introduce extraneous solutions, you should check any solutions in the original equation to see if they are valid or extraneous.

EXAMPLE 7 ***Squaring and Converting to Quadratic Type***

Find all solutions of $\cos x + 1 = \sin x$ in the interval $[0, 2\pi)$.

Solution

It is not clear how to rewrite this equation in terms of a single trigonometric function. See what happens when you square both sides of the equation.

$\cos x + 1 = \sin x$ Original equation

$\cos^2 x + 2\cos x + 1 = \sin^2 x$ Square both sides.

$\cos^2 x + 2\cos x + 1 = 1 - \cos^2 x$ Pythagorean identity

$2\cos^2 x + 2\cos x = 0$ Combine like terms.

$2\cos x(\cos x + 1) = 0$ Factor.

NOTE In Example 7, the general solution would be

$$x = \frac{\pi}{2} + 2n\pi$$
$$x = \pi + 2n\pi$$

where n is an integer.

Setting each factor equal to zero produces the following.

$2\cos x = 0$ and $\cos x + 1 = 0$

$\cos x = 0$ $\cos x = -1$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $x = \pi$

Because you squared the original equation, check for extraneous solutions. Of the three possible solutions, $x = 3\pi/2$ is extraneous. (Try checking this.) Thus, in the interval $[0, 2\pi)$, the only two solutions are $x = \pi/2$ and $x = \pi$.

Exploration

Use a graphing utility to confirm the solutions found in Example 7 in two different ways. Do both methods produce the same x-values? Which method do you prefer? Why?

1. Graph both sides of the equation and find the x-coordinates of the points at which the graphs intersect.

   Left side: $y = \cos x + 1$ *Right side:* $y = \sin x$

2. Graph the equation $y = \cos x + 1 - \sin x$ and find the x-intercepts of the graph.

Exploration Throughout the text, the Exploration features encourage active participation by students, strengthening their intuition and critical thinking skills by exploring mathematical concepts and discovering mathematical relationships. Using a variety of approaches, including visualization, verification, use of graphing utilities, pattern recognition, and modeling, students develop conceptual understanding of theoretical topics.

Notes Notes anticipate students' needs by offering additional insights, pointing out common errors, and describing generalizations.

Historical Notes To help students understand that algebra has a past, historical notes featuring mathematicians and their work and mathematical artifacts are included in each chapter.

242 CHAPTER 2 | Analytic Trigonometry

Hipparchus, considered the most eminent of Greek astronomers, was born about 160 B.C. in Nicaea. He was credited with the invention of trigonometry. He also derived the sum and difference formulas for $\sin(A \pm B)$ and $\cos(A \pm B)$. *(Illustration: The Granger Collection, New York)*

Using Sum and Difference Formulas

In the remainder of this section, you will study a variety of uses of sum and difference formulas. For instance, Examples 1 and 2 show how sum and difference formulas can be used to find exact values of trigonometric functions involving sums or differences of special angles.

EXAMPLE 1 ***Evaluating a Trigonometric Function***

Find the exact value of $\cos 75°$.

Solution

To find the *exact* value of $\cos 75°$, use the fact that $75° = 30° + 45°$. Consequently, the formula for $\cos(u + v)$ yields

$$\begin{aligned}\cos 75° &= \cos(30° + 45°)\\ &= \cos 30° \cos 45° - \sin 30° \sin 45°\\ &= \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) - \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)\\ &= \frac{\sqrt{6} - \sqrt{2}}{4}.\end{aligned}$$

NOTE Try checking the result obtained in Example 1 on your calculator. You will find that $\cos 75° \approx 0.259$.

EXAMPLE 2 ***Evaluating a Trigonometric Function***

Find the exact value of $\sin \frac{\pi}{12}$.

Solution

Using the fact that

$$\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$$

together with the formula for $\sin(u - v)$, you obtain

$$\begin{aligned}\sin \frac{\pi}{12} &= \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)\\ &= \sin \frac{\pi}{3} \cos \frac{\pi}{4} - \cos \frac{\pi}{3} \sin \frac{\pi}{4}\\ &= \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) - \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)\\ &= \frac{\sqrt{6} - \sqrt{2}}{4}.\end{aligned}$$

Exploration

Graph $y = \cos(x + 2)$ and $y = \cos x + \cos 2$ on the same coordinate plane. What can you conclude about the graphs? Is it true that $\cos(x + 2) = \cos x + \cos 2$?

Graph $y = \sin(x + 4)$ and $y = \sin x + \sin 4$ on the same coordinate plane. What can you conclude about the graphs? Is it true that $\sin(x + 4) = \sin x + \sin 4$?

Applications Real-life applications are integrated throughout the text in examples and exercises. These applications offer students constant review of problem-solving skills, and they emphasize the relevance of the mathematics. Many of the applications use recent, real data, and all are titled for easy reference. Photographs with captions in the introduction to the chapter and throughout the text also encourage students to see the link between mathematics and real life.

SECTION P.3 | Graphs of Equations 29

The Midpoint Formula

To find the **midpoint** of the line segment that joins two points in a coordinate plane, you can simply find the average values of the respective coordinates of the two endpoints.

THINK ABOUT THE PROOF

The Distance Formula can be used to prove the Midpoint Formula. Can you see how to do it? The details of the proof are listed in the appendix.

THE MIDPOINT FORMULA

The midpoint of the segment joining the points (x_1, y_1) and (x_2, y_2) is

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

EXAMPLE 4 ***Finding a Segment's Midpoint***

Find the midpoint of the line segment joining the points $(-5, -3)$ and $(9, 3)$, as shown in Figure P.13.

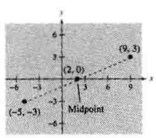


FIGURE P.13

Solution

Let $(x_1, y_1) = (-5, -3)$ and $(x_2, y_2) = (9, 3)$.

$$\begin{aligned}\text{Midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) && \text{Midpoint Formula}\\ &= \left(\frac{-5 + 9}{2}, \frac{-3 + 3}{2}\right) && \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2.\\ &= (2, 0) && \text{Simplify.}\end{aligned}$$

EXAMPLE 5 ***Estimating Annual Sales*** *Real Life*

Ben and Jerry's had annual sales of $132.0 million in 1992 and $148.8 million in 1994. Without knowing any additional information, what would you estimate the 1993 sales to have been? (Source: Ben and Jerry's, Inc.)

Solution

One solution to the problem is to assume that sales followed a linear pattern. With this assumption, you can estimate the 1993 sales by finding the midpoint of the segment connecting the points (1992, 132.0) and (1994, 148.8).

$$\text{Midpoint} = \left(\frac{1992 + 1994}{2}, \frac{132.0 + 148.8}{2}\right) = (1993, 140.4)$$

Hence, you would estimate the 1993 sales to have been about $140.4 million, as shown in Figure P.14. (The actual 1993 sales were $140.3 million.)

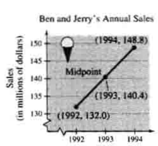
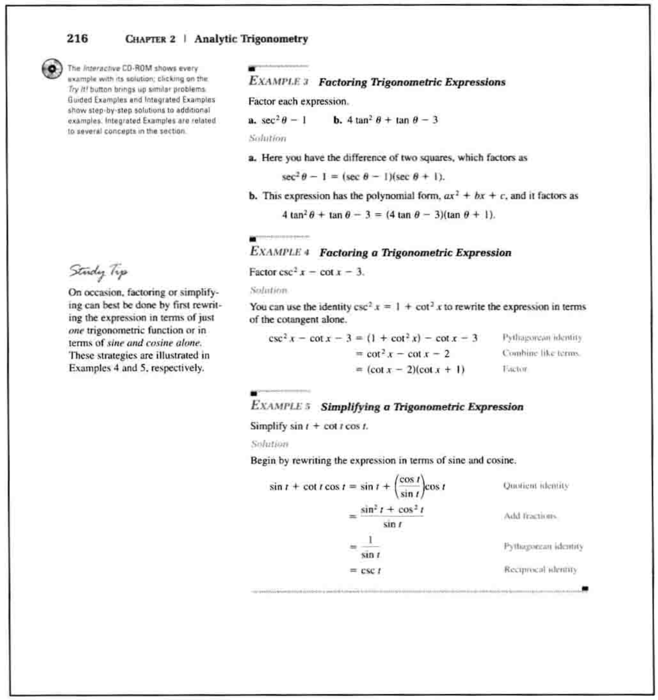


FIGURE P.14

Study Tips Study Tips appear in the margin at point of use and offer students specific suggestions for studying algebra.



216 CHAPTER 2 | Analytic Trigonometry

The *Interactive* CD-ROM shows every example with its solution; clicking on the *Try It!* button brings up similar problems. Guided Examples and Integrated Examples show step-by-step solutions to additional examples. Integrated Examples are related to several concepts in the section.

EXAMPLE 3 ***Factoring Trigonometric Expressions***

Factor each expression.

a. $\sec^2\theta - 1$ **b.** $4\tan^2\theta + \tan\theta - 3$

Solution

a. Here you have the difference of two squares, which factors as

$$\sec^2\theta - 1 = (\sec\theta - 1)(\sec\theta + 1).$$

b. This expression has the polynomial form, $ax^2 + bx + c$, and it factors as

$$4\tan^2\theta + \tan\theta - 3 = (4\tan\theta - 3)(\tan\theta + 1).$$

Study Tip

On occasion, factoring or simplifying can best be done by first rewriting the expression in terms of just *one* trigonometric function or in terms of *sine and cosine alone*. These strategies are illustrated in Examples 4 and 5, respectively.

EXAMPLE 4 ***Factoring a Trigonometric Expression***

Factor $\csc^2 x - \cot x - 3$.

Solution

You can use the identity $\csc^2 x = 1 + \cot^2 x$ to rewrite the expression in terms of the cotangent alone.

$$\csc^2 x - \cot x - 3 = (1 + \cot^2 x) - \cot x - 3 \quad \text{Pythagorean identity}$$
$$= \cot^2 x - \cot x - 2 \quad \text{Combine like terms.}$$
$$= (\cot x - 2)(\cot x + 1) \quad \text{Factor}$$

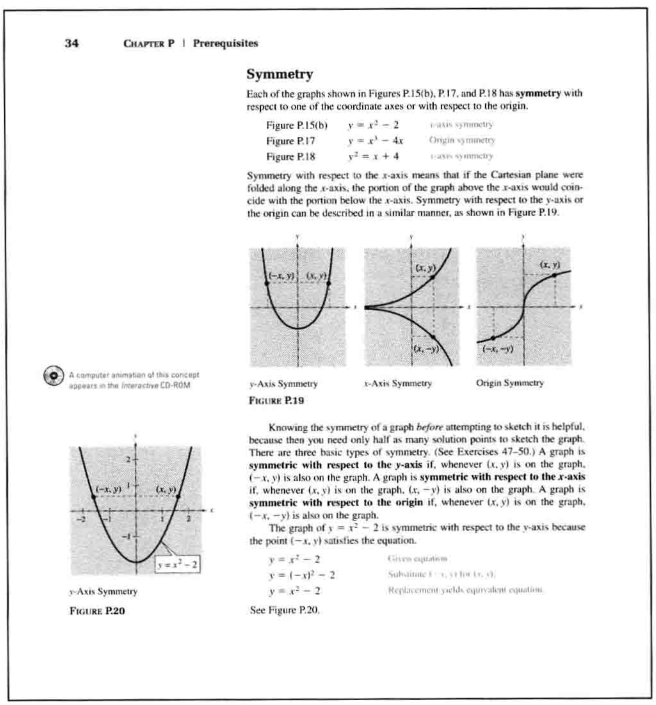
EXAMPLE 5 ***Simplifying a Trigonometric Expression***

Simplify $\sin t + \cot t \cos t$.

Solution

Begin by rewriting the expression in terms of sine and cosine.

$$\sin t + \cot t \cos t = \sin t + \left(\frac{\cos t}{\sin t}\right)\cos t \quad \text{Quotient identity}$$
$$= \frac{\sin^2 t + \cos^2 t}{\sin t} \quad \text{Add fractions.}$$
$$= \frac{1}{\sin t} \quad \text{Pythagorean identity}$$
$$= \csc t \quad \text{Reciprocal identity}$$



34 CHAPTER P | Prerequisites

Symmetry

Each of the graphs shown in Figures P.15(b), P.17, and P.18 has **symmetry** with respect to one of the coordinate axes or with respect to the origin.

Figure P.15(b)	$y = x^2 - 2$	y-axis symmetry
Figure P.17	$y = x^3 - 4x$	Origin symmetry
Figure P.18	$y^2 = x + 4$	x-axis symmetry

Symmetry with respect to the x-axis means that if the Cartesian plane were folded along the x-axis, the portion of the graph above the x-axis would coincide with the portion below the x-axis. Symmetry with respect to the y-axis or the origin can be described in a similar manner, as shown in Figure P.19.

y-Axis Symmetry x-Axis Symmetry Origin Symmetry

FIGURE P.19

A computer animation of this concept appears in the *Interactive* CD-ROM

Knowing the symmetry of a graph *before* attempting to sketch it is helpful, because then you need only half as many solution points to sketch the graph. There are three basic types of symmetry. (See Exercises 47–50.) A graph is **symmetric with respect to the y-axis** if, whenever (x, y) is on the graph, $(-x, y)$ is also on the graph. A graph is **symmetric with respect to the x-axis** if, whenever (x, y) is on the graph, $(x, -y)$ is also on the graph. A graph is **symmetric with respect to the origin** if, whenever (x, y) is on the graph, $(-x, -y)$ is also on the graph.

The graph of $y = x^2 - 2$ is symmetric with respect to the y-axis because the point $(-x, y)$ satisfies the equation.

$$y = x^2 - 2 \quad \text{Given equation}$$
$$y = (-x)^2 - 2 \quad \text{Substitute } (-x, y) \text{ for } (x, y).$$
$$y = x^2 - 2 \quad \text{Replacement yields equivalent equation.}$$

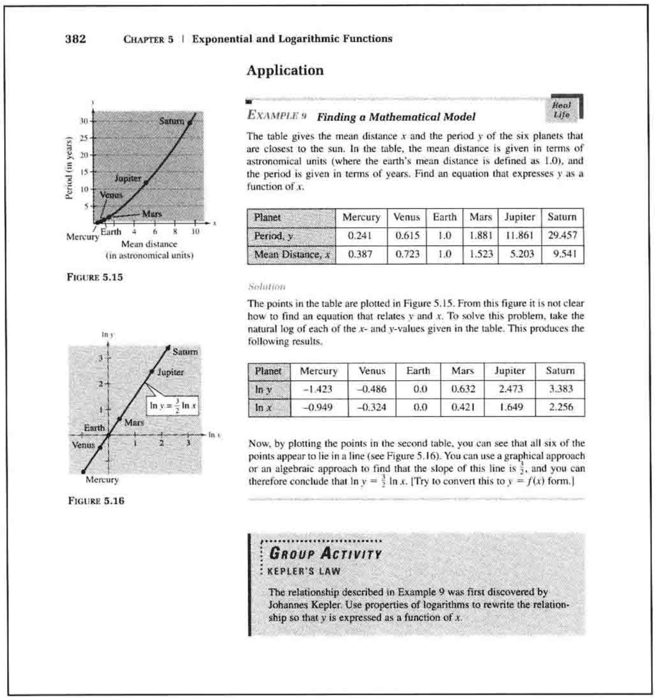
See Figure P.20.

y-Axis Symmetry

FIGURE P.20

CD-ROM The icon refers to additional features of *Interactive Trigonometry* that enhance the text presentation, such as exercises, computer animations, examples, tests, and *TI-82* and *TI-83* graphing calculator emulators.

Examples Each of the over 300 text examples was carefully chosen to illustrate a particular mathematical concept, problem-solving approach, or computational technique, and to enhance students' understanding. The examples in the text cover a wide variety of problem types, including theoretical problems, real-life applications (many with real data), and problems requiring the use of graphing technology. Each example is titled for easy reference, and real-life applications are labeled. Many examples include side comments in color that clarify the steps of the solution.



382 CHAPTER 5 | Exponential and Logarithmic Functions

Application

EXAMPLE 9 ***Finding a Mathematical Model*** *Real Life*

The table gives the mean distance x and the period y of the six planets that are closest to the sun. In the table, the mean distance is given in terms of astronomical units (where the earth's mean distance is defined as 1.0), and the period is given in terms of years. Find an equation that expresses y as a function of x.

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn
Period, y	0.241	0.615	1.0	1.881	11.861	29.457
Mean Distance, x	0.387	0.723	1.0	1.523	5.203	9.541

FIGURE 5.15

Solution

The points in the table are plotted in Figure 5.15. From this figure it is not clear how to find an equation that relates y and x. To solve this problem, take the natural log of each of the x- and y-values given in the table. This produces the following results.

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn
ln y	−1.423	−0.486	0.0	0.632	2.473	3.383
ln x	−0.949	−0.324	0.0	0.421	1.649	2.256

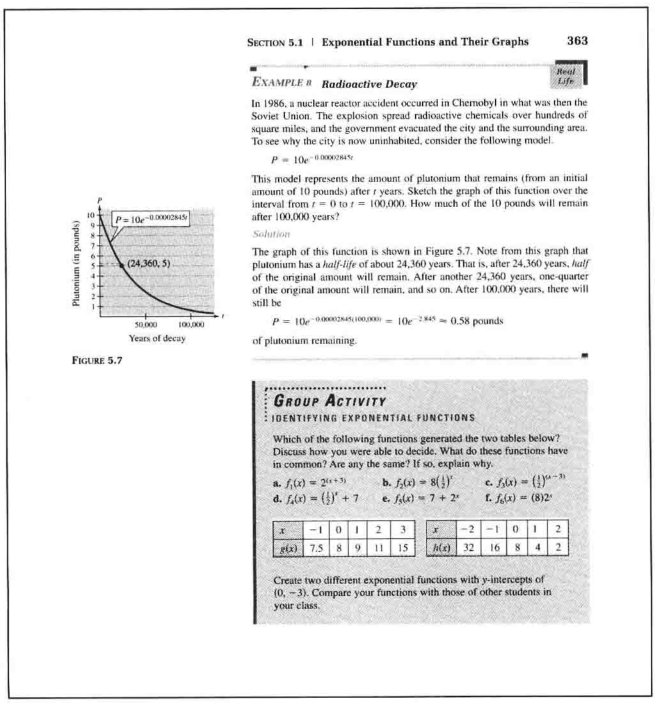
FIGURE 5.16

Now, by plotting the points in the second table, you can see that all six of the points appear to lie in a line (see Figure 5.16). You can use a graphical approach or an algebraic approach to find that the slope of this line is $\frac{3}{2}$, and you can therefore conclude that $\ln y = \frac{3}{2} \ln x$. [Try to convert this to $y = f(x)$ form.]

GROUP ACTIVITY

KEPLER'S LAW

The relationship described in Example 9 was first discovered by Johannes Kepler. Use properties of logarithms to rewrite the relationship so that y is expressed as a function of x.



SECTION 5.1 | Exponential Functions and Their Graphs 363

EXAMPLE 8 ***Radioactive Decay*** *Real Life*

In 1986, a nuclear reactor accident occurred in Chernobyl in what was then the Soviet Union. The explosion spread radioactive chemicals over hundreds of square miles, and the government evacuated the city and the surrounding area. To see why the city is now uninhabited, consider the following model.

$$P = 10e^{-0.00002845t}$$

This model represents the amount of plutonium that remains (from an initial amount of 10 pounds) after t years. Sketch the graph of this function over the interval from $t = 0$ to $t = 100{,}000$. How much of the 10 pounds will remain after 100,000 years?

Solution

The graph of this function is shown in Figure 5.7. Note from this graph that plutonium has a *half-life* of about 24,360 years. That is, after 24,360 years, *half* of the original amount will remain. After another 24,360 years, one-quarter of the original amount will remain, and so on. After 100,000 years, there will still be

$$P = 10e^{-0.00002845(100{,}000)} = 10e^{-2.845} \approx 0.58 \text{ pounds}$$

of plutonium remaining.

FIGURE 5.7

GROUP ACTIVITY

IDENTIFYING EXPONENTIAL FUNCTIONS

Which of the following functions generated the two tables below? Discuss how you were able to decide. What do these functions have in common? Are any the same? If so, explain why.

a. $f_1(x) = 2^{(x+3)}$ **b.** $f_2(x) = 8(\frac{1}{2})^x$ **c.** $f_3(x) = (\frac{1}{2})^{(x-3)}$
d. $f_4(x) = (\frac{1}{2})^x + 7$ **e.** $f_5(x) = 7 + 2^x$ **f.** $f_6(x) = (8)2^x$

x	−1	0	1	2	3
$g(x)$	7.5	8	9	11	15

x	−2	−1	0	1	2
$h(x)$	32	16	8	4	2

Create two different exponential functions with y-intercepts of $(0, -3)$. Compare your functions with those of other students in your class.

Group Activities The Group Activities that appear at the ends of sections reinforce students' understanding by studying mathematical concepts in a variety of ways, including talking and writing about mathematics, creating and solving problems, analyzing errors, and developing and using mathematical models. Designed to be completed as group projects in class or as homework assignments, the Group Activities give students opportunities to do interactive learning and to think, talk, and write about mathematics.

Warm Ups Each section (except Section P.1) contains a set of 10 warm-up exercises that students can use for review and practice of the previously learned skills that are necessary for mastery of the new skills and concepts presented in the section. All warm-up exercises are answered in the back of the text.

The *Interactive* CD-ROM contains step-by-step solutions to all odd-numbered Section and Review Exercises. It also provides Tutorial Exercises, which link to Guided Examples for additional help.

WARM UP

In Exercises 1–4, write each complex number in standard form and give its complex conjugate.

1. $4 - \sqrt{-29}$
2. $-5 - \sqrt{-144}$
3. $-1 + \sqrt{-32}$
4. $6 + \sqrt{-1/4}$

In Exercises 5–10, perform the operations and write the answers in standard form.

5. $(-3 + 6i) - (10 - 3i)$
6. $(12 - 4i) + 20i$
7. $(4 - 2i)(3 + 7i)$
8. $(2 - 5i)(2 + 5i)$
9. $\dfrac{1 + i}{1 - i}$
10. $(3 + 2i)^3$

4.2 Exercises

In Exercises 1–4, determine the number of solutions of the equation in the complex number system.

1. $x^3 - 4x + 5 = 0$
2. $2x^6 + 3x^3 - 10 = 0$
3. $25 - x^4 = 0$
4. $12 - x + 3x^2 - 3x^5 = 0$

In Exercises 5–8, use the discriminant to determine the number of real solutions of the quadratic equation.

5. $2x^2 - 5x + 5 = 0$
6. $2x^2 - x - 1 = 0$
7. $\frac{1}{5}x^2 + \frac{6}{5}x - 8 = 0$
8. $\frac{1}{3}x^2 - 5x + 25 = 0$

In Exercises 9–20, solve the equation. List any complex solutions in the form $a + bi$.

9. $x^2 - 5 = 0$
10. $3x^2 - 1 = 0$
11. $(x + 5)^2 - 6 = 0$
12. $16 - (x - 1)^2 = 0$
13. $x^2 - 8x + 16 = 0$
14. $4x^2 + 4x + 1 = 0$
15. $x^2 + 2x + 5 = 0$
16. $54 + 16x - x^2 = 0$
17. $4x^2 - 4x + 5 = 0$
18. $4x^2 - 4x + 21 = 0$
19. $230 + 20x - 0.5x^2 = 0$
20. $6 - (x - 1)^2 = 0$

Graphical and Analytical Analysis **In Exercises 21–24, find all the zeros of the function. Is there a relationship between the number of real zeros and the number of x-intercepts of the graph? Explain.**

21. $f(x) = x^3 - 4x^2 + x - 4$
22. $f(x) = x^3 - 4x^2 - 4x + 16$

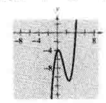


FIGURE FOR 21

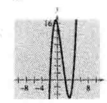


FIGURE FOR 22

Geometry Geometric formulas and concepts are reviewed throughout the text in examples, group activities, and exercises. For reference, common formulas are listed inside the back cover of this text.

In Exercises 95–108, find all solutions of the equation. Check your solutions in the original equation.

95. $\sqrt{x - 10} - 4 = 0$
96. $\sqrt{5 - x} - 3 = 0$
97. $\sqrt[3]{2x + 5} + 3 = 0$
98. $\sqrt[3]{3x + 1} - 5 = 0$
99. $x = \sqrt{11x - 30}$
100. $2x - \sqrt{15 - 4x} = 0$
101. $\sqrt{x + 1} - 3x = 1$
102. $\sqrt{x + 5} = \sqrt{x - 5}$
103. $\sqrt{x} - \sqrt{x - 5} = 1$
104. $\sqrt{x} + \sqrt{x - 20} = 10$
105. $2\sqrt{x + 1} - \sqrt{2x + 3} = 1$
106. $3\sqrt{x} - \dfrac{4}{\sqrt{x}} = 4$
107. $(x - 5)^{2/3} = 16$
108. $(x + 3)^{3/4} = 27$

109. ***Market Research*** The demand equation for a certain product is modeled by $p = 40 - \sqrt{0.01x + 1}$, where x is the number of units demanded per day and p is the price per unit. Approximate the demand if the price is $37.55.

110. ***Market Research*** The demand equation for a certain product is modeled by $p = 40 - \sqrt{0.0001x + 1}$, where x is the number of units demanded per day and p is the price per unit. Approximate the demand if the price is $34.70.

In Exercises 111 and 112, solve for the indicated variable.

111. ***Surface Area of a Cone***
Solve for h: $S = \pi r\sqrt{r^2 + h^2}$

112. ***Inductance***
Solve for Q: $i = \pm\sqrt{\dfrac{1}{LC}}\sqrt{Q^2 - q}$

In Exercises 113 and 114, consider an equation of the form $x + \sqrt{x - a} = b$, where a and b are constants.

113. ***Exploration*** Find a and b if the solution to the equation is $x = 20$. (There are many correct answers.)

114. ***Essay*** Write a short paragraph listing the steps required in solving an equation involving radicals.

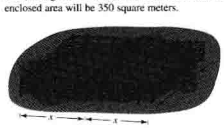
In Exercises 115–120, find all solutions of the equation. Check your solutions in the original equation.

115. $|x + 1| = 2$
116. $|x - 2| = 3$
117. $|2x - 1| = 5$
118. $|3x + 2| = 7$
119. $|x^2 + 6x| = 3x + 18$
120. $|x - 10| = x^2 - 10x$

Think About It **In Exercises 121 and 122, find an equation having the given solutions. (There are many correct answers.)**

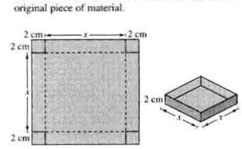
121. $-3, 5$
122. $0, 2, \frac{5}{2}$

123. ***Dimensions of a Corral*** A rancher has 100 meters of fencing to enclose two adjacent rectangular corrals (see figure). Find the dimensions such that the enclosed area will be 350 square meters.



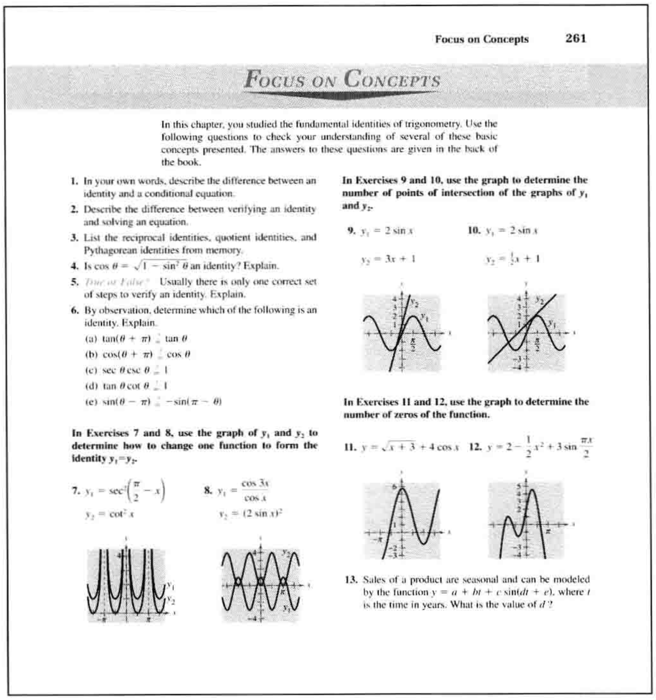
$4x + 3y = 100$

124. ***Dimensions of a Box*** An open box is to be made from a square piece of material by cutting 2-centimeter squares from the corners and turning up the sides (see figure). The volume of the finished box is to be 200 cubic centimeters. Find the size of the original piece of material.



Exercises The exercise sets were completely revised—and expanded by over 20%—for the Fourth Edition. The text now offers nearly 4000 exercises with a broad range of conceptual, computational, and applied problems to accommodate a variety of teaching and learning styles. Included in the section and review exercise sets are multi-part, writing, and more challenging problems with extensive graphics that encourage exploration and discovery, enhance students' skills in mathematical modeling, estimation, and data interpretation and analysis, and encourage the use of graphing technology for conceptual understanding. Applications are labeled for easy reference. The exercise sets are designed to build competence, skill, and understanding; each exercise set is graded in difficulty to allow students to gain confidence as they progress. Detailed solutions to all odd-numbered exercises are given in the *Study and Solutions Guide*; answers to all odd-numbered exercises appear in the back of the text.

Focus on Concepts Each Focus on Concepts feature is a set of exercises that test students' understanding of the basic concepts covered in the chapter. Answers to all questions are given in the back of the text.



Focus on Concepts 261

FOCUS ON CONCEPTS

In this chapter, you studied the fundamental identities of trigonometry. Use the following questions to check your understanding of several of these basic concepts presented. The answers to these questions are given in the back of the book.

1. In your own words, describe the difference between an identity and a conditional equation.
2. Describe the difference between verifying an identity and solving an equation.
3. List the reciprocal identities, quotient identities, and Pythagorean identities from memory.
4. Is $\cos\theta = \sqrt{1-\sin^2\theta}$ an identity? Explain.
5. *True or False?* Usually there is only one correct set of steps to verify an identity. Explain.
6. By observation, determine which of the following is an identity. Explain.
   (a) $\tan(\theta+\pi) \stackrel{?}{=} \tan\theta$
   (b) $\cos(\theta+\pi) \stackrel{?}{=} \cos\theta$
   (c) $\sec\theta\csc\theta \stackrel{?}{=} 1$
   (d) $\tan\theta\cot\theta \stackrel{?}{=} 1$
   (e) $\sin(\theta-\pi) \stackrel{?}{=} -\sin(\pi-\theta)$

In Exercises 7 and 8, use the graph of y_1 and y_2 to determine how to change one function to form the identity $y_1 = y_2$.

7. $y_1 = \sec^2\left(\frac{\pi}{2}-x\right)$, $y_2 = \cot^2 x$
8. $y_1 = \frac{\cos 3x}{\cos x}$, $y_2 = (2\sin x)^2$

In Exercises 9 and 10, use the graph to determine the number of points of intersection of the graphs of y_1 and y_2.

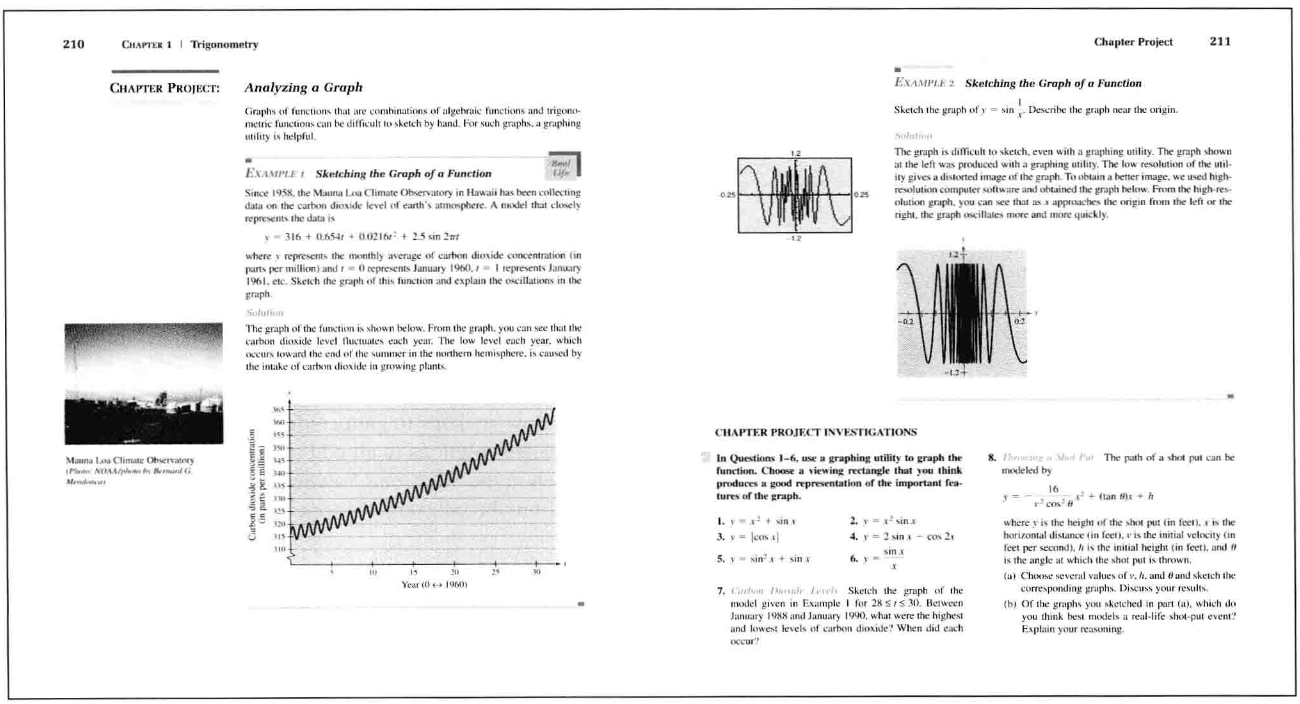
9. $y_1 = 2\sin x$, $y_2 = 3x + 1$
10. $y_1 = 2\sin x$, $y_2 = \frac{1}{2}x + 1$

In Exercises 11 and 12, use the graph to determine the number of zeros of the function.

11. $y = \sqrt{x+3} + 4\cos x$
12. $y = 2 - \frac{1}{2}x^2 + 3\sin\frac{\pi x}{2}$

13. Sales of a product are seasonal and can be modeled by the function $y = a + bt + c\sin(dt + e)$, where t is the time in years. What is the value of d?

Chapter Projects Chapter Projects are extended applications that use real data, graphs, and modeling to enhance students' understanding of mathematical concepts. Designed as individual or group projects, they offer additional opportunities to think, discuss, and write about mathematics. Many projects give students the opportunity to collect, analyze, and interpret data.



210 CHAPTER 1 | Trigonometry

CHAPTER PROJECT: *Analyzing a Graph*

Graphs of functions that are combinations of algebraic functions and trigonometric functions can be difficult to sketch by hand. For such graphs, a graphing utility is helpful.

EXAMPLE 1 *Sketching the Graph of a Function* (Real Life)

Since 1958, the Mauna Loa Climate Observatory in Hawaii has been collecting data on the carbon dioxide level of earth's atmosphere. A model that closely represents the data is

$$y = 316 + 0.654t + 0.0216t^2 + 2.5\sin 2\pi t$$

where y represents the monthly average of carbon dioxide concentration (in parts per million) and $t = 0$ represents January 1960, $t = 1$ represents January 1961, etc. Sketch the graph of this function and explain the oscillations in the graph.

Solution

The graph of the function is shown below. From the graph, you can see that the carbon dioxide level fluctuates each year. The low level each year, which occurs toward the end of the summer in the northern hemisphere, is caused by the intake of carbon dioxide in growing plants.

Mauna Loa Climate Observatory (Photo: NOAA/photo by Bernard G. Mendonca)

Carbon dioxide concentration (in parts per million); Year (0 ↔ 1960)

Chapter Project 211

EXAMPLE 2 *Sketching the Graph of a Function*

Sketch the graph of $y = \sin\frac{1}{x}$. Describe the graph near the origin.

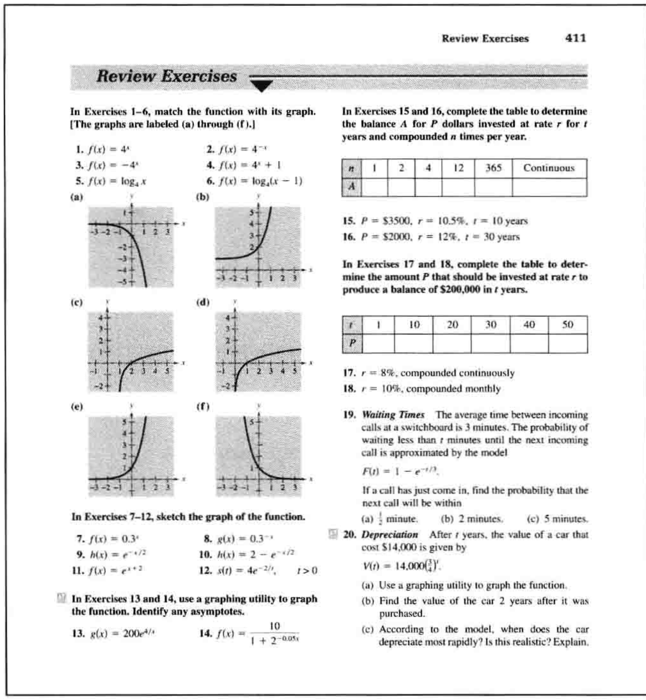
Solution

The graph is difficult to sketch, even with a graphing utility. The graph shown at the left was produced with a graphing utility. The low resolution of the utility gives a distorted image of the graph. To obtain a better image, we used high-resolution computer software and obtained the graph below. From the high-resolution graph, you can see that as x approaches the origin from the left or the right, the graph oscillates more and more quickly.

CHAPTER PROJECT INVESTIGATIONS

In Questions 1–6, use a graphing utility to graph the function. Choose a viewing rectangle that you think produces a good representation of the important features of the graph.

1. $y = x^2 + \sin x$
2. $y = x^2\sin x$
3. $y = |\cos x|$
4. $y = 2\sin x - \cos 2x$
5. $y = \sin^2 x + \sin x$
6. $y = \frac{\sin x}{x}$
7. *Carbon Dioxide Levels* Sketch the graph of the model given in Example 1 for $28 \le t \le 30$. Between January 1988 and January 1990, what were the highest and lowest levels of carbon dioxide? When did each occur?
8. *Throwing a Shot Put* The path of a shot put can be modeled by
$$y = -\frac{16}{v^2\cos^2\theta}x^2 + (\tan\theta)x + h$$
where y is the height of the shot put (in feet), x is the horizontal distance (in feet), v is the initial velocity (in feet per second), h is the initial height (in feet), and θ is the angle at which the shot put is thrown.
   (a) Choose several values of v, h, and θ and sketch the corresponding graphs. Discuss your results.
   (b) Of the graphs you sketched in part (a), which do you think best models a real-life shot-put event? Explain your reasoning.


Review Exercises 411

Review Exercises

In Exercises 1–6, match the function with its graph. [The graphs are labeled (a) through (f).]

1. $f(x) = 4^x$ 2. $f(x) = 4^{-x}$
3. $f(x) = -4^x$ 4. $f(x) = 4^x + 1$
5. $f(x) = \log_4 x$ 6. $f(x) = \log_4(x - 1)$

(a) (b) (c) (d) (e) (f)

In Exercises 7–12, sketch the graph of the function.

7. $f(x) = 0.3^x$ 8. $g(x) = 0.3^{-x}$
9. $h(x) = e^{-x/2}$ 10. $h(x) = 2 - e^{-x/2}$
11. $f(x) = e^{x+2}$ 12. $s(t) = 4e^{-2/t}, \quad t > 0$

In Exercises 13 and 14, use a graphing utility to graph the function. Identify any asymptotes.

13. $g(x) = 200e^{4/x}$ 14. $f(x) = \dfrac{10}{1 + 2^{-0.05x}}$

In Exercises 15 and 16, complete the table to determine the balance A for P dollars invested at rate r for t years and compounded n times per year.

n	1	2	4	12	365	Continuous
A						

15. $P = \$3500$, $r = 10.5\%$, $t = 10$ years
16. $P = \$2000$, $r = 12\%$, $t = 30$ years

In Exercises 17 and 18, complete the table to determine the amount P that should be invested at rate r to produce a balance of \$200,000 in t years.

t	1	10	20	30	40	50
P						

17. $r = 8\%$, compounded continuously
18. $r = 10\%$, compounded monthly

19. *Waiting Times* The average time between incoming calls at a switchboard is 3 minutes. The probability of waiting less than t minutes until the next incoming call is approximated by the model

$F(t) = 1 - e^{-t/3}$.

If a call has just come in, find the probability that the next call will be within

(a) $\frac{1}{2}$ minute. (b) 2 minutes. (c) 5 minutes.

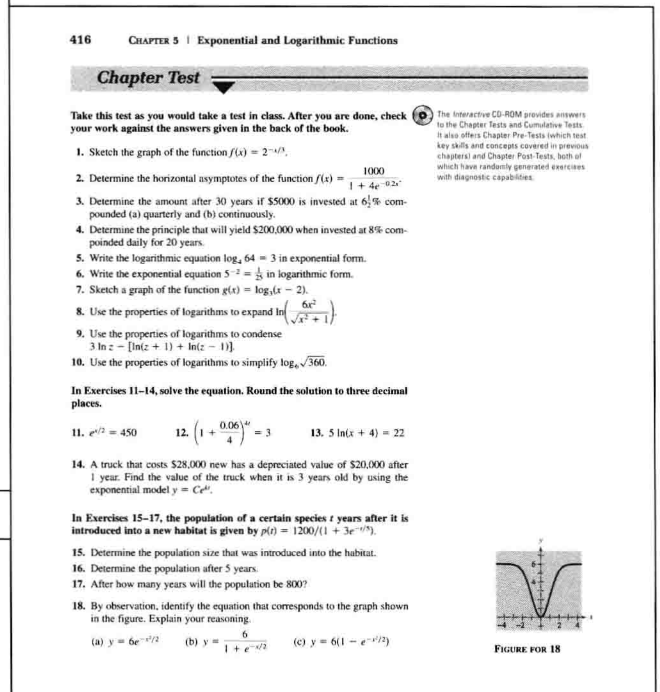
20. *Depreciation* After t years, the value of a car that cost \$14,000 is given by

$V(t) = 14{,}000(\frac{3}{4})^t$.

(a) Use a graphing utility to graph the function.
(b) Find the value of the car 2 years after it was purchased.
(c) According to the model, when does the car depreciate most rapidly? Is this realistic? Explain.

Review Exercises

The Review Exercises at the end of each chapter offer students an opportunity for additional practice. Answers to odd-numbered review exercises are given in the back of the text.


416 CHAPTER 5 | Exponential and Logarithmic Functions

Chapter Test

Take this test as you would take a test in class. After you are done, check your work against the answers given in the back of the book.

The *Interactive* CD-ROM provides answers to the Chapter Tests and Cumulative Tests. It also offers Chapter Pre-Tests (which test key skills and concepts covered in previous chapters) and Chapter Post-Tests, both of which have randomly generated exercises with diagnostic capabilities.

1. Sketch the graph of the function $f(x) = 2^{-x/3}$.
2. Determine the horizontal asymptotes of the function $f(x) = \dfrac{1000}{1 + 4e^{-0.2x}}$.
3. Determine the amount after 30 years if \$5000 is invested at $6\frac{1}{2}\%$ compounded (a) quarterly and (b) continuously.
4. Determine the principle that will yield \$200,000 when invested at 8% compounded daily for 20 years.
5. Write the logarithmic equation $\log_4 64 = 3$ in exponential form.
6. Write the exponential equation $5^{-2} = \frac{1}{25}$ in logarithmic form.
7. Sketch a graph of the function $g(x) = \log_3(x - 2)$.
8. Use the properties of logarithms to expand $\ln\left(\dfrac{6x^2}{\sqrt{x^2+1}}\right)$.
9. Use the properties of logarithms to condense $3 \ln z - [\ln(z + 1) + \ln(z - 1)]$.
10. Use the properties of logarithms to simplify $\log_6 \sqrt{360}$.

In Exercises 11–14, solve the equation. Round the solution to three decimal places.

11. $e^{t/2} = 450$ 12. $\left(1 + \dfrac{0.06}{4}\right)^{4t} = 3$ 13. $5 \ln(x + 4) = 22$

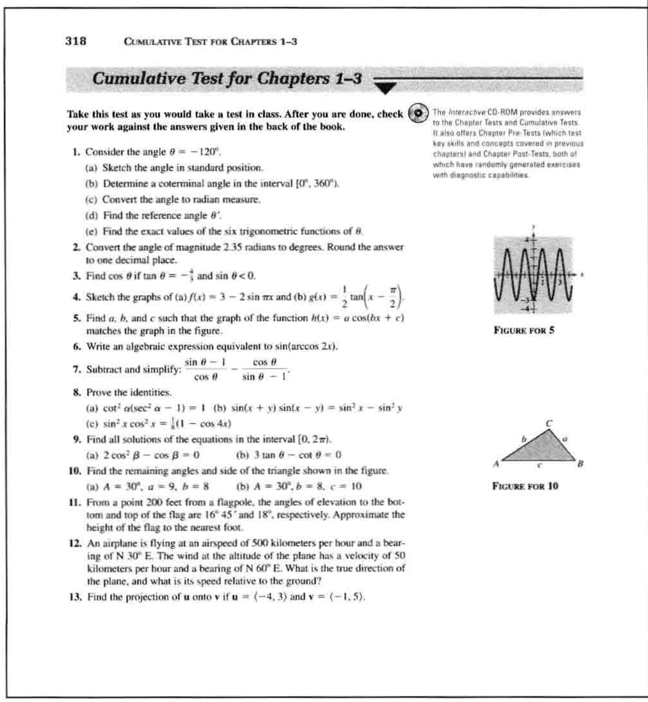
14. A truck that costs \$28,000 new has a depreciated value of \$20,000 after 1 year. Find the value of the truck when it is 3 years old by using the exponential model $y = Ce^{kt}$.

In Exercises 15–17, the population of a certain species t years after it is introduced into a new habitat is given by $p(t) = 1200/(1 + 3e^{-t/5})$.

15. Determine the population size that was introduced into the habitat.
16. Determine the population after 5 years.
17. After how many years will the population be 800?
18. By observation, identify the equation that corresponds to the graph shown in the figure. Explain your reasoning.

(a) $y = 6e^{-x^2/2}$ (b) $y = \dfrac{6}{1 + e^{-x/2}}$ (c) $y = 6(1 - e^{-x^2/2})$

FIGURE FOR 18


318 CUMULATIVE TEST FOR CHAPTERS 1–3

Cumulative Test for Chapters 1–3

Take this test as you would take a test in class. After you are done, check your work against the answers given in the back of the book.

The *Interactive* CD-ROM provides answers to the Chapter Tests and Cumulative Tests. It also offers Chapter Pre-Tests (which test key skills and concepts covered in previous chapters) and Chapter Post-Tests, both of which have randomly generated exercises with diagnostic capabilities.

1. Consider the angle $\theta = -120°$.
(a) Sketch the angle in standard position.
(b) Determine a coterminal angle in the interval $[0°, 360°)$.
(c) Convert the angle to radian measure.
(d) Find the reference angle θ'.
(e) Find the exact values of the six trigonometric functions of θ.
2. Convert the angle of magnitude 2.35 radians to degrees. Round the answer to one decimal place.
3. Find $\cos\theta$ if $\tan\theta = -\frac{4}{3}$ and $\sin\theta < 0$.
4. Sketch the graphs of (a) $f(x) = 3 - 2\sin \pi x$ and (b) $g(x) = \dfrac{1}{2}\tan\left(x - \dfrac{\pi}{2}\right)$.
5. Find a, b, and c such that the graph of the function $h(x) = a\cos(bx + c)$ matches the graph in the figure.
6. Write an algebraic expression equivalent to $\sin(\arccos 2x)$.
7. Subtract and simplify: $\dfrac{\sin\theta - 1}{\cos\theta} - \dfrac{\cos\theta}{\sin\theta - 1}$.
8. Prove the identities.
(a) $\cot^2\alpha(\sec^2\alpha - 1) = 1$ (b) $\sin(x + y)\sin(x - y) = \sin^2 x - \sin^2 y$
(c) $\sin^2 x \cos^2 x = \frac{1}{8}(1 - \cos 4x)$
9. Find all solutions of the equations in the interval $[0, 2\pi)$.
(a) $2\cos^2\beta - \cos\beta = 0$ (b) $3\tan\theta - \cot\theta = 0$
10. Find the remaining angles and side of the triangle shown in the figure.
(a) $A = 30°$, $a = 9$, $b = 8$ (b) $A = 30°$, $b = 8$, $c = 10$
11. From a point 200 feet from a flagpole, the angles of elevation to the bottom and top of the flag are 16° 45′ and 18°, respectively. Approximate the height of the flag to the nearest foot.
12. An airplane is flying at an airspeed of 500 kilometers per hour and a bearing of N 30° E. The wind at the altitude of the plane has a velocity of 50 kilometers per hour and a bearing of N 60° E. What is the true direction of the plane, and what is its speed relative to the ground?
13. Find the projection of **u** onto **v** if **u** = ⟨−4, 3⟩ and **v** = ⟨−1, 5⟩.

FIGURE FOR 5

FIGURE FOR 10

Chapter Tests

Each chapter that is not followed by a Cumulative Test ends with a Chapter Test, an effective tool for student self-assessment.

Cumulative Tests

The Cumulative Tests that follow Chapters 3 and 6 help students judge their mastery of previously covered material, as well as reinforce the knowledge they have been accumulating throughout the text—preparing them for other exams and for future courses.

Supplements

Trigonometry, Fourth Edition, by Larson and Hostetler, is accompanied by a comprehensive supplements package. Most items are keyed to the text.

Printed Resources

FOR THE STUDENT

Study and Solutions Guide by Dianna Zook, Indiana University/Purdue University—Fort Wayne

- Section summaries of key concepts
- Detailed, step-by-step solutions to all odd-numbered exercises
- Practice tests with solutions
- Study strategies

Graphing Technology Keystroke Guide

- Keystroke instructions for many graphing calculators from Texas Instruments, Sharp, Casio, and Hewlett-Packard, including *TI-83, TI-92, HP-38G,* and *Casio CFX-9800G*.
- BestGrapher instructions for both IBM and Macintosh
- Examples with step-by-step solutions
- Extensive graphics screen output
- Technology tips

FOR THE INSTRUCTOR

Instructor's Annotated Edition

- Includes the entire student edition of the text, with the student answers section
- Instructor's Answers section: Answers to all even-numbered exercises, and answers to all Explorations, Technology exercises, Chapter Project exercises, and Group Activities
- Annotations at point of use offer specific teaching strategies and suggestions for implementing Group Activities, point out common student errors, and give additional examples, exercises, class activities, and group activities.

Test Item File and Instructor's Resource Guide

- Printed test bank with nearly 1600 test items (multiple-choice, open-ended, and writing) coded by level of difficulty
- Technology-required test items coded for easy reference
- Bank of chapter test forms with answer keys
- Two final exam test forms
- Notes to the instructor, including materials for alternative assessment and managing the multicultural and cooperative-learning classrooms

Problem Solving, Modeling, and Data Analysis Labs by Wendy Metzger, Palomar College

- Multipart, guided discovery activities and applications
- Keystroke instructions for Derive and *TI-82*
- Keyed to the text by topic
- Funded in part by NSF (National Science Foundation, Instrumentation and Laboratory Improvement) and California Community College Fund for Instructional Improvement

Media Resources

FOR THE STUDENT

Interactive Trigonometry (See pages xvi–xviii for a description, or visit the Houghton Mifflin home page at http://www.hmco.com for a preview.)

- Interactive, multimedia CD-ROM format
- IBM-PC for Windows

Tutor software

- Interactive tutorial software keyed to the text by section
- Diagnostic feedback
- Chapter self-tests
- Guided exercises with step-by-step solutions
- Glossary

Videotapes by Dana Mosely

- Comprehensive, text-specific coverage keyed to the text by section
- Real-life application vignettes introduced where appropriate
- Computer-generated animation
- For media/resource centers
- Additional explanation of concepts, sample problems, and applications
- Instructional graphing calculator videotape also available

FOR THE INSTRUCTOR

Computerized Testing (IBM, Macintosh, Windows)

- New on-line testing
- New grade-management capabilities
- Algorithmic test-generating software provides an unlimited number of tests.
- Nearly 1600 test items
- Also available as a printed test bank

Transparency Package

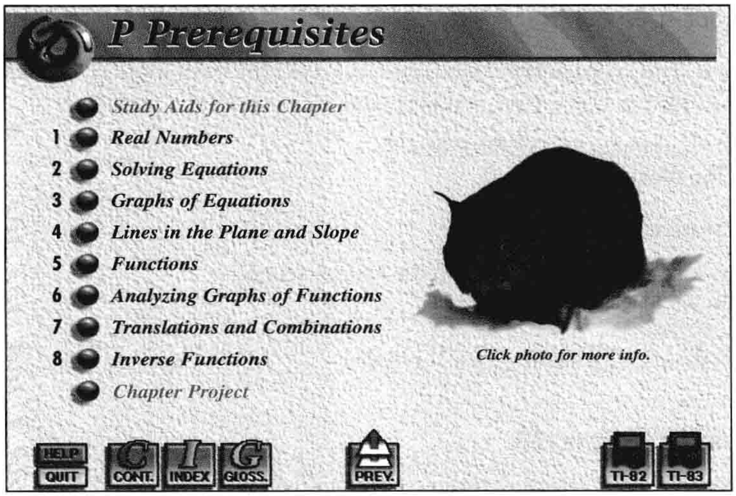
- 40 color transparencies color-coded by topic

Interactive Trigonometry

To accommodate a variety of teaching and learning styles, *Trigonometry* is also available in a multimedia, CD-ROM format. In this interactive format, the text offers the student additional tutorial assistance with

- Complete solutions to all odd-numbered text exercises.
- Chapter pre-tests, self-tests, and post-tests.
- *TI-82* and *TI-83* emulators.
- Guided examples with step-by-step solutions.
- Editable graphs.
- Animations of mathematical concepts.
- Warm-up, section, and tutorial exercises.
- Glossary of key terms.

These and other pedagogical features of the CD-ROM are illustrated by the screen dumps shown below.



Chapter Topics Each chapter begins with an outline of the topics to be covered. Using the buttons at the bottom of the screen, the student can quickly move to the appropriate section.

Introductory Chapter Application Each chapter opens with a real-data application that illustrates the key concepts and techniques to be covered. Clicking on the photo, the student can access additional data and background information that frames the real-world context for a mathematical concept.

Chapter Project Each chapter is accompanied by a Chapter Project. This offers the student the opportunity to synthesize the algebraic techniques and concepts studied in the chapter. Many projects use real data and emphasize data analysis and mathematical modeling.

Study Aids Each section offers the student an array of additional study aids, including Chapter Pre-, Post-, and Self-Tests, Review Exercises, and Focus on Concepts. With diagnostics, complete solutions, or answers, these helpful features promote the focused practice needed to master mathematical concepts. Short, informative video segments are also included.