Mathematics for Technicians Level 1

GRANADA TEC TECHNICIAN SERIES

D. J. Hancox

Mathematics for Technicians 1

D J Hancox

Head of Mathematics Department,
Coventry Technical College

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Preface

This book is the first in a series of books designed to provide the mathematics required for the TEC certificate courses. It covers the work of the TEC level 1 mathematics unit.

The standard unit was modified in 1980 and part of the introductory work, previously included in the unit, was taken out. This work has been included in the first three chapters. At the end of chapter 3 a pre-unit test has been written.

The presentation has been designed to create interest in the subject and uses practical examples from other subject areas wherever possible. Each chapter represents a module from the unit. The student should study the text and worked examples in each section and then work all of the examples in the exercise. The above average student will find further examples in the General Examples section at the end of each chapter. Each chapter has a summary and a self assessment test. There are also phase tests as appropriate and an end of unit examination. Answers are given to all exercises and full solutions are given to the self assessment tests.

This book should be valuable to students in colleges, or studying a similar course in school. The book has been written so that, if necessary, a student can work through the unit with a minimum amount of teacher assistance.

I would like to thank the publishers for their helpful guidance and valuable suggestions, my daughter Jane for typing the manuscript and Les Burrows and Phillip Doughty for preparing the diagrams.

D J Hancox

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Contents

1 Basic arithmetic involving integers and fractions

1.1 INTEGER ADDITION AND SUBTRACTION

Always write the numbers to be added underneath each other keeping the right hand numbers (units) under each other. Add together 18 m and 7 m. This is written 18 m + 7 m.

1 18 7 + 25	8 + 7 = 15, write down 5 under the 7 and write 1 at the top of the next column on the left (tens column) 1 + 1 = 2, write down the 2. 18 m + 7 m = 25 m.

Add together 6587p, 917p, 77p and 1762p.

222 6587 917 77 1762 + 9343	Always add the numbers UP 2 + 7 + 7 + 7 = 23 Check by adding the numbers DOWN 7 + 7 + 7 + 2 = 23 6587p + 917p + 77p + 1762p = 9343p.

Example 1.1 Add together 267 mm and 538 mm

11 267 538 + 805	267 mm + 538 mm = 805 mm

Example 1.2 Find the total volume in litres of 527 ℓ, 1863 ℓ, 62 ℓ, and 2791 ℓ

221 527 1863 62 2791 + 5243	527 ℓ + 1863 ℓ + 62 ℓ + 2791 ℓ = 5243 ℓ

Take 483 g from 972 g, or subtract 483 g from 972 g or 972 g – 483 g

Method 1 The borrow method.

972 483 –	2 – 3; since 2 is less than 3 borrow 1 from the 7 in the tens column to make the 2 up to 12. 12 – 3 = 9
6 12 9~~7~~~~2~~ 483 – 9	6 – 8; since the 6 is less than 8 borrow 1 from the 9 in the hundreds column to make the 6 up to 16. The 9 in the hundreds column then becomes 8. 16 – 8 = 8
16 8 ~~6~~ 12 ~~9~~~~7~~~~2~~ 483 – 489	8 – 4 = 4 972 g – 483 g = 489 g.

Method 2 Equal addition method.

972 483 –	2 – 3; since 2 is less than 3 borrow 1 from the tens column to make the 2 up to 12. Pay this back by adding 1 to the 8 in the tens column. 12 – 3 = 9
12 97~~2~~ 9 4~~8~~3 – 9	7 – 9; since 7 is less than 9 borrow 1 from the hundreds column to make the 7 up to 17. Pay this back by adding 1 to the 4 in the hundreds column. 17 – 9 = 8
17 12 9~~7~~~~2~~ 5 9 ~~4~~~~8~~3 – 489	9 – 5 = 4 972 g – 483 g = 489 g.

Always check subtraction by adding the answer to the number being subtracted.

Example 1.3 A tank contains 864 ℓ of petrol. 395 ℓ are run off from the tank. How many litres remain in the tank?

864 ℓ take away 395 ℓ or 864 ℓ – 395 ℓ

Method 1	Check	Method 2
15 7 ~~5~~ 14 ~~8~~~~6~~~~4~~ 395 – 469	11 395 469 + 864	16 14 8~~6~~~~4~~ 4 10 ~~3~~~~9~~5 – 469

469 ℓ remain in the tank.

The mass of a bolt is 321 g and the mass of a nut is 117 g. The mass of the nut and the bolt is 438 g.

We can write this 321 g + 117 g = 438 g

or 117 g + 321 g = 438 g

In addition, the order of the numbers does not matter. If we now add a washer to the nut and bolt and the mass of the washer is 23 g, the mass of the bolt, nut and washer is 461 g.

We can write this 321 g + 117 g + 23 g = 461 g

or 438 g + 23 g = 461 g

or 321 g + 140 g = 461 g

again the order does not matter in addition.

321 g + 117 g + 23 g = 438 g + 23 g = 321 g + 140 g = 461 g

Example 1.4 Mr Davies has a bank account with a balance of £734. He withdraws £176 and pays in £97. The next week he pays in £137 and withdraws £207. What is the balance of the account after these transactions?

The sum to be worked out is

£734 – £176 + £97 + £137 – £207.

Since we have shown above that the order of addition does not matter we can write this

£734 + £97 + £137 – £176 – £207

which is the same as £(734 + 97 + 137) – £(176 + 207)

1 1		816
734		
97	176	9̸6̸8
137 +	207 +	383 –
968	383	585

The balance at the bank is £585 after the transactions.

EXERCISE 1.1

1. Find the total sum of money received if you are given £17, £25, £117 and £39.
2. A component is made up from five metal bars of lengths 169 mm, 95 mm, 235 mm, 8 mm and 195 mm. Find the total length of the metal bar in the component.
3. Four resistors are placed in series. Their values are 807 Ω, 86 Ω, 284 Ω and 1127 Ω. Calculate the total resistance, this is found by adding these four resistances.
4. The dimensions of a shaft used in an electric motor are shown in fig. 1.1. Calculate the total length of the shaft, all dimensions are in mm.

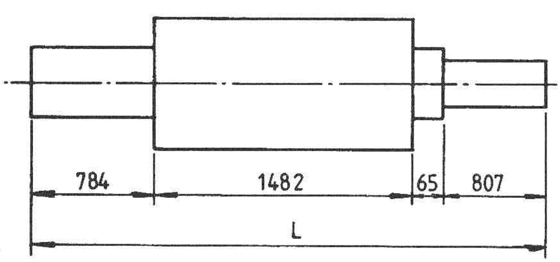

Figure 1.1

5. The following sums of money are deposited in a bank account £1976, £385, £789, £1162 and £4197. Find the total sum of money deposited in the bank account.
6. Six tanks of oil contain 179 ℓ, 1872 ℓ, 97 ℓ, 448 ℓ, 2897 ℓ and 277 ℓ. Calculate the total volume of oil in the tanks.
7. The number of hours worked each week over a period of four weeks by four people are shown in the table below. Find the total number of hours worked by all four people. Check your total by adding up the columns and finding the total.

Week No.	1	2	3	4	Total
Mr Smith	43	38	44	37	
Mr Jones	37	38	36	41	
Mr Brown	38	38	42	36	
Mrs Evans	44	39	40	37	
Total					

8. Mr Jones earns £367 in one month. He pays £98 tax. How much has he left?
9. A component consists of two parts and has a mass of 564 kg. One part has a mass of 273 kg. What is the mass of the other part?
10. The dimensions of a shaft are shown in fig. 1.2. Calculate the dimension L and the dimension x. All of the dimensions are in mm.

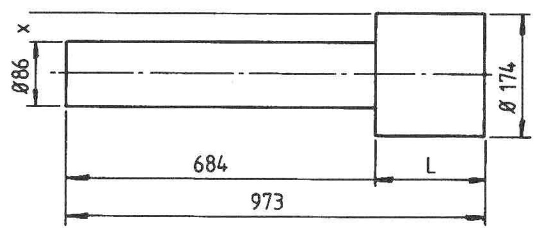

Figure 1.2

11. Two resistors are in series. The total resistance is 1584 Ω. One resistance is 795 Ω. Calculate the size of the other resistor.
12. A tank holds 8756 litres of water when full. If 3767 litres are run off, how much water remains in the tank?
13. The three angles in a triangle add up to 180°. Two angles add up to 137°. Find the size of the third angle.
14. A machine costs £2875 to make. It is sold for £4050. Calculate the profit.
15. The angles of a four sided figure add up to 360°; three of the angles are 74°, 97°, and 101°. Find the fourth angle.
16. An electric motor is sold for £428. The selling price is made up of parts, labour, profit and V.A.T. The cost of the parts is £87, the cost of labour is £176 and the profit is £109. Find the amount of V.A.T.
17. In fig. 1.3 the dimensions are in mm. Calculate the dimensions L, x and d.
18. Five resistors are in series. The total resistance is 1200 Ω. Four of the five resistors have resistances 284 Ω, 278 Ω, 98 Ω and 176 Ω. Find the resistance of the other resistor.
19. A tank contains 9000 litres of petrol. On Monday 1234 ℓ are used, on Tuesday 785 ℓ are used, on Wednesday 1725 ℓ are used and on Friday 984 ℓ are used. How many litres of petrol are left in the tank?

(a)

(b)

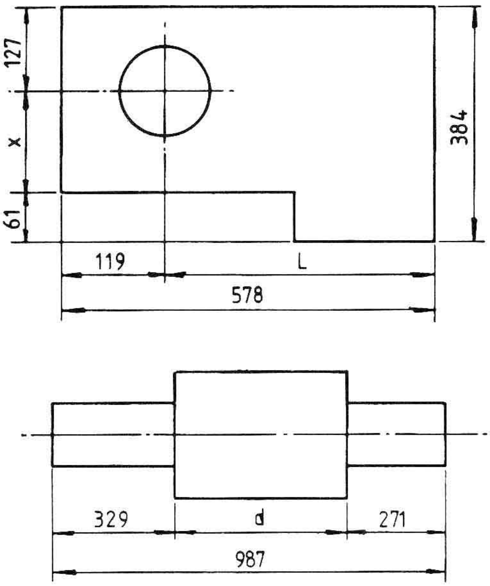

Figure 1.3

1.2 INTEGER MULTIPLICATION AND DIVISION

Multiply 672 m by 9. This is written 672 m x 9
The answer is called the product of these two numbers.

672	9 x 2 = 18; write down 8 and carry 1
9 x	9 x 7 = 63 add 1 carried from units column.
6048	This makes 64; write down 4 and carry 6.
6 1	9 x 6 = 54 add 6 makes 60
	672 m x 9 = 6048 m.

For larger numbers long multiplication is used.
Multiply 78 km by 34

78	Write down 0 under the 4
34 x	multiply 78 by 3 = 234
2340	multiply 78 by 4 = 312
312	
2652	78 km x 34 = 2652 km

Example 1.5 48 boxes are loaded into a van. Each box has a mass of 128 kg. Find the total mass of the 48 boxes.

128	
48 x	
5120	
1024	
6144	128 kg x 48 = 6144 kg

Example 1.6 A rectangular metal plate measures 738 mm by 327 mm. Find the area of the metal plate.

The area of the metal plate = 738 mm x 327 mm: the answer will be in mm x mm = mm^2

738	
327 x	
221400	
14760	
5166	
241326	The area of the metal plate is 241 326 mm^2

When numbers have more than four digits they are written with a half space between every three digits. Hence 241 362 has a half space between the 1 and the 3.

In multiplication it can be seen that 738 x 327 is the same as 327 x 738. The order of multiplication does not matter. The answer is the same.

Also 7 x 8 x 3 = 56 x 3 = 7 x 24 = 168
That is 7 x 8 x 3 = (7 x 8) x 3 = 7 x (8 x 3) = 168

Example 1.7 An electric fire is rated at 2250 W. It is used for 7 hours on Monday, 9 hours on Tuesday, 5 hours on Wednesday, 7 hours on Thursday and 9 hours on Friday.
How much electricity, in W hours is used in the five days?

Monday it uses	2250 x 7 = 15 750 W hours
Tuesday it uses	2250 x 9 = 20 250 W hours
Wednesday it uses	2250 x 5 = 11 250 W hours
Thursday it uses	2250 x 7 = 15 750 W hours
Friday it uses	2250 x 9 = 20 250 W hours
	Total 83 250 W hours

The total amount of electricity used can also be found by adding the hours to find the total hours. Multiplying the total by the rating of the fire.
2250 x (7 + 9 + 5 + 7 + 9) = 2250 x 37

2250	
37 x	
67500	
15750	
83250	The total electricity used is 83 250 W hours

It can be seen that (2250 x 7) + (2250 x 9) + (2250 x 5) + (2250 x 7) + (2250 x 9) = 2250 x (7 + 9 + 5 + 7 + 9) = 83 250 W hours.

£36 is to be divided between 4 men. Each man will receive £9

4 | 36	divisor | dividend
9	quotient

If the numbers are large long division is used. £79 571 is to be divided between 34 men.

```
   2340
34|79571
   68
   115
   102
    137
    136
     11
```

34 into 79 goes 2 (34 x 2 = 68). Write down 2 above the 9 and write 68 under 79. 79 - 68 = 11. Bring down the 5 to make 115. 34 into 115 goes 3 (34 x 3 = 102). Write down 3 above the 5 and write 102 under 115. 115 - 102 = 13. Bring down the 7 to make 137. 34 into 137 goes 4 (34 x 4 = 136). Write down 4 above the 7 and write 136 under 137. 137 - 136 = 1. Bring down the 1 to make 11. Write down 0 above the 1. The remainder is 11. Each man will receive £2340 and there will be £11 left over.

Example 1.8 A roll of electric cable is 750 m long. How many 23 m lengths can be cut from the cable?
750 m ÷ 23

```
   32
23|750
   69
    60
    46
    14
```

32 lengths of cable 23 m long can be cut from the roll and 14 m is left over.

EXERCISE 1.2

1. A casting has a mass of 32 kg. What is the mass of 53 such castings?
2. An electric motor costs £394. Find the cost of 36 such motors.
3. 29 resistors are placed in series. Each resistor has a resistance of 195Ω. Find the total resistance.
4. A rectangular field is 278 m long and 194 m wide. Find the area of the field in m^2, given that area = length x width.
5. A screw has a mass of 17 g. What is the mass of 925 such screws?
6. A forging has a mass of 178 kg. A lorry delivers 15 forgings to one firm, 23 to another firm and 19 to a third firm. Find the total mass of these forgings.
7. A firm buys a fleet of 28 cars. Each car costs £4786. Find the total cost of these cars.
8. A steel bar is 2654 mm long. How many lengths each 21 mm long can be cut from the bar?
9. Mr Jones receives £7858 for a 52 week year. How many pounds does he receive each week?
10. A circuit has 17 equal resistors connected in series. The total resistance is 2669Ω. Find the resistance of each resistor.
11. 93 identical castings have a total mass of 10 974 kg. What is the mass of each casting?
12. How many 16 ℓ cans can be filled from a tank containing 17 857 ℓ of oil?
13. A firm pays £17 901 for 27 identical electric motors. How much does each motor cost?
14. A box of bolts has a mass of 2898 g. The box has a mass of 317 g and each bolt has a mass of 29 g. How many bolts are in the box?

1.3 NEGATIVE INTEGERS

Negative integers are integers with a negative sign attached to them, for example −7. In the previous work with integers no sign has been attached because they have all been positive integers. With positive integers the + sign is usually omitted.

Negative numbers are always difficult to understand. +£3 means you have £3 and (−£3) means you owe £3.

+ have £3	=	have £3
+ owe £3	=	owe £3
− owe £3	=	have £3
− have £3	=	owe £3

That is ++ leads to +
+− leads to −
−− leads to +
−+ leads to −

For example
$6 + 3 = 9; 6 + (-3) = 6 - 3 = 3$
$6 - 3 = 3; (-6) + 3 = -3$
$(-6) + (-3) = -9; 6 - (-3) = 6 + 3 = 9$
$(-6) - 3 = -6 - 3 = -9; (-6) - (-3) = -6 + 3 = -3$
$6 \times 3 = 18; 6 \times (-3) = -18$
$(-6) \times 3 = -18; (-6) \times (-3) = 6 \times -(-3) = 6 \times 3 = 18$
$6 \div 3 = 2; 6 \div (-3) = -2$
$(-6) \div 3 = -2; (-6) \div (-3) = 2$

Example 1.9 Find the value of the following
(a) $(-5) - (-6)$, (b) $(-5) \times 6$, (c) $(-3) \times (-8)$, (d) $(-12) \div (-4)$, (e) $(-18) \times (-6) \div 3$, (f) $(-16) \times (-3) \div (-4)$

(a) $(-5) - (-6) = (-5) + 6 = 1$
(b) $(-5) \times 6 = -30$
(c) $(-3) \times (-8) = 24$
(d) $(-12) \div (-4) = 3$
(e) $(-18) \times (-6) \div 3 = 108 \div 3 = 36$
(f) $(-16) \times (-3) \div (-4) = 48 \div (-4) = -12$

EXERCISE 1.3

Find the values of the following

1. $+9+5$ 2. $-8-3-5$ 3. $13-7$ 4. $-7+12-3$ 5. $16-8-3$ 6. $9-(+5)$ 7. $-4-(-9)$ 8. $9-(-7)$ 9. $-3-(+4)$ 10. $6 \times (-7)$ 11. $(-5) \times 9$ 12. $(-5) \times (-7)$ 13. $4 \times (-5) \times (-3) \times 6$ 14. $8 \div (-2)$ 15. $(-8) \div 4$ 16. $(-16) \div (-2)$
17. $\dfrac{(-4) \times 6}{-3}$ 18. $\dfrac{6 \times (-5) \times 3}{3 \times 10}$

1.4 LOWEST COMMON MULTIPLES (L.C.M.) AND HIGHEST COMMON FACTOR (H.C.F.)

3, 7 and 5 will all divide into 105. 105 is the smallest number that 3, 7 and 5 will divide into. It is called the lowest common multiple of these numbers. It is found by multiplying 3 by 7 by 5.
The L.C.M. of 3, 6 and 8 is 24. This is not 3 x 6 x 8 since 3 divides into 3 and 6 and 2 divides into 6 and 8. The L.C.M. can be found as follows

3	368
2	128
	114

Divide by 3. 3 into 3 goes 1, 3 into 6 goes 2, 3 will not divide into 8; hence the 8 is left. Divide by 2. 2 will not divide into 1; hence leave the 1, 2 into 2 goes 1, 2 into 8 goes 4.
$3 \times 2 \times 1 \times 1 \times 4 = 24$

Example 1.10 Find the L.C.M. of 18, 30, 42 and 48

6	18	30	42	48
	3	5	7	8

$6 \times 3 \times 5 \times 7 \times 8 = 5040$
The L.C.M. of 18, 30, 42 and 48 is 5040.

The largest number which will divide into 12, 18 and 24 is 6. 6 is called the highest common factor (H.C.F.) of 12, 18 and 24.

Example 1.11 Find the H.C.F. of 42, 126, 210 and 294

2	42	126	210	294
3	21	63	105	147
7	7	21	35	49
	1	3	5	7

The H.C.F. is $2 \times 3 \times 7 = 42$
Note: the factors 2, 3, 7 must divide into all the numbers.

EXERCISE 1.4

Find the L.C.M. of each of the following sets of numbers:

1. 2, 3 and 5 2. 6, 8 and 10 3. 20 and 25 4. 10, 15 and 40 5. 12, 42, 60 and 70 6. 21, 63, 105 and 147

Find the H.C.F. of each of the following sets of numbers:

7. 2, 8 and 20 8. 15, 25 and 40 9. 14, 35 and 42 10. 12, 30 and 42 11. 42, 210 and 294 12. 52, 260, 338 and 364

1.5 FRACTIONS

$\frac{1}{2}$ is a fraction. The 2 tells us that the whole has been divided into 2 equal parts. The 1 tells us that we are considering 1 part. Thus $\frac{3}{4}$ means the whole has been divided into 4 equal parts and we are considering 3 of these parts. A fraction always consists of two numbers

$$\text{Fraction} = \frac{\text{Numerator (top)}}{\text{Denominator (bottom)}}$$

$\frac{1}{2}$ means 1 whole divided into 2 equal parts. This is the same as 2 wholes divided into 4 equal parts $\frac{2}{4}$ or 3 divided into 6 equal parts $\frac{3}{6}$.

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$$

That is the fraction remains the same if the top and bottom numbers of the fraction are multiplied or divided by the same number.

$$\frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2};\frac{1}{6} = \frac{1 \times 3}{6 \times 3} = \frac{3}{18}$$

The fraction can be divided top and bottom by any number. It cannot be divided or multiplied by 0.

$\frac{1}{2}$ does not equal $\frac{1 \times 0}{2 \times 0}$; $1 \times 0 = 0$ and $2 \times 0 = 0$.

A fraction $\frac{0}{0}$ does not have a value that can be determined.

If the numerator is less than the denominator the fraction is called a proper fraction. The value of the fraction is less than 1 whole.
If the denominator is less than the numerator the fraction is called an improper fraction. The value of the fraction is greater than 1 whole.
$\frac{3}{4}$ is a proper fraction. $\frac{4}{3}$ is an improper fraction.

Example 1.12 Reduce $\frac{42}{210}$ to its lowest terms

$$\frac{42}{210} = \frac{21 \times 2}{105 \times 2} = \frac{21}{105}$$

$$\frac{21}{105} = \frac{7 \times 3}{35 \times 3} = \frac{7}{35}$$

$$\frac{7}{35} = \frac{1 \times 7}{5 \times 7} = \frac{1}{5}$$

Hence $\frac{42}{210}$ reduced to its lowest terms is $\frac{1}{5}$.

Example 1.13 (a) Express as a mixed number $\frac{27}{8}$ and (b) express $2\frac{3}{7}$ as an improper fraction.

(a) $\frac{27}{8} = \frac{24 + 3}{8} = \frac{24}{8} + \frac{3}{8} = 3\frac{3}{8}$

(b) $2\frac{3}{7} = \frac{14}{7} + \frac{3}{7} = \frac{17}{7}$

EXERCISE 1.5

Reduce the following fractions to their lowest terms.

1. $\frac{16}{24}$ 2. $\frac{21}{35}$ 3. $\frac{26}{65}$ 4. $\frac{132}{198}$ 5. $\frac{210}{2310}$

Express the following fractions as mixed numbers.

6. $\frac{8}{5}$ 7. $\frac{17}{5}$ 8. $\frac{23}{6}$ 9. $\frac{27}{11}$ 10. $\frac{31}{9}$

Express the following as improper fractions.

11. $1\frac{1}{3}$ 12. $1\frac{3}{7}$ 13. $2\frac{5}{8}$ 14. $6\frac{7}{10}$ 15. $5\frac{3}{7}$

1.6 ADDITION AND SUBTRACTION OF FRACTIONS

To add and subtract fractions they must be changed to the same kind. That is the denominator must be the same in each fraction.

Example 1.14 Add together the fractions $\frac{5}{6}$, $\frac{2}{9}$ and $\frac{5}{12}$.
We first find the smallest number (L.C.M.) that 6, 9 and 12 will divide into.

$$\begin{array}{r|l} 3 & 6\ 9\ 12 \\ \hline 2 & 2\ 3\ 4 \\ \hline & 1\ 3\ 2 \end{array} \qquad 3 \times 2 \times 1 \times 3 \times 2 = 36$$

$$\frac{5}{6}+\frac{2}{9}+\frac{5}{12}=\frac{5\times 6}{6\times 6}+\frac{2\times 4}{9\times 4}+\frac{5\times 3}{12\times 3}=\frac{30}{36}+\frac{8}{36}+\frac{15}{36}$$

$$=\frac{53}{36}=1\frac{17}{36}$$

Example 1.15 Subtract $\frac{3}{16}$ from $\frac{5}{12}$

$$\begin{array}{r|l} 4 & 16\ \ 12 \\ \hline & \ 4\ \ \ 3 \end{array} \qquad 4 \times 4 \times 3 = 48$$

The L.C.M. of 16 and 12 is 48

$$\frac{5}{12}-\frac{3}{16}=\frac{5\times 4}{12\times 4}-\frac{3\times 3}{16\times 3}=\frac{20}{48}-\frac{9}{48}=\frac{11}{48}$$

If fractions and whole numbers are involved the whole numbers must be added and subtracted first. The fractions are then added and subtracted. If the final fraction is greater than 1 reduce it to a fraction less than 1 and add the whole ones to the whole number part. If the final fraction is negative borrow from the whole number part to make the fraction positive.

Example 1.16 Add $\frac{5}{8}$ to $5\frac{2}{3}$ and subtract $3\frac{3}{4}$

$$\frac{5}{8}+5\frac{2}{3}-3\frac{3}{4}=2+\frac{5}{8}+\frac{2}{3}-\frac{3}{4}=2+\frac{15}{24}+\frac{16}{24}-\frac{18}{24}$$

$$=2+\frac{13}{24}=2\frac{13}{24}$$

Example 1.17 Add $\frac{1}{12}$ to $3\frac{1}{4}$ and subtract $1\frac{5}{6}$

$$\frac{1}{12}+3\frac{1}{4}-1\frac{5}{6}=2+\frac{1}{12}+\frac{1}{4}-\frac{5}{6}=2+\frac{1}{12}+\frac{3}{12}-\frac{10}{12}$$

$$=2-\frac{6}{12}=1+\frac{12}{12}-\frac{6}{12}=1+\frac{6}{12}$$

$$=1\frac{6}{12}=1\frac{1}{2}$$

Example 1.18 Two resistors of 3 Ω and 7 Ω are connected in parallel. Find the total resistance. If the resistors are R_1 and R_2 and the total resistance R then

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$$

$$\frac{1}{\text{total resistance}}=\frac{1}{3}+\frac{1}{7}=\frac{7}{21}+\frac{3}{21}=\frac{10}{21}$$

$$\frac{1}{\text{total resistance}}=\frac{10}{21}$$

$$\text{Total resistance}=\frac{21}{10}=2\frac{1}{10}$$

EXERCISE 1.6

Reduce the following to a single fraction.

1. $\frac{1}{2}+\frac{1}{4}+\frac{3}{8}$ 2. $\frac{2}{3}+\frac{1}{9}+\frac{5}{12}$ 3. $\frac{1}{10}+\frac{7}{10}+\frac{9}{50}+\frac{1}{5}$ 4. $\frac{5}{8}-\frac{3}{8}$

5. $\frac{1}{3}-\frac{5}{16}$ 6. $\frac{3}{4}-\frac{1}{6}$

Work out the following.

7. $1\frac{3}{4}+1\frac{1}{2}+3\frac{2}{3}$ 8. $2\frac{3}{8}-1\frac{1}{6}$ 9. $\frac{3}{4}+4\frac{1}{3}-\frac{3}{16}$ 10. $5\frac{1}{6}-2\frac{7}{8}$

11. $2\frac{3}{16}-2\frac{3}{10}+\frac{5}{8}$ 12. $3\frac{9}{20}+1\frac{3}{8}-2\frac{7}{10}+1\frac{3}{5}$

13. A steel bar is 5 m long. Three lengths $1\frac{3}{4}$ m, $1\frac{1}{8}$ m and $1\frac{1}{2}$ m are cut from the bar. What length of steel bar is left?

14. Two resistors of 2 Ω and 5 Ω are connected in parallel. Find the total resistance.

15. On Monday Mr Brown has $12\frac{1}{2}$ ℓ of petrol in his petrol tank. On Tuesday he uses $5\frac{1}{4}$ ℓ of petrol. On Wednesday he uses $3\frac{7}{10}$ ℓ of petrol. On Thursday he puts $27\frac{3}{4}$ ℓ of petrol in the tank and uses $10\frac{3}{10}$ ℓ of petrol. On Friday he uses $7\frac{1}{5}$ ℓ of petrol. How much petrol is left in the tank?

16. Three resistors of 3 Ω, 8 Ω and 6 Ω are connected in parallel. Find the total resistance.

1.7 MULTIPLICATION AND DIVISION OF FRACTIONS

When a fraction is multiplied by another fraction the numerators are multiplied together and the denominators are multiplied together.

$$\frac{1}{2}\times\frac{3}{4}=\frac{1\times 3}{2\times 4}=\frac{3}{8}$$

Whole numbers and fractions must be written as improper fractions first.

$$1\frac{7}{8}=\frac{15}{8}$$

Whole numbers are treated as whole numbers over 1.

$$8=\frac{8}{1}$$

When a fraction is divided by another fraction, the second fraction (or divisor) is turned upside down and multiplied by the first fraction.

$$\frac{3}{7} \div \frac{1}{2} = \frac{3}{7} \times \frac{2}{1} = \frac{6}{7}$$

In multiplication and division of fractions common factors at the top and bottom can be cancelled.

$$\frac{3}{4} \times \frac{8}{9} = \frac{\cancel{3}^{1} \times 1}{\cancel{4}_{1} \times 1} \times \frac{\cancel{4}^{1} \times 2}{\cancel{3}_{1} \times 3} = \frac{1 \times 1 \times 1 \times 2}{1 \times 1 \times 1 \times 3} = \frac{2}{3}$$

This is usually written $\frac{\cancel{3}^{1}}{\cancel{4}_{1}} \times \frac{\cancel{8}^{2}}{\cancel{9}_{3}} = \frac{2}{3}$

Sometimes 'of' is used for multiplication. $\frac{3}{4}$ of $\frac{8}{9}$ means $\frac{3}{4} \times \frac{8}{9}$.

Example 1.19 Find the value of $\frac{3}{4} \times \frac{5}{6} \times \frac{2}{7}$

$$\frac{\cancel{3}^{1}}{\cancel{4}_{2}} \times \frac{5}{\cancel{6}_{2}} \times \frac{\cancel{2}^{1}}{7} = \frac{1 \times 5 \times 1}{2 \times 2 \times 7} = \frac{5}{28}$$

Example 1.20 Find the value of $\frac{3}{5} \div 3$

$$\frac{3}{5} \div 3 = \frac{3}{5} \div \frac{3}{1} = \frac{\cancel{3}^{1}}{5} \times \frac{1}{\cancel{3}_{1}} = \frac{1}{5}$$

Example 1.21 Find the value of $1\frac{1}{3} \times 3\frac{1}{8} \div 4\frac{1}{6}$

$$1\frac{1}{3} \times 3\frac{1}{8} \div 4\frac{1}{6} = \frac{4}{3} \times \frac{25}{8} \div \frac{25}{6} = \frac{\cancel{4}^{1}}{\cancel{3}_{1}} \times \frac{\cancel{25}^{1}}{\cancel{8}_{\cancel{2}_{1}}} \times \frac{\cancel{6}^{\cancel{2}^{1}}}{\cancel{25}_{1}} = \frac{1}{1} = 1$$

EXERCISE 1.7

Find the value of the following

1. $\frac{1}{3} \times \frac{2}{7} \times \frac{1}{5}$ 2. $\frac{3}{4}$ of 5 3. $\frac{7}{12} \times \frac{3}{14} \times \frac{4}{5}$ 4. $\frac{1}{7} \div \frac{4}{5}$ 5. $\frac{3}{5} \div \frac{2}{7}$

6. $\frac{3}{4} \times \frac{1}{3} \div \frac{1}{2}$ 7. $2\frac{3}{4} \times 2\frac{2}{3}$ 8. $5\frac{2}{5} \div 1\frac{4}{5}$ 9. $3\frac{1}{5} \times 3\frac{1}{8} \times 1\frac{3}{4}$

10. $3\frac{3}{5} \times 2\frac{1}{2} \div \frac{3}{4}$

1.8 RATIO AND PROPORTION

A metal bar is cut into two parts. One part is twice as long as the other. We say the bar has been cut into two parts in the ratio 2:1. If the bar is 6 m long it is cut into one part 4 m and one part 2 m. This is the same as the fraction $\frac{1}{2}$.

As with fractions both parts of the ratio can be multiplied by the same quantity and the ratio remains the same.
1:2 is the same as 3:6 or 5:10.

The density of lead is approximately 11 g/cm^3. This means that 1 cm^3 has a mass of 11 g. 5 cm^3 has a mass of 55 g.

The mass is proportional to the volume. As the volume increases the mass increases in the same proportion. The ratio of the volume is 1:5 ($\frac{1}{5}$), the ratio of the mass is 11:55 ($\frac{1}{5}$).

Sometimes the proportion is reversed, called inverse proportion. If an aircraft travels at 300 km/h it will take 2 hours to travel 600 km. If the speed is doubled the time is halved. The ratio of the speed is 1:2, the ratio of the time is 2:1.

Example 1.22 The electrical resistance of a wire 25 cm long is 3 Ω. Find the resistance of a similar piece of wire 100 cm long.
The resistance of the wire is proportional to its length. Hence the resistance of the wire is in the same ratio as the lengths of wire.
The ratio of the lengths of wire is 25:100 or 1:4
The resistance of 100 cm of wire $= 3 \times \frac{4}{1} = 12\ \Omega$

Example 1.23 An alloy has a composition by mass of 85 parts copper, 10 parts tin and 5 parts zinc. Find the mass of each metal in 2000 g of the alloy.
The metals are in the ratio 85:10:5. There are 85 + 10 + 5 parts or 100 parts. The metals are therefore in the ratio $\frac{85}{100} : \frac{10}{100} : \frac{5}{100}$ or $\frac{17}{20} : \frac{2}{20} : \frac{1}{20}$

Thus in 2000 g the mass of copper $= \frac{17}{20} \times 2000 = 1700$ g
the mass of tin $= \frac{2}{20} \times 2000 = 200$ g
the mass of zinc $= \frac{1}{20} \times 2000 = 100$ g

The alloy consists of 1700 g of copper, 200 g of tin and 100 g of zinc.

EXERCISE 1.8

1. A drawing is made to scale of 1:50. Find the length of a component which is shown on the drawing as 25 mm.
2. Divide 75 kg in the ratio 2:3.
3. A gear wheel has 45 teeth and revolves at 150 rev/min. It meshes with a wheel having 27 teeth, find the speed of this wheel.
4. A resistance wire of length 10 m has a resistance of 5 Ω. If the resistance is proportional to its length find the resistance of
(i) a wire 30 m long
(ii) a wire 2 m long
5. A special cutting oil is to be made up of oils A and B in the ratio 3:1. If 40 litres of cutting oil are required, calculate the

quantities of each oil needed.
6. An alloy has a composition by mass of 75 parts copper, 15 parts tin and 10 parts zinc. Find the mass of each metal in 1000 g of the alloy.
7. A right angled triangle has sides in the ratio 3:4:5. If the shortest side is 15 m long find the lengths of the other two sides.
8. A resistance wire of length 3 m has a resistance of 33 Ω. Find the length of wire needed for a resistance of 77 Ω.

1.9 GENERAL EXAMPLES

The order of working out mixed problems involving +, −, x, and ÷ will be clear if units are used. If the order of evaluation is not clear then brackets should be used. The calculations in the brackets are then worked out first. Otherwise x and ÷ should be worked out first and + and − last.

$7 + 3 \times 5 = 7 + 15 = 22$ but $(7 + 3) \times 5 = 50$

Example 1.24 Find the value of $\dfrac{3\frac{2}{5} - 3\frac{1}{2} \times \frac{2}{3}}{1 - (\frac{5}{8} \times \frac{7}{15})}$

$$3\tfrac{1}{2} \times \tfrac{2}{3} = \tfrac{7}{\not 2_1} \times \tfrac{\not 2^1}{3} = \tfrac{7}{3} = 2\tfrac{1}{3}$$

$$\tfrac{5}{8} \times \tfrac{7}{15} = \tfrac{7}{24}$$

Hence $\dfrac{3\frac{2}{5} - 3\frac{1}{2} \times \frac{2}{3}}{1 - (\frac{5}{8} \times \frac{7}{15})} = \dfrac{3\frac{2}{5} - 2\frac{1}{3}}{1 - \frac{7}{24}} = \dfrac{1 + \frac{2}{5} - \frac{1}{3}}{\frac{24}{24} - \frac{7}{24}}$

$$= \dfrac{1 + \frac{6}{15} - \frac{5}{15}}{\frac{17}{24}} = \dfrac{1\frac{1}{15}}{\frac{17}{24}} = 1\tfrac{1}{15} \div \tfrac{17}{24} = \tfrac{16}{\not{15}_5} \times \tfrac{\not{24}^8}{17} = \tfrac{128}{85}$$

$$= 1\tfrac{43}{85}$$

EXERCISE 1.9

1. Four resistors are placed in series. Their values are 197 Ω, 89 Ω, 346 Ω and 1385 Ω. Calculate the total resistance.
2. A machine is sold for £9875 and the profit was £1788. Find the cost of making the machine.
3. Calculate the dimension x in fig. 1.4, all dimensions are in mm.
4. A casting has a mass of 177 kg. What is the mass of 47 such castings?
5. How many 27 ℓ cans can be filled from a tank containing 29 185 ℓ of petrol?
6. Find the H.C.F. and the L.C.M. of 12, 36, 42 and 60.
7. (a) Express $\frac{60}{16}$ as a mixed number in its lowest terms.

(b) Express $4\frac{7}{8}$ as an improper fraction.

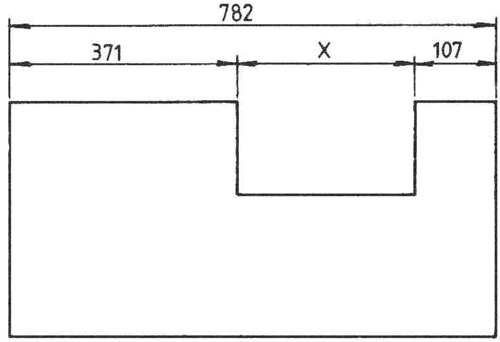

Figure 1.4

8. Work out the following
(a) $6\frac{7}{8} - 3\frac{3}{4} + 5\frac{3}{16}$ (b) $5\frac{2}{3} + 3\frac{5}{6} - 4\frac{7}{12}$
9. Three resistors are connected in parallel, their values are 6 Ω, 5 Ω and 3 Ω. Find the total resistance.
10. Find the value of
(a) $4\frac{7}{8} \times 3\frac{2}{3} \div 2\frac{3}{4}$ (b) $\frac{11}{12}$ of 14
11. If 12 men working 7 hours a day complete an order in $5\frac{1}{2}$ days, calculate the number of men required to complete the same order in 7 days working at a rate of 6 hours a day.
12. Find the value of
(a) $\dfrac{4\frac{3}{4} - 2\frac{1}{2} \div \frac{5}{8}}{\frac{5}{7} \times 2\frac{4}{5}}$ (b) $5\frac{7}{8} - 1\frac{3}{10} \times 2\frac{1}{2} + \frac{3}{4}$ of 7

SUMMARY

1. With addition and subtraction always write the numbers underneath each other keeping the right hand digits (the units) under each other.
2. Addition and multiplication can be worked out in any order, the answer is the same e.g. $7 + (6 + 5) = (7 + 6) + 5$; $7 \times (6 \times 5) = (7 \times 6) \times 5$; $7 + 6 = 6 + 7$; $7 \times 6 = 6 \times 7$.
3. The value of a fraction remains the same if both the numerator and the denominator are multiplied or divided by the same number.
4. The L.C.M. of a set of numbers is the smallest number into which each of the numbers of the set will divide exactly.
5. To add or subtract fractions express each fraction with their lowest common denominator and then add the resulting numerators.
6. To multiply fractions multiply the numerators together and then multiply the denominators together.
7. To divide fractions, invert the divisor and then proceed as in multiplication.
8. The H.C.F. of a set of numbers is the greatest number which is a factor of each of the numbers.

9. The sequence of operations in arithmetic is: (a) brackets, (b) multiply and divide, (c) add and subtract.
10. Two quantities are in direct proportion if they increase or decrease at the same rate.
11. Two quantities are inversely proportional if one is doubled the other is halved etc.

SELF ASSESSMENT PAPER No. 1

Instructions: Section A. Answer ALL questions.
Section B. Answer fully.

Time allowed: Section A. 20 minutes (2 marks each question)
Section B. 20 minutes (10 marks each question)
Total 40 minutes

Marks gained: 20+ pass with credit, 16–20 pass, less than 16 fail, repeat chapter 1.

Section A.

Find:

1. 237 + 491 + 37 + 5842 2. 3957 – 1488
3. 1972 – 785 + 37 – 84 + 192 4. 194 x 73
5. 18473 ÷ 43 6. The H.C.F. of 84, 72, 168
7. The L.C.M. of 14, 42, 84
8. $\frac{92}{16}$ as a mixed number in its lowest terms
9. $9\frac{3}{8} - 5\frac{1}{4} + 2\frac{1}{16}$ 10. $3\frac{3}{8} \times 2\frac{2}{3} \div 1\frac{1}{4}$

Section B

1. (a) The cost of electricity is calculated on a two part tariff. The first 85 units cost $3\frac{1}{2}$p a unit. The remainder cost $\frac{3}{4}$p a unit. Calculate the cost of 500 units.
(b) Two resistors 3 Ω and 2 Ω are connected in parallel. Find the total resistance.
2. (a) A metal plate has to be riveted along an edge 1199 mm long. The distance between the rivet centres is to be 43 mm and the distance between the centres of the rivet and the edge of the plates is 19 mm. How many rivets are required?
(b) An alloy consists of 3 parts copper to 2 parts zinc by mass. Find the mass of copper and zinc in a casting of this alloy which has a mass of 70 kg.

ANSWERS

Exercise 1.1 1. £198, 2. 702 mm, 3. 2304 Ω, 4. 3138 mm, 5. £8509, 6. 5770 ℓ, 7. Totals across 162, 152, 154, 160. Totals down 162, 153, 162, 151. Grand total 628. 8. £269, 9. 291 kg, 10. ℓ = 289 mm, x = 44 m, 11. 789 Ω, 12. 4989 ℓ, 13. 43°, 14. £1175, 15. 88°, 16. £56, 17. ℓ = 459 mm, x = 196 mm, d = 387 mm, 18. 364 Ω, 19. 4272 ℓ.

Exercise 1.2 1696 kg, 2. £14184, 3. 5655 Ω, 4. 53932 m^2, 5. 15725 g, 6. 10146 kg, 7. £134008, 8. 126, 9. £151, 10. 157 Ω, 11. 118 kg, 12. 1116, 13. £663, 14. 89.

Exercise 1.3 1. 14, 2. –16, 3. 6, 4. 2, 5. 5, 6. 4, 7. 5, 8. 16, 9. –7, 10. –42, 11. –45, 12. 35, 13. 360, 14. –4, 15. –2, 16. 8, 17. 8, 18. –3.

Exercise 1.4 1. 30, 2. 120, 3. 100, 4. 120, 5. 420, 6. 2205, 7. 2, 8. 5, 9. 7, 10. 6, 11. 42, 12. 26.

Exercise 1.5 1. $\frac{2}{3}$, 2. $\frac{3}{5}$, 3. $\frac{2}{5}$, 4. $\frac{2}{3}$, 5. $\frac{1}{11}$, 6. $1\frac{3}{5}$, 7. $3\frac{2}{5}$, 8. $3\frac{5}{6}$, 9. $2\frac{5}{11}$, 10. $3\frac{4}{9}$, 11. $\frac{4}{3}$, 12. $\frac{10}{7}$, 13. $\frac{21}{8}$, 14. $\frac{67}{10}$, 15. $\frac{38}{7}$.

Exercise 1.6 1. $1\frac{1}{8}$, 2. $1\frac{7}{36}$, 3. $1\frac{9}{50}$, 4. $\frac{1}{4}$, 5. $\frac{1}{48}$, 6. $\frac{7}{12}$, 7. $6\frac{11}{12}$, 8. $1\frac{5}{24}$, 9. $4\frac{43}{48}$, 10. $2\frac{7}{24}$, 11. $\frac{41}{80}$, 12. $3\frac{29}{40}$, 13. $\frac{5}{8}$ m, 14. $1\frac{3}{7}$ Ω, 15. $13\frac{4}{5}$ ℓ, 16. $1\frac{3}{5}$ Ω.

Exercise 1.7 1. $\frac{2}{105}$, 2. $3\frac{3}{4}$, 3. $\frac{1}{10}$, 4. $\frac{5}{28}$, 5. $2\frac{1}{10}$ 6. $\frac{1}{2}$, 7. $7\frac{1}{3}$, 8. 3, 9. $17\frac{1}{2}$, 10. 12.

Exercise 1.8 1. 1250 mm, 2. 30 kg, 45 kg, 3. 250 rev/min, 4. (i) 15 Ω, (ii) 1 Ω, 5. 30 ℓ of A, 10 ℓ of B, 6. 750 g of copper, 150 g of tin, 100 g of zinc, 7. 20 m, 25 m, 8. 7 m.

Exercise 1.9 1. 2017, 2. £8087, 3. 304 mm, 4. 8319 kg, 5. 1080, 6. H.C.F. = 6, L.C.M. = 1260, 7. (a) $3\frac{3}{4}$, (b) $\frac{39}{8}$, 8. (a) $8\frac{5}{16}$, (b) $4\frac{11}{12}$, 9. $1\frac{3}{7}$ Ω, 10 (a) $6\frac{1}{2}$, (b) $12\frac{5}{6}$, 11. 11 men, 12. (a) $\frac{3}{8}$, (b) $7\frac{7}{8}$.