Stress Analysis and Experimental Techniques

An Introduction

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Preface

The intention of this book is to present a brief general introduction to the treatment of stress analysis techniques. Today, stress analysis has become a leading part in Mechanical and Civil Engineering disciplines. A large number of techniques are coming day-by-day in Applied Mechanics for measurement of stresses and strains. Still newer approaches are being invented. As neither of the experimental stress analysis techniques is purely experimental, an emphasis on basic theoretical concepts of solid mechanics is necessary. Basic course of solid mechanics covers the related topics like shear force and bending moment diagrams, torsion, buckling behaviour of columns and pressure vessel theory. In fact, there are several principles used for measurement of stresses and strains. These include the methods based on opto-mechanical principles, piezo-electric principles, brittle-lacquers and so on. For example, the techniques like photoelasticity, moiré fringe analysis and interferometry use the principles of opto-mechanics. Here the stress analysis is carried out using the properties of fringe patterns formed.

This book covers the basic principles of solid mechanics along with the commonly used experimental techniques in stress analysis laboratory. There are in total thirteen chapters and four appendices presented in the book.

In the first chapter, basic introduction to stress analysis, simple and complex stresses, graphical and algebraic approaches to obtain principal stresses as well as fundamental concepts of theory of elasticity are presented. Chapter-2 gives a brief introduction to shear force and bending moment diagrams in beams, which are of relative importance in obtaining the deflections and stresses in beams. Chapter-3 presents concepts of bending and shear stresses in beam sections. Chapter-4 describes deflection analysis of beams by various methods including the theorems of reciprocal deflections. Chapter-5 deals with torsional analysis of shafts. It includes study of stresses in circular and noncircular shafts and helical springs. Its final application is to apply the concept for thin walled-sections to know its static characteristics.

Chapter-6 gives basic principles of stability analysis of columns. The peculiar stiffness-based failure mode known as buckling is explained with columns of various end-conditions. Euler and Rankine formulae are generalized. Chapter-7 describes stress analysis of thin, thick and compound cylindrical (spherical) pressure vessels subjected to internal and external pressure loads. The basic concept of curved-beams is also presented. Chapter-8 deals with finite element analysis (numerical) approach to obtain stresses in various members. One-dimensional, two-dimensional triangular and isoparametric elements are explained and some applications are also given. Chapter-9 presents the basic concept of photoelasticity. Types of polariscopes in use and distinction between the isoclinic and isochromatic fringe patterns as well as necessary stress separation techniques are illustrated one after the other. An introduction to three-dimensional photoelasticity is also presented. Chapter-10 begins with basic grid method. Development of moiré fringe analysis and concept of interferometry are presented. Chapter-11 deals with several types of strain gauges in use. Principles of operation of mechanical, opto-mechanical, acoustical, pneumatic, electrical, piezoelectric and optical fiber Bragg grating-based gauges are explained. Chapter-12 is exclusively dedicated to resistance type of strain gauges. Types of resistance gauges and their mounting arrangements for stress measurement are explained. Basic circuits for analysis are also presented. Chapter-13 gives an introduction to brittle-coating technique. Various available types of coatings, definition of state of stress in the coatings and effects of loading conditions like direct loading, relaxation and refrigeration are presented. Brittle-coating failure law and some definitions of constant parameter loci are explained. Appendix-A gives the important theories of failure used in stress analysis work. Appendix-B explains the basic concepts of fracture mechanics necessary in experimental mechanics. Appendix-C presents the some basic computer programs needed in writing finite element analysis codes for stress evaluations. Brief introduction to carry-out some case studies with available equipment in stress analysis lab are provided in appendix-D.

The book aims to cater the needs of many students studying a one-semester stress analysis course. Most of the techniques have been presented very concisely and the book may serve as a brief guide for those in this field. Suggestions for improvement of the book are always cherished. I am thankful to M/s NAROSA Publishing House, New Delhi in bringing this volume in a nice format. I thank all my family members for their help and patience throughout the manuscript preparation.

J. Srinivas

Contents

Chapter 1

Introduction to Stress Analysis

1.1 INTRODUCTION

The basic structure of matter is characterized by non-uniformity and discontinuity with respect to its various subdivisions: molecules, atoms, and subatomic particles. Often it is concerned with matter which is homogeneous and continuously distributed over the volume. In such an assumption, the smallest element cut from the body possesses the same properties as the body. Random fluctuations in the properties of the material are thus of no consequence. In such continuum mechanics approach, the solid elastic materials are treated as though they are continuous media, rather than composed of discrete molecules. Of all the matter, solids resist continuous shear, tension, and compression. In contrast with rigid-body statics and dynamics, which treat the external behavior of bodies, the mechanics of solids is concerned with the relationships of external effect (forces and moments) to internal stresses and strains. Two different approaches used in solid mechanics are the mechanics of materials or elementary theory (also called the technical theory) and the theory of elasticity. The mechanics of materials focuses mainly on approximate solutions of practical problems. On the other hand, the theory of elasticity concerns itself largely with more mathematical analysis to determine the exact stress and strain distributions in a loaded body. External forces acting on a body may be classified as surface forces and body forces. A surface force is of the concentrated type when it acts at a point; a surface force may also be distributed uniformly or nonuniformly over a finite area. Body forces are associated with the mass of a body, rather than its surfaces, and are distributed throughout the volume of a body. Gravitational, magnetic and inertia forces are all body forces. They are specified in terms of force per unit volume. All forces acting on a body, including the reactive forces caused by supports and body forces, are considered to be external forces. Internal forces are the forces that hold together the particles forming the body. In the International System of Units (SI), force is measured in Newton (N). Because the Newton is a small quantity, the kilo-Newton (kN) is often used in practice. The literature dealing with various aspects of solid mechanics is voluminous.

Scope of Treatment

The usual objective of mechanics of materials and theory of elasticity is the examination of the load-carrying capacity of a body from three standpoints: strength, stiffness (deformation characteristics) and stability. These are the laws of forces, the laws of material deformation and the conditions of geometric compatibility. The principal topics under the general heading of mechanics of solids may be summarized as follows:

1. Analysis of the stresses and deformations within a body subject to a prescribed system of forces. This is accomplished by solving the governing equations that describe the stress and strain fields (theoretical stress analysis). It is often advantageous, where the shape of the structure or conditions of loading preclude a theoretical solution or where verification is required, to apply the laboratory techniques of experimental stress analysis.
2. Determination by theoretical analysis or by experiment of the limiting values of load that a structural element can sustain without suffering damage, failure, or compromise of function.
3. Determination of the body shape and selection of the materials which are most efficient for resisting a prescribed system of forces under specified conditions of operation like temperature, vibration, and ambient pressure.

A design factor of safety commonly heard in 'mechanics of material', reflects the consequences of failure; for example, the possibility that failure will result in loss of human life or injury or in costly repairs or danger to other components of the overall system. For the aforementioned reasons, the design factor of safety is also sometimes called the factor of ignorance. The uncertainties encountered during the design phase may be of such magnitude as to lead to a design carrying extreme weight, volume, or cost penalties. It may then be advantageous to perform thorough tests or more exacting analysis, rather to rely on overly large design factors of safety. The true factor of safety, usually referred to simply as the factor of safety, can only be determined after the member is constructed and tested. This factor is the ratio of the maximum load the member can sustain under severe testing without failure to the maximum load actually carried under normal service conditions (working load). When a linear relationship exists between the load and the stress produced by the load, the factor of safety n is expressed as n=maximum usable stress/allowable stress. Maximum usable stress represents either the yield stress or the ultimate stress. The allowable stress is the working stress. The factor of safety must be greater than 1.0 if failure is to be avoided. Values for factor of safety, selected by the designer on the basis of experience and judgment are 1.5 or greater. For the majority of applications, appropriate factors of safety are found in various construction and manufacturing codes. In scope of experimental stress analysis subject which is not 100% experimental, it is often first necessary to know the principles of theory of elasticity such as: state of stress at a point, the variation of stress throughout an elastic body etc.

1.2 DEFINITION OF STRESS

Consider a body in equilibrium subjected to a system of external forces, as shown in Fig. 1.1. Under the action of these forces, internal forces will be developed within the body.

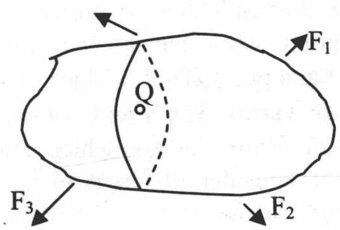

Fig. 1.1 Body under several forces

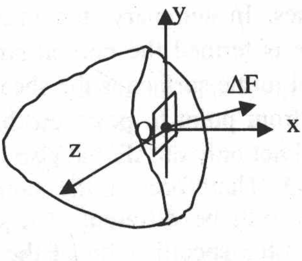

Fig. 1.2 Isolated part of body

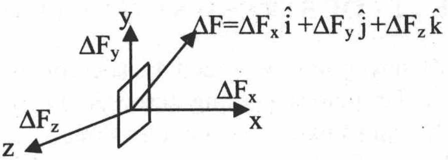

Fig. 1.3 Components of ΔF

In order to examine internal forces at some interior point Q, we use an imaginary plane to cut the body at a section passing through Q, dividing the body into two parts. As the forces acting on the entire body are in equilibrium, the forces acting on one part alone must also be in equilibrium: The internal forces, applied to both parts, are distributed continuously over the cut surface. Figure 1.2 shows the isolated left part of the body. An element of area ΔA located at point Q on the cut surface is acted on by force ΔF. Let the origin of coordinates be placed at point Q, with x-axis as normal and y, z axes as tangential to area ΔA. In general, ΔF does not lie along x, y or z. Hence, decomposing ΔF into components parallel to x, y and z (as seen in Fig. 1.3), we define the normal stress σ_x and the shearing stresses τ_{xy} and τ_{xz} as follows:

$$\sigma_x = \lim_{\Delta A_x \to 0} \frac{\Delta F_x}{\Delta A_x} \quad \text{and} \quad \tau_{xy} = \lim_{\Delta A_x \to 0} \frac{\Delta F_y}{\Delta A_x}, \quad \tau_{xz} = \lim_{\Delta A_x \to 0} \frac{\Delta F_z}{\Delta A_x} \tag{1.1}$$

At the same location in y and z directions, we can define:

$$\sigma_y = \lim_{\Delta A_y \to 0} \frac{\Delta F_y}{\Delta A_y} \quad \text{and} \quad \tau_{yx} = \lim_{\Delta A_y \to 0} \frac{\Delta F_x}{\Delta A_y}, \quad \tau_{yz} = \lim_{\Delta A_y \to 0} \frac{\Delta F_z}{\Delta A_y} \tag{1.2}$$

$$\sigma_z = \lim_{\Delta A_z \to 0} \frac{\Delta F_z}{\Delta A_z} \quad \text{and } \tau_{zx} = \lim_{\Delta A_z \to 0} \frac{\Delta F_x}{\Delta A_z}, \; \tau_{zy} = \lim_{\Delta A_z \to 0} \frac{\Delta F_y}{\Delta A_z} \tag{1.3}$$

Here ΔA_x, ΔA_y and ΔA_z are the components of area ΔA along the x, y and z directions. These definitions provide the stress components at a point Q to which the area ΔA is reduced in the limit. Stress is defined adequately for engineering purposes. In summary, the intensity of force perpendicular or normal to the surface is termed the normal stress (σ) at a point, while the intensity of force parallel to the surface is the shearing stress (τ\) at a point. These values obtained differ from point to point on the surface as ΔF varies. The stress components depend not only on ΔF, but also on the orientation of the plane on which it acts at point Q. Therefore, at the same point, as we consider different planes, the stresses will be different. The complete description of stress at a point thus requires the specification of the stress on all planes passing through the point. Stress has units of Newton per square meter or Pascal (Pa) and sometimes Mega Pascal (MPa) is commonly used.

1.3 COMPONENTS OF STRESS: STRESS TENSOR

To define stress at any point, we need to specify only stress components in mutually perpendicular planes passing through the point. These three planes perpendicular to the coordinate axes, contain three hidden sides of infinitesimal cube. The general case of a three-dimensional state of stress is shown in Fig. 1.4. When we move from point Q to Q′, values of stress will in general change. Also body forces may also exist.

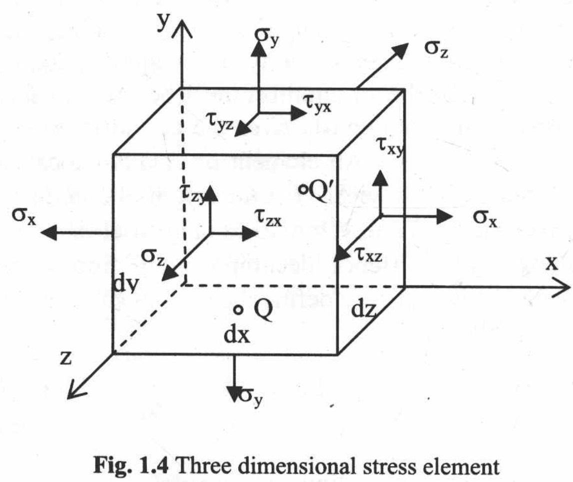

Fig. 1.4 Three dimensional stress element

Consider the stresses to be identical at points Q and Q′ and uniformly distributed on each face, represented by a single vector acting at the center of each face. In accordance with the foregoing, a total of nine scalar stress components define the state of stress at a point. The stress components can be assembled in the following matrix form, wherein each row represents the group of stresses acting on a plane passing through Q:

$$[\tau_{ij}] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \tag{1.4}$$

This array represents a tensor of second rank requiring two indexes to identify its elements or components. A vector is a tensor of first rank; a scalar is of zero rank. The double subscript notation is interpreted as follows: The first subscript indicates the direction of a normal to the plane or face on which the stress component acts; the second subscript relates to the direction of the stress itself. A face or plane is usually identified by the axis normal to it; for example, the x faces are perpendicular to the x axis.

Sign Convention

We observe that both stresses labeled τ_{yx} tend to twist the element in a clockwise direction. It would be convenient, therefore, if a sign convention were adopted under which these stresses carried the same sign.

When both the outer normal and the stress component face in a positive direction relative to the coordinate axes, the stress is positive. When both the outer normal and the stress component face in a negative direction relative to the coordinate axes, the stress is positive. When the normal points in a positive direction while the stress points in a negative direction (or vice versa), the stress is negative. In accordance with this sign convention, tensile stresses are always positive and compressive stresses always negative.

Equality of Shearing Stresses

We now examine properties of shearing stress by studying the equilibrium of forces acting on the cubic element shown in Fig. 1.4. As the stresses acting on opposite faces (which are of equal area) are equal in magnitude, but opposite in direction, translational equilibrium in all directions is assured:

That is: $\Sigma F_x=0$, $\Sigma F_y=0$ and $\Sigma F_z=0$ (1.5)

Rotational equilibrium is established by taking moments of the x-, y-, and z-directed forces about point Q. That is for example:

$$\Sigma M_z=0 \Rightarrow (-\tau_{xy}\,dy\,dz)\,dx + (\tau_{yx}\,dx\,dz)\,dy = 0 \text{ (or) } \tau_{yx}=\tau_{xy} \tag{1.6}$$

Likewise $\Sigma M_x=0$ gives: $\tau_{yz}=\tau_{zy}$ and $\Sigma M_y=0$ gives: $\tau_{xz}=\tau_{zx}$ (1.7)

Hence, the subscripts for the shearing stresses are commutative and the stress tensor is symmetric. This means that shearing stresses on mutually perpendicular planes of the element are equal.

Special Cases of Stress

Under particular circumstances, the general state of stress (Fig. 1.4) reduces to following simpler stress states.

(a) Triaxial Stress. When an element subjected to only stresses σ_1, σ_2 and σ_3 and acting in mutually perpendicular directions, it is said to be in a state of triaxial stress. Such a state of stress can be written as

$$\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} \tag{1.8}$$

The absence of shearing stresses indicates that the stresses are principal stresses for the element. A special case of triaxial stress, known as spherical or dilatational stress, occurs if all principal stresses are equal.

(b) Two-dimensional or Plane Stress. In this case, only the x and y faces of the element are subjected to stress and all the stresses act parallel to the x and y axes as shown in Fig. 1.5.

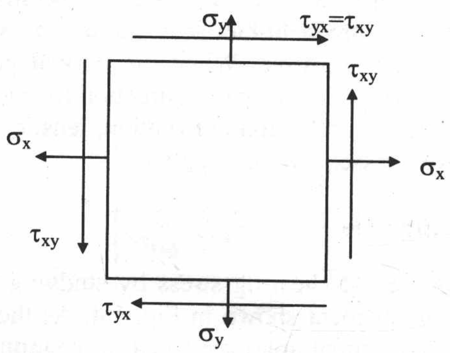

Fig. 1.5 Plane stress element

The plane stress matrix is written as:

$$\begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} \tag{1.9}$$

Although the three-dimensional nature of the element under stress should not be forgotten, for the sake of convenience we usually draw only a two-dimensional view of the plane stress element. Just like triaxial system, when only two normal