# maths for economics

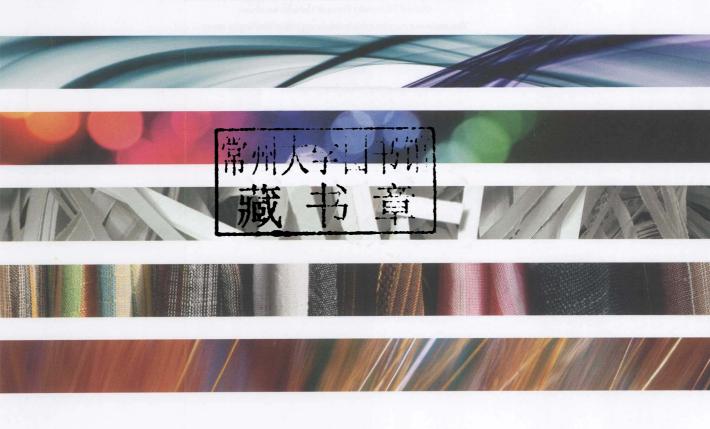
geoff renshaw
3rd edition



# Maths for Economics

**Third Edition** 

Geoff Renshaw
with contributions from Norman Ireland





## OXFORD

Great Clarendon Street, Oxford 0x2 6DP

Oxford University Press is a department of the University of Oxford. It furthers the University's objective of excellence in research, scholarship, and education by publishing worldwide in

Oxford New York

Auckland Cape Town Dar es Salaam Hong Kong Karachi Kuala Lumpur Madrid Melbourne Mexico City Nairobi New Delhi Shanghai Taipei Toronto

With offices in

Argentina Austria Brazil Chile Czech Republic France Greece Guatemala Hungary Italy Japan Poland Portugal Singapore South Korea Switzerland Thailand Turkey Ukraine Vietnam

Oxford is a registered trade mark of Oxford University Press in the UK and in certain other countries

Published in the United States by Oxford University Press Inc., New York

© Geoff Renshaw 2012

The moral rights of the author have been asserted Database right Oxford University Press (maker)

First published 2005 Second edition 2009

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, without the prior permission in writing of Oxford University Press, or as expressly permitted by law, or under terms agreed with the appropriate reprographics rights organization. Enquiries concerning reproduction outside the scope of the above should be sent to the Rights Department, Oxford University Press, at the address above

You must not circulate this book in any other binding or cover and you must impose the same condition on any acquirer

British Library Cataloguing in Publication Data Data available

Library of Congress Cataloging in Publication Data

Data available

Typeset by Graphicraft Limited, Hong Kong Printed in Italy on acid-free paper by L.E.G.O. S.p.A. – Lavis TN

ISBN 978-0-19-960212-4

1 3 5 7 9 10 8 6 4 2

To my wife, Irene, for her unstinting moral and practical support; and to my mother and father, to whom I owe everything.

# **About the author**



Geoff Renshaw was formerly a lecturer and is now an associate fellow in the Economics Department at Warwick University. He has lectured mainly in the areas of international economics, national and international economic policy, and political economy, but also taught maths to economists for more than thirty years. His teaching philosophy has always been to remember his first encounters with new ideas and techniques, and keep in mind how difficult they seemed then, even though they may seem obvious now. Geoff has always endeavoured to keep things simple and down to earth and infect his students with his own enthusiasm for economics.

Geoff was educated at Oxford and the London School of Economics. Before becoming an academic he

worked in the research department of the Trades Union Congress. Most of his career has been spent at Warwick University, but he has also taught at Washington University, St Louis, and at Birmingham University.

He has also been a consultant to the International Labour Organization (a UN agency) and spent two years in Geneva working on international trade and economic relations between industrialized and developing countries, in addition to a year in Budapest, where he headed a project on the Hungarian labour market. Geoff has also consulted for the United Nations Industrial Development Organization, and has spent time in Vienna and Warsaw working on the Polish economy.

Geoff has published several books on industrial adjustment, north–south trade and development, and multinational corporations.

Outside of economics and politics Geoff enjoys studying the English language, practising DIY on houses and cars, and thinking up new inventions—none successful as yet. He is married with three children.

#### About the contributor

Norman Ireland has been a professor of economics at the University of Warwick since 1990, and was Chair of the Department of Economics from 1994 to 1999. He was joint Managing Editor of the *International Journal of Industrial Organization* from 1986 to 1992, and is currently a member of the editorial board of the *Journal of Comparative Economics*. He has published two books in the field of industrial organization and a number of articles across several fields of economics, but particularly in industrial economics, public economics, and economic theory. He has always been involved in teaching mathematics for economists, or mathematical economics, at various levels from first year undergraduate to Master's level.

# **About the book**

This book is intended for courses in maths for economics taken in the first year, or in some cases in the second year, of undergraduate degree programmes in economics whether they be single honours or combined honours courses. It has its origins in lectures that I gave for many years to first year economics students at Warwick University.

Students arriving to study economics at British universities are highly diverse in their prior exposure to both maths and economics. Some have studied maths to the age of 17 or 18 (GCE AS and A2 level in the UK) and arrive at university with some degree of competence and confidence in maths. Others have studied maths only to the age of 16 (GCSE level) and many of these have forgotten, or perhaps never fully understood, basic mathematical techniques. There are also many students from abroad, whose backgrounds in maths are highly varied. Moreover, some students beginning economics at university have previously studied economics or business studies at school, while others have not.

The degree courses taken at university by students of economics are also highly diverse. In some courses, economics is the sole or main subject. Consequently maths for economics is prioritized and sufficient space is created in the curriculum to allow it to be explored in some depth. In other degree courses economics forms only a part, and sometimes a small part, of a combined-subject programme that includes subjects such as business studies, philosophy, politics, and international studies. Then, the crowded curriculum often leaves little space for studying maths for economics.

This diversity, in both students' prior knowledge of maths and economics and their course requirements, creates a challenge for anyone attempting to write a textbook that will meet the needs of as many students and their courses as possible. This book seeks to respond to this challenge and thereby enable every reader, whether mathematically challenged or mathematically gifted, and whether they are specializing in economics or not, to realize their true potential in maths for economics, and thereby develop the tools to study economics more effectively and more rewardingly.

More specifically, in responding to this challenge I have attempted to give the book four core structural characteristics:

#### 1 Confidence building

Recognizing that many economics students found maths difficult and unrewarding at school, and have often forgotten much of what they once knew, part 1 of the book is devoted entirely to revision and consolidation of basic skills in arithmetic, algebraic manipulation, solving equations and curve sketching. Part 1 starts at the most elementary level and terminates at GCSE level or a little above. It should be possible for every student to find a starting point in part 1 that matches his or her individual needs, while more advanced students can of course proceed directly to part 2. More guidance on finding the appropriate starting point is given in the chapter map on pages xviii—xix.

### 2 Steady learning gradient

Many textbooks in this area develop their subject matter at a rapid pace, thus imposing a steep learning curve on their readers. This often leaves students with a weak maths background feeling

lost, while even students who are relatively strong in maths sometimes fail to grasp concepts fully and to understand the economic analysis behind the various techniques and applications they are learning. To avoid these pitfalls, I have tried to give this book a carefully calibrated learning gradient that starts from the most basic level but gradually increases in mathematical sophistication as the book progresses. Consequently, no reader need be lost or left behind, and hopefully will go beyond a rote-learning approach to mathematics to achieve (perhaps for the first time) true understanding.

#### 3 Comprehensive explanation

Many textbooks skim briefly over a wide range of mathematical techniques and their economic applications, leaving students able to solve problems in a mechanical way but feeling frustrated by their lack of real understanding.

In this book I explain concepts and techniques in a relatively leisurely and detailed way, using an informal style, trying to anticipate the misconceptions and misunderstandings that the reader can so easily fall victim to, and avoiding jumps in the chain of reasoning, however small. Wherever possible, every step is illustrated by means of a graph or diagram, based on the adage that 'one picture is worth a thousand words'. Many of the explanations are by means of worked examples, which most students find easier to understand than formal theoretical explanations. There is extensive cross-referencing both within and between chapters, making it easy for the reader to quickly refresh their understanding of earlier concepts and rules when they are reintroduced later. There is also a glossary which defines all of the key terms in maths and economics used in the book.

# 4 Economic applications and progress exercises

As soon as it is introduced, every core mathematical technique is immmediately applied to an economic problem, but in a way that requires no prior knowledge of economics. While this is challenging to the reader because it requires grappling with mathematics and economics simultaneously, I feel that it is essential to renew and reinforce the reader's motivation. Additionally, progress exercises have been strategically positioned in every chapter. I regard these as an integral part of the book, *not* an optional extra. Their answers are at the end of the book, while much supplementary material can be found at the book's Online Resource Centre (www.oxfordtextbooks.co.uk/orc/renshaw3e/). For more ambitious readers and lengthier courses, the final part of the book contains some relatively advanced topics, and there is a further supplementary chapter W21 at the Online Resource Centre.

## How to use the book

#### To the student

Of course, the way in which you use this book will be primarily dictated by the requirements of your course and the instructions of your lecturer or tutor. However, at university you are expected to undertake a significant amount of independent study, much of which needs to be self-directed. The chapter map is intended to help you with this. You will see that the book caters for three levels of prior maths knowledge, labelled A, B, and C in the map. Even if you don't feel you fit neatly into any of these categories, studying the flow chart should help you to choose your own personal route through the book.

Although much effort has gone into making this book as user-friendly as possible, studying maths and its application to economics can never be light reading. In a single study session of 1–2 hours you should not expect to get through more than a few sections of a single chapter. To achieve a full understanding you may need to re-read some sections, and even whole chapters. It is usually better to re-read something that you don't fully understand, rather than pressing on in the hope that enlightenment will dawn later. You should always take notes as you read. It also greatly helps understanding if you work through with pencil and paper all the steps in any chain of mathematical reasoning. <u>Tedious</u>, but worth the effort. 'No pain, no gain' is just as true of mental exercise as it is of physical exercise.

Above all, it is essential that you attempt the progress exercises at the end of each section, as this is the only reliable way of testing your understanding. Worked answers to most of the questions are at the end of the book, with further answers on the book's website, or Online Resource Centre, which is at

www.oxfordtextbooks.co.uk/orc/renshaw3e/

There you will also find more exercises with answers, and a wide range of additional material such as how to use Excel® to plot graphs.

#### To the lecturer or tutor

At first sight this book may appear excessively long for many courses in maths for economics, which, in today's crowded syllabuses, are often quite short. However, this length is deceptive, for two reasons. First, explanations are quite detailed, facilitating independent study and thereby economizing on teaching time. For example, at Warwick University those who need to study part 1 of the book do so as an intensive revision programme, most of which is independent study, in the first two weeks of term.

Second, the range of material covered between the first and last chapters is so wide that it is extremely unlikely that any course would find the whole book appropriate for study. Rather, there are at least three overlapping study programmes within the book, as outlined in the chapter map. If none of these three suggested study programmes is suitable, the map may help you to design a path through the book that matches your syllabus requirements and the characteristics of your students.

Please note too that the book's website, or Online Resource Centre (see address above), contains much useful supplementary material, including a bank of exercises and answers reserved (by means of a password) for lecturers which can therefore be used for setting tests and examinations. There is also an additional chapter, W21, of more advanced material written by Professor Norman Ireland (see main contents pages).

# Chapter map: alternative routes through the book

Choose A, B, or C as your starting point, then follow the arrows

(A) You have forgotten almost all of the maths you ever knew and want to make a completely fresh start. (B) You have passed GCSE maths or an equivalent exam taken at age 16+, but you have done no maths since and now feel the need for some revision.

(C) You have passed AS/A2 maths or equivalent exams taken at age 17+ and 18+ and are fairly confident in your maths knowledge at this level.

#### Part one Foundations

Chapter 1. This starts from the lowest possible level and aims to rebuild basic knowledge and self-confidence. Be sure to complete the progress exercises and the **self-test** at the end of the chapter.

Take the **self-test** at the end of chapter 1; answers are at the end of the book. If you struggle with this, read chapter 1 and complete the progress exercises before going on.

Chapters 2–5. These revise the algebra component of GCSE maths or equivalent maths exam taken at age 16+. Chapters 3–5 contain in addition some economic applications. In chapter 5, sections 5.5–5.9 go a little beyond GCSE maths and you can skip them

In chapter 5, sections 5.5–5.9 go a little beyond GCSE maths and you can skip them if you wish, but be sure to study sections 5.10–5.12 on inequalities as these are important in economics.

#### Part two Optimization with one independent variable

**Chapters 6 and 7**. These introduce differentiation, a powerful mathematical technique widely used in economics. You may find these chapters a little difficult initially, but hard work at this stage will pay off later in your studies.

Chapters 8 and 9. These apply to economics the techniques of differentiation learned in chapters 6 and 7. Chapter 8 is concerned with a firm's costs, the demand for its product and its profit-seeking behaviour. Chapter 9 is devoted to the concept of elasticity.

If you are joining the book at this point because you have passed AS/A2 maths or equivalent exams, you will find that you are already familiar with all the pure maths used in these chapters. However, you may feel the need to browse chapters 6 and 7 for revision purposes. You should also study the economic applications in chapters 3–5 (see detailed contents pages). This will also help you to tune in to the book's notation and style.

#### Part three Mathematics of finance and growth

Chapter 10. This important chapter introduces the key concept of present discounted value, and also how to calculate growth rates, effective interest rates and repayments of a loan. The maths is fairly simple and mostly covered in the GCSE syllabus, though its economic application will of course be new.

This chapter is not closely linked to any other chapters and can be read at any time.

Chapters 11–13. These chapters explain the maths of logarithmic and exponential functions, which are used widely in economics. These concepts are covered in AS/A2 maths, though less fully. If you find you know the maths already, skip to the economic applications in sections 11.8, 12.9–12.11, and 13.8–13.10.

#### Part four **Optimization with two or more independent variables**

Chapters 14–17. These four chapters are, in a sense, the core of the book. The maths in these chapters will be new to all students, but is a natural extension of part 3 and you should find it no more difficult than earlier chapters.

Chapters 14 and 15 introduce functions with two or more independent variables, their derivatives, and maximum/minimum values. This material, although new to all students, is a natural extension of chapters 6 and 7 (and earlier chapters).

Chapter 16 explains the Lagrange multiplier, an optimization technique with many important uses in economics. Chapter 17 introduces some new but quite simple mathematical concepts and techniques: homogeneous functions, Euler's theorem, and the proportionate differential.

The economic applications—to cost minimization, profit maximization, and consumer choice among others—take up about one-half of chapters 14–16, and most of chapter 17.

#### Part five Some further topics

There are four chapters in this part, each of which can be studied independently of one another and of the rest of the book. All chapters contain economic applications.

**Chapter 18** introduces the mathematical technique of integration, with some applications to economics. The maths will be familiar if you have taken AS/A2 maths, but will also be well within the capacity of any student who has progressed this far in the book.

**Chapter 19** is concerned with matrix algebra, which some students of AS/A2 maths will have met before, but which again will be fairly readily understood by any sufficiently motivated student.

Chapter 20 introduces difference and differential equations, which will be new to all students but which are in part merely an extension of work in chapter 13.

Finally, **chapter 21** develops three relatively advanced topics as a taster for students who want to carry their study of mathematical economics further. Owing to space constraints this chapter is located on the book's Online Resource Centre **www.oxfordtextbooks.co.uk/orc/renshaw3e/**.

# **Guided tour of the textbook features**

Maths can seem like a daunting topic if you have not studied it for a while, and you may be somewhat surprised to find how much maths there seems to be in university economics courses. However, once you have overcome your initial fears you will find that the maths techniques used in mainstream economics are quite straightforward and that using them can even be enjoyable! This guided tour shows you how best to utilize this textbook and get the most out of your study, whether or not you have studied maths at A-level.

# OBJECTIVES

- Having completed this chapter you should be able to:

  Manipulate an equation by performing elementary operations o any variable on one side of the equation.
- Solve linear equations containing one unknown.

#### **Objectives**

Each chapter begins with a bulleted list of learning objectives outlining the main concepts and ideas you will encounter in the chapter. These serve as helpful signposts for learning and revision.

#### **Progress exercises**

At the end of each main section of each chapter you will have the opportunity to complete a progress exercise, designed to test your understanding of key concepts before moving on. You are strongly recommended to complete these exercises to help reinforce your understanding and identify any areas requiring further revision. Solutions to the progress exercises are at the end of the book, with further materials at the Online Resource Centre at www.oxfordtextbooks.co.uk/orc/renshaw3e/.

#### 2

#### Progress exercise 3.1

- My electricity supplier's tariff (= payment or charging so I pay £9.50 per quarter, irrespective of how much electr for every kilowatt used between 8 am and midnight (t pence for every kilowatt used between midnight and 8 a
- (a) Calculate my bill if I use 500 units at the day rate and 2
- (b) I estimate that my washing machine and dishwasher ear I use them only between 8 am and midnight. If I buy tim to use them only between midnight and 8 am, by how
- 2. The bus fare from my home to the university is £1.75 is ticket (allowing unlimited travel on this route) costs £23

#### Examples

You understand the theory, but how is it used in practice? Examples play a key role in the book, from short illustrative examples that demonstrate a formula in use to more involved worked examples that show step by step how an individual problem is solved.

#### EXAMPLE 10.18

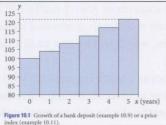
If I deposit €100 in a bank that pays interest at a nominal rate of will I get back after 5 years if interest is added (a) annually; (b) m

#### Answer

(a) Using rule 10.4,  $y = a(1+r)^x$  with a = 100, r = 0.1 (because 10  $y = a(1+r)^x = 100(1+0.1)^5 = 161.05$ 

## enset in the second of the sec

lator) equalled en rose by 4% t 5 years. Then nula  $y = a(1 + r)^x$ , 4, and x = 0, 1, 2, he price index in that is, the base he next five). in this example he example 10.9, rice index will be 10.5 above, and 0.1. The only



#### **Graphs and diagrams**

There is an old saying: one picture is worth a thousand words. Reflecting this, verbal explanations are reinforced by numerous graphs and diagrams that will help you understand both the maths techniques and the economic applications.

#### 4.13 Economic application 1: supply ar

At the beginning of this chapter we said that we needed to develop s in order to be able to analyse non-linear relationships in econom equipped for this analysis, as we first need to look at some other in ext chapter. However, to conclude this chapter on quadratic equationships that are likely to be qua-

#### **Economic applications**

Each key mathematical idea in the text is applied to an economic situation, so that you can immediately see the usefulness and relevance of maths in solving economic problems, and its significance in economic methodology.

#### \*

#### **RULE 10.4** The compound growth formula

The formula is:  $y = a(1+r)^x$ 

whore

a = 'principal' (initial sum invested, or initial value of the v r = annual proportionate interest rate (for example, if the i proportionate interest rate is r = 10 + 100 = 0.1)

x = number of years

y = future compounded value (= value of initial sum + cum

#### Rules

Each chapter highlights the most important rules (key definitions and relationships) that you need to know to complete the maths that underpins the economics. You should memorize and revise the rules in each chapter before moving on to the next topic.



**Hint** Beware of a mistake that is often made in handling gr The mistake is to assume that if a variable grows at 4% per is  $4\% \times 5 = 20\%$ . This would be correct if growth followed section 10.4); but, as explained above, growth almost invar growth formula (rule 10.4).

#### Hints

Hint boxes have been included throughout the text to alert you to common mistakes and misunderstandings, so that you can proceed with your studies with confidence.



#### Summary of sections 3.1-3.7

In sections 3.1–3.4 we showed how any equation could be mentary operations (adding, multiplying and so on) to be the key distinction between variables and parameters in any one unknown can be solved (rule 3.2).

In sections 3.5 and 3.6 we introduced the idea of a function For any linear function y = ax + b the graph of this function referred to as a linear function. The slope is given by the content of the slope is given by the slo

#### **Summaries**

**End-of-chapter checklists** 

The central points and concepts covered in each chapter are distilled into summaries at the end of chapters. These provide a mechanism for you to reinforce your understanding and can be used as a revision tool.

#### Checklist

Be sure to test your understanding of this chapter by attempting the progress exercises (answers are at the end of the book). The Online Resource Centre contains further exercises and materials relevant to this chapter www.oxfordtextbooks.co.uk/orc/renshaw3e/.

changesIndex n pressing

Fraction

verting f

and vice

by a gir

The overall objective of this chapter was to refresh and renew your understanding of the basic rules that govern the manipulation of numbers. SpecificThe topics in each chapter are presented in checklist form at the end of every chapter to allow you to reflect on your learning and 'tick' each topic as you master it, before moving on if you wish to the further exercises on the Online Resource Centre.

# **Guided tour of the Online Resource Centre**

#### www.oxfordtextbooks.co.uk/orc/renshaw3e/

The Online Resource Centre that accompanies this book contains a further chapter, chapter W21, which due to space limitations had to be omitted from the book itself. The chapter, written by Norman Ireland of Warwick University, provides an introduction to some more advanced topics which should help undergraduate students intending to take further modules in mathematical economics in their second or later years of study, as well as postgraduate students.

The Online Resource Centre also provides students and lecturers with ready-to-use teaching and learning resources. These are free of charge and are designed to maximize the learning experience. Below is a brief outline of what you will find.



#### For students

#### Solutions to progress exercises

Restribution: Mariths for Economics

Chapter 4. Question of a guider 4. Quadratic equations

Expanded couldnot be serviced progress exercises

Exercise 4.1 Question 1

Looking at figure 4.4 in the book, we see that to say that (a + b)<sup>2</sup> = a<sup>2</sup> + b<sup>2</sup> would be a nervo because through only the house result in the corner canner is (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as at Thus the corner canner is (a + c)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which we usually write as (a + b)<sup>2</sup> = a<sup>2</sup> - ab + ab, which w

# Renahes: Maths for Economics Chapter 2: Algebra Further exercises Exercise WS2.1 1. Write in their simplest forms: (a) 3e -e + 1s (b) s + 2e - 5e - (-b) (c) 3x + 4xy - xy

#### **Further exercises**

The best way to master a topic area is through practice, practice, and more practice! A bank of questions, with answers, additional to the progress exercises in the book itself, has been provided for each chapter in the book to allow you to further test your understanding of the topics.



# A P Gentron started the Excel Gentron started the Excel Location limits Column limits Description of the Column limits Description of the

# Parallel State | State



#### 'Ask the author' forum

If you are struggling with a particular problem, or just cannot seem to get your head around a specific technique or idea, then you can submit your question to the author via the interactive online forum created for this text. As well as replying directly to you by email, Geoff will post his responses to all questions and comments from both students and lecturers on this site.

#### Instructions on how to use Excel® and Maple

An introduction to the use of Excel® and Maple software for graph plotting and solving equations has been created for both students and instructors, and includes demonstrations of how these software programs can assist in the use of maths for economics.

#### For adopting lecturers

#### **Test exercises for instructors**

One of the greatest burdens facing instructors is the need to continually prepare fresh assessment material. To aid this task, a suite of additional exercises, with answers, has been created. As these are password protected and hence not available to students, they are suitable for use by instructors in assignments and examinations. Lecturers who have adopted the book are assigned a password.

#### **Graphs from the text**

Again for instructors only, all graphs from the book have been provided in high-resolution format for downloading into presentation software or for use in assignments and exam material.

# **Acknowledgements**

In preparing the third edition of this book I am again greatly indebted to the OUP editing and production team for their limitless encouragement and advice and their unfailing enthusiasm for the project. Specifically I warmly thank Kirsty Reade, my commissioning editor; my production editor, Joanna Hardern; Charlotte Dobbs, text designer; and Gemma Barber who designed the book's cover. I would also like to thank Peter Hooper, Kirsten Shankland, and Helen Tyas for their parts in the development of this new edition. For their immensely hard work and relentless attention to detail I am very grateful to the copy-editor, Mike Nugent, and the proofreader, Paul Beverley. I also thank June Morrison for compiling a very comprehensive and well-structured index. For their past and, I hope, future management of the book's Online Resource Centre, I am grateful to Fiona Loveday, Fiona Goodall, and Sarah Brett. Most of those named above also worked on the second edition of the book, for which I take this opportunity to thank them again. However, I should also like to repeat my thanks to the many others who have been involved in various ways and at various times in the production of this book. In particular I thank Tim Page and Jane Clayton, two former OUP staff without whom this book would almost certainly never have seen the light of day.

Amongst my colleagues at Warwick University and elsewhere, I am especially grateful to Norman Ireland who, having regrettably declined to become a co-author, agreed to write a lengthy and extremely valuable chapter, as well as setting numerous exercises and offering much general encouragement and support. I also owe a huge debt to Peter Law, whose meticulous checking and painstaking comments on many of the chapters saved me from a large number of small errors and a small number of large errors. Jeff and Ann Round also gave me valuable and very patient advice. Peter Hammond, despite having co-authored a book with which this one attempts to compete, was also very patient and helpful on a number of points. I am also grateful to those users of previous editions who have taken the trouble to email me, sometimes in praise and sometimes to point out errors. Both types of communication are very welcome.

As ever I am profoundly grateful to my wife Irene, who has been unfailingly patient and supportive throughout the three editions of this book. The late Mary Pearson greatly encouraged my labours, as did Lavinia McPherson – her 100 years notwithstanding. As always, the remaining shortcomings of this book are entirely my responsibility.

# **Contents**

About to About to How to Chapter Guided Guided	d contents he author he book use the book r map tour of the textbook features tour of the Online Resource Centre wledgements	it xin xv xvi xvi xvi xxi xxi
	Part One Foundations	
1 2 3 4 5	Arithmetic Algebra Linear equations Quadratic equations Some further equations and techniques	43 63 109 134
	Part Two Optimization with one independent variable	
6 7 8 9	Derivatives and differentiation  Derivatives in action  Economic applications of functions and derivatives  Elasticity	165 184 213 256
N.	Part Three Mathematics of finance and growth	
10 11 12 13	Compound growth and present discounted value The exponential function and logarithms Continuous growth and the natural exponential function Derivatives of exponential and logarithmic functions and their applications	297 328 342 368
	Part Four Optimization with two or more independent variables	
14 15 16 17	Functions of two or more independent variables  Maximum and minimum values, the total differential, and applications  Constrained maximum and minimum values  Returns to scale and homogeneous functions; partial elasticities;  growth accounting; logarithmic scales	389 441 479 519



#### Part Five Some further topics

18	Integration	551
19	Matrix algebra	577
20	Difference and differential equations	597
W21	Extensions and future directions (on the Online Resource Centre)	
Answers to progress exercises		623
Answers to chapter 1 self-test		657
Glossar	y control of the Cont	658
Index		667