Problems for physics students

WITH HINTS AND ANSWERS

K. F. RILEY

Problems for physics students

WITH HINTS AND ANSWERS

K. F. RILEY

Lecturer in Physics, Cavendish Laboratory
Fellow of Clare College, Cambridge

CAMBRIDGE UNIVERSITY PRESS

Cambridge

London New York New Rochelle

Melbourne Sydney

Published by the Press Syndicate of the University of Cambridge
The Pitt Building, Trumpington Street, Cambridge CB2 1RP
32 East 57th Street, New York, NY 10022, USA
296 Beaconsfield Parade, Middle Park, Melbourne 3206, Australia

First published 1982

Printed in Great Britain at the University Press, Cambridge

Library of Congress catalogue card number: 82-4575

British Library Cataloguing in Publication Data

Riley, K. F.
Problems for physics students with hints and answers

1. Physics – Problems, exercises, etc.
I. Title .
530'.076 QC32

ISBN 0 521 24921 X hard covers
ISBN 0 521 27073 1 paperback

DJ

PREFACE

This book aims to provide a set of problems which will test a student's understanding of the principles which are usually taught in a tertiary level physics course. In the United Kingdom, for example, the topics covered are those typically met in an A-level physics syllabus. Most of the material is thus what is known as 'classical physics', although all A-level courses contain some 'modern physics', i.e. nuclei, atoms and photons. A few A-level syllabuses, notably the Nuffield one, also contain introductory ideas on the more advanced topics of electronics, entropy and the Schrödinger equation, and questions on these have been included. In countries which adopt a less specialized school curriculum than that found in the UK, the relevant course level is that of junior college or the early years of university.

The difficulty of the questions varies widely, from straightforward application of a single basic idea to quite complex situations involving the use of several ideas at once, some of them not immediately apparent. It is hoped that these more demanding problems will not only stretch the best of pre-university students, but prove of value to those already at university during the earlier parts of their physics courses.

In the first part (sections A–T) of the book I have tried to group problems on similar areas of physics together in one section, the separate sections being alphabetically labelled. In the later part, however, in particular in sections U, V, X and Y, a deliberate attempt has been made to include problems involving ideas from several areas. Within each section similar ideas have in general been grouped together, with those groups which in my view are the more straightforward placed earlier. The questions considered to be the most demanding have been marked with an asterisk (*). It will thus be apparent that the questions under a single letter form a generally increasingly difficult set of problems on one area of physics, and that those with the same number, but different letters, form a roughly uniform set of questions on a variety of topics. A student may therefore take advantage of this structure to meet his or her individual aims or needs.

It seems appropriate here to say a little about the choice of format for the problems posed. Consider the following question and response.

Q. A uniform solid cylinder is set spinning about its axis and is then gently placed, with its axis horizontal, on a rough horizontal table. What happens?

A. At first slipping occurs, but eventually this stops and the cylinder rolls smoothly along the table.

The answer given is, of course, correct. But is it clear that the physics principles underlying the analysis of the situation have been understood? The reply to this latter question has to be 'no'; they may have been, but it is not clear that they have. Were the original question to have asked whether the final linear speed of the cylinder depends upon its mass, its initial angular speed, its radius, the coefficient of friction or the acceleration due to gravity, or what fraction of the initial kinetic energy of the cylinder is dissipated against sliding friction, then answers of no, yes, yes, no and no, or $\frac{2}{3}$, respectively, would be clear indicators that the principles had not only been understood but also correctly applied.

I would not for a minute claim that qualitative discussions of physical problems are not important or difficult; in some cases they are the only discussions that are possible. However, for the reasons illustrated by the previous paragraph, and because it is clearly more practicable when the written word, rather than a face-to-face discussion, is the medium of interaction with the student, I have chosen to make the large majority of problems in this book quantitative. This has the additional advantage that intermediate answers can be provided in a compact form, and so enable the student to locate more readily the part of the analysis in which the reasoning has been at fault, in those cases in which the problem has not been correctly solved. Even though it is not possible in a book like the present one to provide the corresponding answers, since anything offered will inevitably be found by some to be either incomplete or misleading, the importance of descriptive physics is recognized in sections Y and Z, where a significant number of qualitative questions are posed for the student to consider.

Despite the decision to make the questions quantitative, either algebraic or numerical, the mathematical techniques involved are not difficult and should be well within the capabilities of anyone who has studied mathematics beyond O-level; nor are the techniques the main points of the questions, except in the case of the data-handling exercises of section W.

Just as important a purpose of this book as testing, is that of instruction. Not of course in the basic ideas of physics, for which standard texts and teachers are the proper agents, but in the ability to pose to one's self the kind of question which will make it clear which ideas are involved. In doing this by means of hints for the problems, a sometimes difficult balance has to be struck between being so helpful that there is nothing left to the problem, and being so oblique that the hint is merely one more baffling aspect. I hope that in the majority of cases such a balance has been found. The same kind of considerations have applied to the intermediate answers which appear mixed in with the hints. Clearly they do not make the same kind of qualitative suggestions to the student as do the hints, but they should serve to indicate where a calculation has gone off the rails.

The hints and final answers are to be found in separate sections towards the end of the book. A letter H in square brackets at the end of a question indicates that, if needed, a hint or intermediate answer is available for that question or part-question. In some cases answers are given in the questions themselves. Separate listings have been used so as to enable a student requiring help to obtain it without 'accidentally' noticing the final answer.

In order to make the book self-contained for its own purposes, I have included the values of standard constants on the very last page of the book, an alphabetical list of symbols used in the questions, hints and answers (these I have tried to keep in accord with the recommendations of *SI Units, Signs, Symbols and Abbreviations* published by the Association for Science Education), and a list of formulae and relationships such as is used in some A-level courses in the United Kingdom.

It is a pleasure to record my sincere thanks to Heather Cuff, Belinda Powell and Sue Arnold for their patient and careful typing of a difficult text.

My thanks also go to the Cambridge University Tutorial Representatives for permission to base many of the problems on questions set in the Cambridge Colleges' Examination. The suggested answers and hints are of course my own, as they are for the original problems included, and in no way represent solutions officially approved by the Cambridge Colleges. Also my own are all errors and ambiguities, and I would be most grateful to have them brought to my attention.

Finally I wish to place on record my appreciation of the help given by the staff and advisors of the Cambridge University Press with the presentation of this book.

K. F. R.

Cambridge, 1982

CONTENTS

A PHYSICAL DIMENSIONS

In this section symbols not explicitly defined have the meanings indicated in the table of symbols and units (p. 171) or in the list of constants given on the last page of the book.

A1 According to Bohr's theory of the hydrogen atom, the ionization energy is $m_e e^4/8\epsilon_0^2 h^2$. Show that this expression has the dimensions of an energy and a value of 13.5 eV. [H]

A2 The Wiedemann–Franz law states that under certain conditions the electrical conductivity σ of a metal is related to its thermal conductivity λ by the equation

$$\frac{\lambda}{\sigma T}=\frac{\pi^2}{3}\left(\frac{k}{e}\right)^2.$$

Show that this equation is dimensionally acceptable, and estimate the thermal conductivity of copper at room temperature, given that its electrical conductivity is $5.6\times 10^7\,\mathrm{S\,m^{-1}}$. [H]

A3 Examine the following (supposed) equations for dimensional plausibility:

(i) The velocity v of surface waves of wavelength λ on a liquid of density ρ, under the influence of both gravity and surface tension γ:

$$v^2=\frac{g\lambda}{2\pi}+\frac{2\pi\gamma}{\rho\lambda}.$$

(ii) The energy flux S (the magnitude of the Poynting vector) associated with an electromagnetic wave in a vacuum, the electric field strength of the wave being E, and the associated magnetic flux density B:

$$S=\frac{1}{2}\left[\left(\frac{\epsilon_0}{\mu_0}\right)^{1/2}E^2+\left(\frac{\mu_0}{\epsilon_0}\right)^{1/2}B^2\right].$$

(iii) The relativistic Schrödinger equation (the Klein–Gordon equation) for a spinless particle, which gives the development of the wave function ψ describing the motion of a π-meson of mass m_π and arbitrary energy;

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}+\frac{4\pi^2 m_\pi^2 c^2}{h^2}\psi.$$

The symbols x, y, z and t indicate the usual space and time coordinates.

[H (ii), (iii)]

A4 (i) Assuming that the flow of liquid in a tube becomes turbulent at a critical velocity v which depends upon the viscosity η and density ρ of the liquid, and the radius r of the tube, find how v varies with these quantities.

(ii) If turbulence sets in when the velocity of flow exceeds $4.0\,\mathrm{m\,s^{-1}}$ for water flowing in a tube of radius 5.0 mm, at what flow velocity will it occur for olive oil, which has a density of $9.3\times10^2\,\mathrm{kg\,m^{-3}}$ and a viscosity of $1.0\times10^{-2}\,\mathrm{kg\,m^{-1}\,s^{-1}}$ when it flows in a tube of radius 15 mm? The viscosity of water is $1.0\times10^{-3}\,\mathrm{kg\,m^{-1}\,s^{-1}}$.

A5 The power required by a helicopter when hovering depends only upon the vertical thrust F its blades provide, their length l, and the density ρ of air. By what factor is the power requirement increased when it takes on a load which doubles its weight?

[H]

A6 The radiation emitted per unit time by unit area of a 'black body' at temperature T is σT^4, where σ is the Stefan–Boltzmann constant which has a value of $5.7\times10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$. The constant σ can also be expressed in terms of k, h and c as

$$\sigma=\mu k^\alpha h^\beta c^\gamma,$$

where μ, α, β and γ are dimensionless constants. Determine the numerical value of μ.

[H]

A7 A cobweb of length l, mass m, and under a tension F is contained in a glass case at a temperature T. Because it is struck by air molecules it has random vibrations with energy a few times kT. Determine how the amplitude of these motions can depend upon the quantities mentioned.

A8 The radius R of the fireball of an atomic bomb exploded in an atmosphere of density ρ may be assumed to depend upon ρ, upon the time t after the explosion and upon the energy E released by the bomb.

(i) Check this as far as possible from the following measurements made on an explosion at sea-level:

t/ms	R/m
0.24	19.9
0.66	31.9
1.22	41.0
4.61	67.3
15.0	106.5
53.0	175.0

(ii) Assume the unknown dimensionless constant to have a value close to unity and hence estimate the energy of the explosion. (Ref. G. I. Taylor, *Proc. Roy. Soc.* A, **201**, 175, 1950.)

A9 A new system of units is devised in which unit length is 1 m, but the units of time and mass are so chosen that c and G are both of magnitude unity. What is the new unit of mass in kg? [H]

A10 Elementary particle physicists prefer to work with a system of units in which c and m_p have unit value and h has the value 2π. Find the unit of time in this system. [H]

A11* The periodic time T for a moon of mass m_1 to describe an elliptical orbit of major axis a about a planet of mass m_2 depends on m_1, m_2, a and G.

(i) Determine as far as possible the relationship between these quantities.

(ii) How would the periodic time compare with that T' of another system with masses $2m_1$ and $2m_2$ but the same major axis? [H]

A12* An incompressible fluid flows through a small hole of diameter d in a thin plane sheet. The volume flow rate R depends on d, on the viscosity η and density ρ of the fluid, and on the pressure difference p between the two sides of the sheet.

(i) Find the most general relationship between these quantities.

(ii) Measurement of the flow rate R_1 for a pressure difference p_1 is made using a particular fluid. What can be predicted for a fluid of twice the density and one-third the viscosity? [H (i), (ii)]

A13* (i) The envelope of a jet formed by a liquid escaping from an orifice has a form periodic with distance from the orifice. If it is assumed that the wavelength of this envelope depends only upon the surface tension σ, the density ρ, the pressure p due to the head of liquid behind the orifice, and on the area A of the orifice, obtain by dimensional analysis as much information as possible about the relationship between these quantities.

In a particular experiment with water the following values were obtained:

Head of water/m	0.2	0.4	0.6	0.8	1.0
Wavelength/mm	3.5	4.1	4.8	5.6	6.5

Determine:

(ii) the wavelength at a head of 0.3 m for a liquid whose surface tension is one-third that of water, but has the same density;

(iii) the head needed for a liquid of surface tension one-half and density twice that of water in order to produce a wavelength of 5.0 mm. [H (ii), (iii)]

B LINEAR MECHANICS AND STATICS

B1 An empty cylindrical beaker of mass $m = 100$ g, radius $r = 30$ mm, and negligible wall thickness has its centre of gravity a height $h = 100$ mm above its base.

(i) To what depth x should it be filled with water (density $10^3\,\text{kg m}^{-3}$) so as to make it as stable as possible?

(ii) What is then the height of the centre of gravity of the partially-filled beaker above its base?

(iii) Explain the connection between the two values. [H (i)]

B2 A ladder, 5 m long and of mass M, rests on a rough horizontal floor and against a smooth vertical wall. The maximum distance its foot may be from the wall before slipping occurs is 4 m. When its foot is 3 m from the base of the wall, what is:

(i) The maximum mass that can be placed anywhere on the ladder without causing slipping?

(ii) The maximum distance up the ladder a (point) man of mass $5M$ can safely go? [H]

B3 A light rope carries a mass of 100 kg on one end and is wound around a horizontal cylindrical bar, the coefficient of static friction μ between the two being 0.05.

(i) By considering the equilibrium of a small portion of the rope in contact with the bar, show that the force on the free end needed to support the mass depends exponentially on the number of turns of the rope in contact with the bar.

(ii) Find the minimum number of turns required if the only available counterweight has mass 1 kg. [H (i)]

B4 A particle of mass m carries an electric charge Q and is subject to the combined action of gravity and a uniform horizontal electric field of strength E. It is projected in the vertical plane parallel to the field at a positive angle θ to the horizontal. Show that the horizontal distance it has travelled when it is next level with its starting point will be a maximum if $\tan 2\theta = -mg/EQ$. [H]

B5 An engine of mass M works with a constant tractive force F against a resistance proportional to the square of its speed. The maximum speed it can reach is U. Calculate (i) the time it takes, and (ii) the distance covered, whilst it accelerates from rest to a speed $\frac{1}{2}U$. [H (i), (ii)]

B6 A free-standing wall of height h and thickness t is made of bricks of density ρ_W, and rests on a rough floor. A wind of speed V blows against the wall.

(i) Assuming that the air is stopped when it reaches the wall, show that the wall will topple over if V exceeds $(\rho_W g/\rho_A h)^{1/2} t$ where ρ_A is the density of the air.

(ii) If $\rho_W = 3 \times 10^3$ kg m^{-3}, $\rho_A = 1.25$ kg m^{-3}, $h = 2$ m and $t = 0.1$ m, find the critical value of V, and determine the minimum value of the coefficient of friction between the floor and the wall for the wall to topple rather than slide. [H (ii)]

B7* Calculate the power theoretically extractable from the wind by a windmill, using the following model.

Let the wind far up- and down-stream of the mill be one-dimensional with speeds V and αV respectively ($0 \leqslant \alpha \leqslant 1$), and let the wind speed at the sails, which sweep out an area A, be v.

(i) By equating the power absorbed by the mill to the rate of loss of kinetic energy of the wind, show that $v/V = \frac{1}{2}(1 + \alpha)$.

(ii) Show that the power obtainable is proportional to $A\rho V^3$ where ρ is the density of the air, and that its maximum value is 16/27 of the initial power available in the wind. [H (i), (ii)]

B8 (*a*) A bicycle stands at rest, prevented from falling sideways but able to move forwards or backwards, and with its pedals in their highest and lowest positions. A man crouches beside the bicycle and applies a horizontal force directed towards the back wheel to the lower pedal. (i) Which way does the bicycle move? (ii) In which sense does the chain-wheel rotate (as viewed by the man)? (iii) Which way does the lower pedal move relative to the ground? Explain your answers carefully.

(*b*) A chair and a jug are placed on the platform of a weighing machine and a barrel of beer is then placed on the chair with its tap above the jug. When the tap is open and beer runs into the jug, does the machine register a higher, a lower, or the same reading as before the tap was turned on? [H *a,b*]

B9 A body of mass M moves with kinetic energy E, but without rotation. Because of an internal spring mechanism it divides into two non-rotating rigid bodies of masses αM and $(1-\alpha)M$ which now move in directions making equal angles θ on either side of the original direction of motion. Show that the spring mechanism must have released energy at least as great as $E \tan^2\theta$. [H]

B10 Both stages of a two-stage rocket are propelled by ejecting gas backwards with speed u relative to the rocket. The rocket starts from rest in a field-free region, and its two stages have masses M_1 and M_2 when filled with fuel, and masses m_1 and m_2 when empty. The empty first stage, of mass m_1, is detached before the second stage is fired.

(i) Find the final velocity of the empty second stage.

(ii) Show that it exceeds by $u \ln [M_2(m_1+m_2)/m_2(m_1+M_2)]$ that achieved by a single-stage rocket of mass M_1+M_2 when fully fuelled and mass m_1+m_2 when empty. [H (i)]

B11 Two steel balls of masses M and m are suspended by vertical strings so as to be just in contact with their centres at the same height. The ball of mass M is pulled to one side, keeping its centre in the vertical plane which originally contained the centres. It is released from rest when its height is h above the original position.

(i) Show that, whatever the value of m, the second ball cannot rise to a height above its equilibrium position greater than $4h$.

(ii) A similarly-suspended third steel ball of mass μ is now added (just touching that of mass m when at rest) and the ball of mass M is again drawn aside and released. Show that the kinetic energy transferred to the third ball is a maximum if m is chosen to have the value $(M\mu)^{1/2}$. Assume that all collisions are elastic. [H (i), (ii)]

B12* An aeroplane flies at constant speed V relative to the air and completes a level circular course in time T on a windless day. Show that if a steady wind of speed kV blows in a fixed horizontal direction, then the time for the course is increased by approximately $\frac{3}{4}k^2T$, provided that $k \ll 1$. [H]

B13* Two small smooth blocks A and B with equal masses are free to slide on a frozen lake. They are joined by an elastic rope of length $2^{1/2}L$, of negligible mass, and having the property that it stretches very little when it becomes taut. At time $t=0$, A is at rest at $x=y=0$ and B is at $x=L, y=0$ moving in the positive y-direction with speed V. What are the positions and velocities of A and B at times (i) $t=2L/V$, and (ii) $t=100L/V$? [H]

C CIRCULAR AND ROTATIONAL MOTION

C1 A wheel, set with its axis vertical, has on one of its horizontal spokes a point mass mounted so that it can slide freely. The mass is connected to the centre of the wheel by a light spring. When the wheel is turned at angular frequency ω the spring is f times its unstretched length. Find the general relationship between ω and f if $f = f_0$ when $\omega = \omega_0$. [H]

C2 A test-tube 100 mm long is filled with water and spun (in a horizontal plane) in a centrifuge at 300 rev s^{-1}. What is the hydrostatic pressure on the outer end of the tube if the inner end is at a distance of 50 mm from the axis of rotation? [H]

C3 A body suspended at rest from a fixed point by a light elastic string of unstretched length l_0 produces an extension l_1. Show that, if it then moves in a horizontal circular path (as a conical pendulum), the period of revolution is $2\pi[(l_0 \cos\theta + l_1)/g]^{1/2}$ where θ is the angle the string makes with the vertical. [H]

C4* (i) Show that, if the inclination θ to the vertical of the inextensible string of a conical pendulum is small, the period of the pendulum is independent of θ.

A small ring is threaded on the string and held so that it provides a point of oscillation. The conical pendulum is set in motion with angular velocity ω_0 and angular momentum J, and its length is then slowly shortened by sliding the ring downwards.

(ii) Find expressions for the pendulum's kinetic and potential energies in terms of J and its angular velocity ω.

(iii) Determine directly the work done in sliding the ring down the string.

Assume that the inclination of the string to the vertical is always small.

[H (i), (ii)]

C5* A cylindrical vessel of height 0.12 m and radius 0.06 m is two-thirds filled with a liquid. The vessel is rotated with constant angular velocity ω about its axis, which is vertical.

(i) Show that if surface tension is neglected the free surface of the liquid is part of a paraboloid of revolution ($z = cr^2$).

(ii) Estimate the greatest angular velocity of rotation for which the liquid does not spill over the edge of the vessel. [H (i), (ii)]

C6 One end of a uniform rod is placed at the edge of a very rough table and the rod is released from rest in an almost vertical position. As it falls away from the table it loses contact with the table when the reaction along the rod becomes zero. Show that this happens when the rod is inclined at an angle $\cos^{-1}(3/5)$ to the vertical. [H]

C7 A horizontal turntable in the form of a uniform disc of mass 150 kg is mounted on a light, frictionless vertical axis at its centre. Two men each of mass 75 kg stand at opposite ends of a diameter, and they and the turntable are at rest. The men then move round the table in the same direction and at the same constant speed. Calculate the angle they have turned through in space when they have made one complete circuit of the table. [H]

C8 A uniform laminar disc of radius a and mass M is held in frictionless bearings with its axis horizontal. The disc is at rest when a fly, also of mass M and moving in a horizontal line in the plane of the disc, lands without significant slipping on the lowest point of the perimeter of the disc at time $t = 0$. In the subsequent motion the disc turns through half of a revolution (carrying the fly with it) before coming to rest.

(i) Find the angular velocity, ω_0, of the system just after the fly has landed.

(ii) Show that the fly is level with the axis of the disc at time $t = T$, where

$$T = \frac{1}{\omega_0}\int_0^{\frac{1}{2}\pi} \sec\left(\tfrac{1}{2}\theta\right)\, d\theta .$$

[H (i)]

C9 Water flows at a horizontal velocity V into the top buckets of a mill-wheel of radius r and is tipped out when the buckets reach the bottom.

(i) If the mass flow rate of the water is M, find the torque on the wheel when it has angular velocity ω.