

*SCHAUM'S OUTLINE OF*  
**THEORY AND PROBLEMS**  
*of*  
**FLUID MECHANICS**  
**and HYDRAULICS**

SECOND EDITION

BY  
RANALD V. GILES, B.S., M.S. in C.E.

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Schaum's Outline of Theory and Problems of  
FLUID MECHANICS AND HYDRAULICS

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## Preface

This book is designed primarily to supplement standard textbooks in fluid mechanics and hydraulics. It is based on the author's conviction that clarification and understanding of the basic principles of any branch of mechanics can be accomplished best by means of numerous illustrative problems.

The previous edition of this book has been very favorably received. In this second edition many chapters have been revised and enlarged to keep pace with the most recent concepts, methods and terminology. Attention is focused earlier on Dimensional Analysis by placing this expanded material in Chapter 5. The most extensive revisions are in the chapters on Fundamentals of Fluid Flow, Fluid Flow in Pipes, and Flow in Open Channels.

The subject matter is divided into chapters covering duly-recognized areas of theory and study. Each chapter begins with statements of pertinent definitions, principles and theorems together with illustrative and descriptive material. This material is followed by graded sets of solved and supplementary problems. The solved problems illustrate and amplify the theory, present methods of analysis, provide practical examples, and bring into sharp focus those fine points which enable the student to apply the basic principles correctly and confidently. Free body analysis, vector diagrams, the principles of work and energy and of impulse-momentum, and Newton's laws of motion are utilized throughout the book. Effort has been made to present original problems developed by the author during many years of teaching the subject. Numerous proofs of theorems and derivations of formulas are included among the solved problems. The large number of supplementary problems serve as a complete review of the material of each chapter.

In addition to the use of this book by engineering students of fluid mechanics, it should be of considerable value as a reference book to the practicing engineer. He will find well-detailed solutions to many practical problems and he can refer to the summary of the theory when necessity arises. Also, the book should serve the professional engineer who must review the subject for licensing examinations or other reasons.

I wish to thank my colleague, Robert C. Stiefel, who carefully checked the solutions to the many new problems. I also wish to express my gratitude to the staff of Schaum Publishing Company, particularly to Henry Hayden and Nicola Miracapillo, for their valuable suggestions and helpful cooperation.

RANALD V. GILES

Philadelphia, Pa.  
June, 1962



## SYMBOLS and ABBREVIATIONS

The following tabulation lists the letter symbols used in this book. Because the alphabet is limited, it is impossible to avoid using the same letter to represent more than one concept. Since each symbol is defined when it is first used, no confusion should result.

<i>a</i>	acceleration in ft/sec <sup>2</sup> , area in ft <sup>2</sup>	<i>l</i>	mixing length in ft
<i>A</i>	area in ft <sup>2</sup>	<i>L</i>	length in ft
<i>b</i>	weir length in ft, width of water surface in ft, bed width of open channel in ft	<i>L<sub>E</sub></i>	equivalent length in ft
<i>c</i>	coefficient of discharge, celerity of pressure wave in ft/sec (acoustic velocity)	<i>m</i>	roughness factor in Bazin formula, weir factor for dams
<i>c<sub>c</sub></i>	coefficient of contraction	<i>M</i>	mass in slugs or lb sec <sup>2</sup> /ft, molecular weight
<i>c<sub>v</sub></i>	coefficient of velocity	<i>n</i>	roughness coefficient, exponent, roughness factor in Kutter's and Manning's formulas
<i>C</i>	coefficient (Chezy), constant of integration	<i>N</i>	rotational speed in rpm
<i>CG</i>	center of gravity	<i>N<sub>s</sub></i>	specific speed in rpm
<i>C<sub>p</sub></i>	center of pressure, power coefficient for propellers	<i>N<sub>u</sub></i>	unit speed in rpm
<i>C<sub>D</sub></i>	coefficient of drag	<i>N<sub>F</sub></i>	Froude number
<i>C<sub>F</sub></i>	thrust coefficient for propellers	<i>N<sub>M</sub></i>	Mach number
<i>C<sub>L</sub></i>	coefficient of lift	<i>N<sub>W</sub></i>	Weber number
<i>C<sub>T</sub></i>	torque coefficient for propellers	<i>p</i>	pressure in lb/ft <sup>2</sup> , wetted perimeter in ft
<i>C<sub>1</sub></i>	Hazen-Williams coefficient	<i>p'</i>	pressure in lb/in <sup>2</sup>
cfs	cubic feet per second	<i>P</i>	force in lb, power in ft lb/sec
<i>d, D</i>	diameter in feet	<i>P<sub>u</sub></i>	unit power in ft lb/sec
<i>D<sub>1</sub></i>	unit diameter in in.	psf	lb/ft <sup>2</sup>
<i>e</i>	efficiency	psia	lb/in <sup>2</sup> , absolute
<i>E</i>	bulk modulus of elasticity in lb/ft <sup>2</sup> or lb/in <sup>2</sup> , specific energy in ft lb/lb	psig	lb/in <sup>2</sup> , gage
<i>f</i>	friction factor (Darcy) for pipe flow	<i>q</i>	unit flow in cfs/unit width
<i>F</i>	force in lb, thrust in lb	<i>Q</i>	volume rate of flow in cfs
<i>g</i>	gravitational acceleration in ft/sec <sup>2</sup> = 32.2 ft/sec <sup>2</sup>	<i>Q<sub>u</sub></i>	unit discharge in cfs
gpm	gallons per minute	<i>r</i>	any radius in ft
<i>h</i>	head in ft, height or depth in ft, pressure head in ft	<i>r<sub>o</sub></i>	radius of pipe in ft
<i>H</i>	total head (energy) in ft or ft lb/lb	<i>R</i>	gas constant, hydraulic radius in ft
<i>H<sub>L</sub>, h<sub>L</sub></i>	lost head in ft (sometimes LH)	<i>R<sub>E</sub></i>	Reynolds Number
hp	horsepower = $wQH/550 = 0.746$ kw	<i>S</i>	slope of hydraulic grade line, slope of energy line
<i>I</i>	moment of inertia in ft <sup>4</sup> or in <sup>4</sup>	<i>S<sub>o</sub></i>	slope of channel bed
<i>I<sub>xy</sub></i>	product of inertia in ft <sup>4</sup> or in <sup>4</sup>	sp gr	specific gravity
<i>k</i>	ratio of specific heats, isentropic (adiabatic) exponent, von Karman constant	<i>t</i>	time in sec, thickness in in., viscosity in Saybolt sec
<i>K</i>	discharge factors for trapezoidal channels, lost head factor for enlargements, any constant	<i>T</i>	temperature, torque in ft lb, time in sec
<i>K<sub>c</sub></i>	lost head factor for contractions	<i>u</i>	peripheral velocity of rotating element in ft/sec
		<i>u, v, w</i>	components of velocity in X, Y and Z directions

$v$	volume in $\text{ft}^3$ , local velocity in $\text{ft}/\text{sec}$ , relative velocity in hydraulic machines in $\text{ft}/\text{sec}$	$x$	distance in ft
$v_s$	specific volume = $1/w = \text{ft}^3/\text{lb}$	$y$	depth in ft, distance in ft
$v_*$	shear velocity in $\text{ft}/\text{sec} = \sqrt{\tau/\rho}$	$y_c$	critical depth in ft
$V$	average velocity in $\text{ft}/\text{sec}$ (or as defined)	$y_N$	normal depth in ft
$V_c$	critical velocity in $\text{ft}/\text{sec}$	$Y$	expansion factors for compressible flow
$w$	specific (unit) weight in $\text{lb}/\text{ft}^3$	$z$	elevation (head) in ft
$W$	weight in lb, weight flow in $\text{lb}/\text{sec} = wQ$	$Z$	height of weir crest above channel bot- tom, in ft

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$\alpha$ (alpha)	angle, kinetic-energy correction factor
$\beta$ (beta)	angle, momentum correction factor
$\delta$ (delta)	boundary layer thickness in ft
$\Delta$ (delta)	flow correction term
$\epsilon$ (epsilon)	surface roughness in ft
$\eta$ (eta)	eddy viscosity
$\theta$ (theta)	any angle
$\mu$ (mu)	absolute viscosity in $\text{lb sec}/\text{ft}^2$ (or poises)
$\nu$ (nu)	kinematic viscosity in $\text{ft}^2/\text{sec}$ (or stokes) = $\mu/\rho$
$\pi$ (pi)	dimensionless parameter
$\rho$ (rho)	density in $\text{lb sec}^2/\text{ft}^4$ or $\text{slugs}/\text{ft}^3 = w/g$
$\sigma$ (sigma)	surface tension in $\text{lb}/\text{ft}$ , intensity of tensile stress in psi
$\tau$ (tau)	shear stress in $\text{lb}/\text{ft}^2$
$\phi$ (phi)	speed factor, velocity potential, ratio
$\psi$ (psi)	stream function
$\omega$ (omega)	angular velocity in $\text{rad}/\text{sec}$

### Conversion Factors

1 cubic foot	= 7.48 U.S. gallons	= 28.32 liters
1 U.S. gallon	= 8.338 pounds of water at 60°F	
1 cubic foot per second	= 0.646 million gallons per day	
	= 448.8 gallons per minute	
1 pound-second per square foot ( $\mu$ )	= 478.7 poises	
1 square foot per second ( $\nu$ )	= 929 square centimeters per second	
1 horsepower	= 550 foot-pounds per second	= 0.746 kilowatts
30 inches of mercury	= 34 feet of water	= 14.7 pounds per square inch

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# Chapter 1

## Properties of Fluids

### FLUID MECHANICS and HYDRAULICS

Fluid mechanics and hydraulics represent that branch of applied mechanics dealing with the behavior of fluids at rest and in motion. In the development of the principles of fluid mechanics, some fluid properties play principal roles, others only minor roles or no roles at all. In fluid statics, specific weight is the important property, whereas in fluid flow, density and viscosity are predominant properties. Where appreciable compressibility occurs, principles of thermodynamics must be considered. Vapor pressure becomes important when negative pressures (gage) are involved, and surface tension affects static and flow conditions in small passages.

### DEFINITION of a FLUID

Fluids are substances which are capable of flowing and which conform to the shape of containing vessels. When in equilibrium, fluids cannot sustain tangential or shear forces. All fluids have some degree of compressibility and offer little resistance to change of form.

Fluids may be divided into liquids and gases. The chief differences between liquids and gases are (a) liquids are practically incompressible whereas gases are compressible and often must be so treated and (b) liquids occupy definite volumes and have free surfaces whereas a given mass of gas expands until it occupies all portions of any containing vessel.

### AMERICAN ENGINEERING SYSTEM of UNITS

Three selected reference dimensions (fundamental dimensions) are length, force and time. In this book the corresponding three fundamental units used will be the foot of length, the pound of force (or pound weight), and the second of time. All other units may be derived from these. Thus unit volume is the  $\text{ft}^3$ , unit acceleration is the  $\text{ft}/\text{sec}^2$ , unit work is the  $\text{ft lb}$ , and unit pressure is the  $\text{lb}/\text{ft}^2$ . Should data be given in other units, they must be converted to the foot-pound-second system before applying them to the solution of problems.

The unit for mass in this system, the *slug*, is derived from the units of force and acceleration. For a freely falling body in vacuum the acceleration is that of gravity ( $g = 32.2 \text{ ft}/\text{sec}^2$  at sea level) and the only force acting is its weight. From Newton's second law,

$$\text{force in pounds} = \text{mass in slugs} \times \text{acceleration in ft/sec}^2$$

Then 
$$\text{weight in pounds} = \text{mass in slugs} \times g(32.2 \text{ ft/sec}^2)$$

or 
$$\text{mass } M \text{ in slugs} = \frac{\text{weight } W \text{ in pounds}}{g(32.2 \text{ ft/sec}^2)} \quad (1)$$

### SPECIFIC WEIGHT

The specific weight  $w$  of a substance is the weight of a unit volume of the substance. For liquids,  $w$  may be taken as constant for practical changes of pressure. The specific (unit) weight of water for ordinary temperature variations is 62.4 lb/ft<sup>3</sup>. See Appendix, Tables 1C and 2, for additional values.

The specific weights of gases may be calculated by using the *equation of state* of a gas

$$\text{or} \quad \frac{pv_s}{T} = R \quad (\text{Boyle's and Charles' laws}) \quad (2)$$

where pressure  $p$  is absolute pressure in lb/ft<sup>2</sup>, specific volume  $v_s$  is the volume per unit weight in ft<sup>3</sup>/lb, temperature  $T$  is the absolute temperature in degrees Rankine (460° + degrees Fahrenheit), and  $R$  is the *gas constant* in feet/degree Rankine. Since  $w = 1/v_s$ , the above equation may be written

$$w = \frac{p}{RT} \quad (3)$$

**MASS DENSITY** of a BODY  $\rho$  (rho) = mass per unit volume =  $w/g$ .

In the engineering system of units, the mass density of water is 62.4/32.2 = 1.94 slugs/ft<sup>3</sup> or lb sec<sup>2</sup>/ft<sup>4</sup>. In the metric system the density of water is 1 g/cm<sup>3</sup> at 4°C. See Appendix, Table 1C.

### SPECIFIC GRAVITY of a BODY

The specific gravity of a body is that pure number which denotes the ratio of the weight of a body to the weight of an equal volume of a substance taken as a standard. Solids and liquids are referred to water (at 39.2°F = 4°C) as standard, while gases are often referred to air free of CO<sub>2</sub> or hydrogen (at 32°F = 0°C and 1 atmosphere = 14.7 lb/in<sup>2</sup> pressure) as standard. For example,

$$\begin{aligned} \text{specific gravity of a substance} &= \frac{\text{weight of the substance}}{\text{weight of equal volume water}} \\ &= \frac{\text{specific weight of substance}}{\text{specific weight of water}} \end{aligned} \quad (4)$$

Thus if the specific gravity of a given oil is 0.750, its specific weight is 0.750(62.4 lb/ft<sup>3</sup>) = 46.8 lb/ft<sup>3</sup>.

The specific gravity of water is 1.00 and of mercury is 13.57. The specific gravity of a substance is the same in any system of measures. See Appendix, Table 2.

### VISCOSITY of a FLUID

The viscosity of a fluid is that property which determines the amount of its resistance to a shearing force. Viscosity is due primarily to interaction between fluid molecules.

Referring to Fig. 1-1, consider two large, parallel plates at a small distance  $y$  apart, the space between the plates being filled with a fluid. Consider the upper plate acted on by a constant force  $F$  and hence moving at a con-

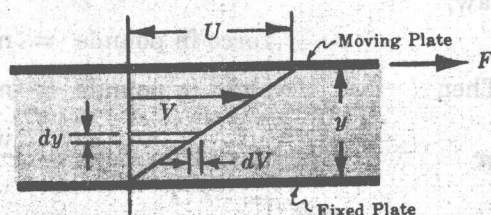


Fig. 1-1



stant velocity  $U$ . The fluid in contact with the upper plate will adhere to it and will move at velocity  $U$ , and the fluid in contact with the fixed plate will have velocity zero. If distance  $y$  and velocity  $U$  are not too great, the velocity variation (gradient) will be a straight line. Experiments have shown that force  $F$  varies with the area of the plate, with velocity  $U$ , and inversely with distance  $y$ . Since by similar triangles,  $U/y = dV/dy$ , we have

$$F \propto \frac{AU}{y} = A \frac{dV}{dy} \quad \text{or} \quad \frac{F}{A} = \tau \propto \frac{dV}{dy}$$

where  $\tau = F/A =$  shear stress. If a proportionality constant  $\mu$  (mu), called the *absolute (dynamic) viscosity*, is introduced,

$$\tau = \mu \frac{dV}{dy} \quad \text{or} \quad \mu = \frac{\tau}{dV/dy} \quad (5)$$

The units of  $\mu$  are  $\frac{\text{lb sec}}{\text{ft}^2}$ , since  $\frac{\text{lb/ft}^2}{(\text{ft/sec})/\text{ft}} = \frac{\text{lb sec}}{\text{ft}^2}$ . Fluids which follow the relation of equation (5) are called *Newtonian fluids* (see Problem 9).

Another viscosity coefficient, the *kinematic coefficient of viscosity*, is defined as

$$\text{Kinematic coefficient } \nu \text{ (nu)} = \frac{\text{absolute viscosity } \mu}{\text{mass density } \rho}$$

$$\text{or} \quad \nu = \frac{\mu}{\rho} = \frac{\mu}{w/g} = \frac{\mu g}{w} \quad (6)$$

The units of  $\nu$  are  $\frac{\text{ft}^2}{\text{sec}}$ , since  $\frac{(\text{lb sec/ft}^2)(\text{ft/sec}^2)}{\text{lb/ft}^3} = \frac{\text{ft}^2}{\text{sec}}$ .

Viscosities are reported in handbooks as poises and stokes (cgs units) and on occasion as Saybolt seconds, from viscosimeter measurements. Conversions to the ft-lb-sec system are illustrated in Problems 6-8. A few values of viscosities are given in Tables 1 and 2 of the Appendix.

Viscosities of liquids decrease with temperature increases but are not affected appreciably by pressure changes. The absolute viscosity of gases increases with increase in temperature but is not appreciably changed due to pressure. Since the specific weight of gases changes with pressure changes (temperature constant), the kinematic viscosity varies inversely as the pressure. However, from the equation above,  $\mu g = w \nu$ .

## VAPOR PRESSURE

When evaporation takes place within an enclosed space, the partial pressure created by the vapor molecules is called vapor pressure. Vapor pressures depend upon temperature and increase with it. See Table 1C for values for water.

## SURFACE TENSION

A molecule in the interior of a liquid is under attractive forces in all directions, and the vector sum of these forces is zero. But a molecule at the surface of a liquid is acted on by a net inward cohesive force which is perpendicular to the surface. Hence it requires work to move molecules to the surface against this opposing force, and surface molecules have more energy than interior ones.

The surface tension of a liquid is the work that must be done to bring enough molecules from inside the liquid to the surface to form one new unit area of that surface ( $\text{ft lb/ft}^2$ ).



This work is numerically equal to the tangential contractile force acting across a hypothetical line of unit length on the surface (lb/ft).

In most problems of introductory fluid mechanics, surface tension is not of particular importance. Table 1C gives values of surface tension  $\sigma$  (sigma) for water in contact with air.

## CAPILLARITY

The rise or fall of a liquid in a capillary tube (or in some equivalent circumstance, such as in porous media) is caused by surface tension and depends on the relative magnitudes of the cohesion of the liquid and the adhesion of the liquid to the walls of the containing vessel. Liquids rise in tubes they wet (adhesion > cohesion) and fall in tubes they do not wet (cohesion > adhesion). Capillarity is important when using tubes smaller than about  $\frac{3}{8}$  inch in diameter.

## FLUID PRESSURE

Fluid pressure is transmitted with equal intensity in all directions and acts normal to any plane. In the same horizontal plane the pressure intensities in a liquid are equal. Measurements of unit pressures are accomplished by using various forms of gages. Unless otherwise stated, gage or relative pressures will be used throughout this book. Gage pressures represent values above or below atmospheric pressure.

**UNIT PRESSURE or PRESSURE** is expressed as force divided by area. In general,

$$p \text{ (lb/ft}^2 \text{ or psf)} = \frac{dP \text{ (lb)}}{dA \text{ (ft}^2\text{)}}$$

For conditions where force  $P$  is uniformly distributed over an area, we have

$$p \text{ (psf)} = \frac{P \text{ (lb)}}{A \text{ (ft}^2\text{)}} \quad \text{and} \quad p' \text{ (psi)} = \frac{P \text{ (lb)}}{A \text{ (in}^2\text{)}}$$

## DIFFERENCE in PRESSURE

Difference in pressure between any two points at different levels in a liquid is given by

$$p_2 - p_1 = w(h_2 - h_1) \quad \text{in psf} \quad (7)$$

where  $w$  = unit weight of the liquid (lb/ft<sup>3</sup>) and  $h_2 - h_1$  = difference in elevation (ft).

If point 1 is in the free surface of the liquid and  $h$  is positive downward, the above equation becomes

$$p = wh \quad \text{(in psf gage)} \quad (8)$$

To obtain the lb/in<sup>2</sup> pressure unit, we use

$$p' = \frac{p}{144} = \frac{wh}{144} \quad \text{(in psi gage)} \quad (9)$$

These equations are applicable as long as  $w$  is constant (or varies so slightly with  $h$  as to cause no significant error in the result).

**PRESSURE VARIATIONS in a COMPRESSIBLE FLUID**

Pressure variations in a compressible fluid are usually very small because of the small unit weights and the small differences of elevation being considered in hydraulic calculations. Where such differences must be recognized for small changes in elevation  $dh$ , the law of pressure variation may be written

$$dp = -w dh \quad (10)$$

The negative sign indicates that the pressure decreases as the altitude increases, with  $h$  positive upward. For applications, see Problems 29-31.

**PRESSURE HEAD  $h$** 

Pressure head  $h$  represents the height of a column of homogeneous fluid that will produce a given intensity of pressure. Then

$$h \text{ (ft of fluid)} = \frac{p \text{ (lb/ft}^2\text{)}}{w \text{ (lb/ft}^3\text{)}} \quad (11)$$

**BULK MODULUS of ELASTICITY ( $E$ )**

The bulk modulus of elasticity ( $E$ ) expresses the compressibility of a fluid. It is the ratio of the change in unit pressure to the corresponding volume change per unit of volume.

$$E = \frac{dp'}{-dv/v} = \frac{\text{lb/in}^2}{\text{ft}^3/\text{ft}^3} = \text{lb/in}^2 \quad (12)$$

**COMPRESSION of GASES**

Compression of gases may occur according to various laws of thermodynamics. For the same mass of gas subjected to two different conditions,

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} = WR \quad \text{and} \quad \frac{p_1}{w_1 T_1} = \frac{p_2}{w_2 T_2} = R \quad (13)$$

where  $p$  = absolute pressure in lb/ft<sup>2</sup>,  $v$  = volume in ft<sup>3</sup>,  $W$  = weight in lb,  
 $w$  = specific weight in lb/ft<sup>3</sup>,  $R$  = gas constant in ft/degree Rankine,  
 $T$  = absolute temperature in degrees Rankine ( $460 + ^\circ\text{F}$ ).

**FOR ISOTHERMAL CONDITIONS** (constant temperature) the above expression (13) becomes

$$p_1 v_1 = p_2 v_2 \quad \text{and} \quad \frac{w_1}{w_2} = \frac{p_1}{p_2} = \text{constant} \quad (14)$$

Also,

$$\text{Bulk Modulus } E = p \text{ (in psf)} \quad (15)$$

**FOR ADIABATIC or ISENTROPIC CONDITIONS** (no heat exchanged) the above expressions become

$$p_1 v_1^k = p_2 v_2^k \quad \text{and} \quad \left(\frac{w_1}{w_2}\right)^k = \frac{p_1}{p_2} = \text{constant} \quad (16)$$

Also 
$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k} \quad (17)$$

and 
$$\text{Bulk Modulus } E = kp \text{ (in psf)} \quad (18)$$

where  $k$  is the ratio of the specific heat at constant pressure to the specific heat at constant volume. It is known as the adiabatic exponent.

Table 1A in the Appendix lists some typical values of  $R$  and  $k$ . For many gases,  $R$  times molecular weight is about 1544.

## PRESSURE DISTURBANCES

Pressure disturbances imposed on a fluid move in waves. These pressure waves move at a velocity equal to that of sound through the fluid. The velocity, or celerity, in ft/sec is expressed as

$$c = \sqrt{E/\rho} \quad (19)$$

where  $E$  must be in lb/ft<sup>2</sup>. For gases, this acoustic velocity is

$$c = \sqrt{kp/\rho} = \sqrt{kgRT} \quad (20)$$

## Solved Problems

1. Calculate the specific weight  $w$ , specific volume  $v_s$  and density  $\rho$  of methane at 100°F and 120 psi absolute.

Solution:

From Table 1A in the Appendix,  $R = 96.3$ .

$$\text{Specific weight } w = \frac{p}{RT} = \frac{120 \times 144}{96.3(460 + 100)} = 0.321 \text{ lb/ft}^3$$

$$\text{Specific volume } v_s = \frac{1}{w} = \frac{1}{0.321} = 3.11 \text{ ft}^3/\text{lb}$$

$$\text{Density } \rho = \frac{w}{g} = \frac{0.321}{32.2} = .00997 \text{ slugs/ft}^3$$

2. If 200 ft<sup>3</sup> of oil weighs 10,520 lb, calculate its specific weight  $w$ , density  $\rho$  and specific gravity.

Solution:

$$\text{Specific weight } w = \frac{10,520 \text{ lb}}{200 \text{ ft}^3} = 52.6 \text{ lb/ft}^3$$

$$\text{Density } \rho = \frac{w}{g} = \frac{52.6 \text{ lb/ft}^3}{32.2 \text{ ft/sec}^2} = 1.63 \text{ slugs/ft}^3$$

$$\text{Specific gravity} = \frac{w_{\text{oil}}}{w_{\text{water}}} = \frac{52.6 \text{ lb/ft}^3}{62.4 \text{ lb/ft}^3} = 0.843$$

3. At 90°F and 30.0 psi absolute the specific volume  $v_s$  of a certain gas was 11.4 ft<sup>3</sup>/lb. Determine the gas constant  $R$  and the density  $\rho$ .

Solution:

$$\text{Since } w = \frac{p}{RT}, \text{ then } R = \frac{p}{wT} = \frac{pv_s}{T} = \frac{(30.0 \times 144)(11.4)}{(460 + 90)} = 89.5.$$

$$\text{Density } \rho = \frac{w}{g} = \frac{1/v_s}{g} = \frac{1}{v_s g} = \frac{1}{11.4 \times 32.2} = .00272 \text{ slugs/ft}^3.$$

4. (a) Find the change in volume of 1.00 ft<sup>3</sup> of water at 80°F when subjected to a pressure increase of 300 psi. (b) From the following test data determine the bulk modulus of elasticity of water: at 500 psi the volume was 1.000 ft<sup>3</sup> and at 3500 psi the volume was 0.990 ft<sup>3</sup>.

Solution:

- (a) From Table 1C in the Appendix,  $E$  at 80°F is 325,000 psi. Using formula (12),

$$dv = -\frac{v dp'}{E} = -\frac{1.00 \times 300}{325,000} = -.00092 \text{ ft}^3$$

- (b) The definition associated with formula (12) indicates that *corresponding* changes in pressure and volume must be considered. Here an increase in pressure corresponds to a decrease in volume.

$$E = -\frac{dp'}{dv/v} = -\frac{(3500 - 500)}{(0.990 - 1.000)/1.000} = 3 \times 10^5 \text{ psi}$$

5. A cylinder contains 12.5 ft<sup>3</sup> of air at 120°F and 40 psi absolute. The air is compressed to 2.50 ft<sup>3</sup>. (a) Assuming isothermal conditions, what is the pressure at the new volume and what is the bulk modulus of elasticity? (b) Assuming adiabatic conditions, what is the final pressure and temperature and what is the bulk modulus of elasticity?

Solution:

- (a) For isothermal conditions,  $p_1 v_1 = p_2 v_2$   
Then  $(40 \times 144)12.5 = (p_2' \times 144)2.50$  and  $p_2' = 200$  psi absolute

The bulk modulus  $E = p' = 200$  psi.

- (b) For adiabatic conditions,  $p_1 v_1^k = p_2 v_2^k$  and Table 1A in the Appendix gives  $k = 1.40$ .

$$\text{Then } (40 \times 144)(12.5)^{1.40} = (p_2' \times 144)(2.50)^{1.40} \text{ and } p_2' = 381 \text{ psi absolute}$$

The final temperature is obtained by using equation (17):

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k}, \quad \frac{T_2}{(460 + 120)} = \left(\frac{381}{40}\right)^{0.40/1.40}, \quad T_2 = 1105^\circ \text{ Rankine} = 645^\circ \text{F}$$

The bulk modulus  $E = kp' = 1.40 \times 381 = 533$  psi.

6. From the International Critical Tables, the viscosity of water at 20°C (68°F) is .01008 poises. Compute (a) the absolute viscosity in lb sec/ft<sup>2</sup> units. (b) If the specific gravity at 20°C is 0.998, compute the value of the kinematic viscosity in ft<sup>2</sup>/sec units.

Solution:

The poise is measured in dyne sec/cm<sup>2</sup>. Since 1 lb = 444,800 dynes and 1 ft = 30.48 cm, we obtain

$$1 \frac{\text{lb sec}}{\text{ft}^2} = \frac{444,800 \text{ dyne sec}}{(30.48 \text{ cm})^2} = 478.7 \text{ poises}$$



$$(a) \quad \mu \text{ in lb sec/ft}^2 = .01008/478.7 = 2.11 \times 10^{-5}$$

$$(b) \quad \nu \text{ in ft}^2/\text{sec} = \frac{\mu}{\rho} = \frac{\mu}{w/g} = \frac{\mu g}{w} = \frac{2.11 \times 10^{-5} \times 32.2}{0.998 \times 62.4} = 1.091 \times 10^{-5}$$

7. Convert 15.14 poises to kinematic viscosity in  $\text{ft}^2/\text{sec}$  units if the liquid has specific gravity 0.964.

Solution:

The steps illustrated in Problem 6 may be taken or an additional factor may be established for water from  $\frac{1}{478.7} \times \frac{32.2}{62.4} = .001078$ . Hence  $\nu$  in  $\text{ft}^2/\text{sec} = \frac{15.14 \times .001078}{\text{sp gr} = 0.964} = .0169$ .

8. Convert a viscosity of 510 Saybolt seconds at  $60^\circ\text{F}$  to kinematic viscosity  $\nu$  in  $\text{ft}^2/\text{sec}$  units.

Solution:

Two sets of formulas are given to establish this conversion when the Saybolt Universal Viscometer is used:

$$\begin{aligned} (a) \quad & \text{for } t \leq 100, \quad \mu \text{ in poises} = (.00226t - 1.95/t) \times \text{sp gr} \\ & \text{for } t > 100, \quad \mu \text{ in poises} = (.00220t - 1.35/t) \times \text{sp gr} \\ (b) \quad & \text{for } t \leq 100, \quad \nu \text{ in stokes} = (.00226t - 1.95/t) \\ & \text{for } t > 100, \quad \nu \text{ in stokes} = (.00220t - 1.35/t) \end{aligned}$$

where  $t$  = Saybolt second units. To convert stokes ( $\text{cm}^2/\text{sec}$ ) to  $\text{ft}^2/\text{sec}$  units, divide by  $(30.48)^2$  or 929.

$$\text{Using group (b), and since } t > 100, \quad \nu = \left( .00220 \times 510 - \frac{1.35}{510} \right) \frac{1}{929} = .001205 \text{ ft}^2/\text{sec}.$$

9. Discuss the shear characteristics of the fluids for which the curves have been drawn in Fig. 1-2.

Solution:

- The Newtonian fluids behave according to the law  $\tau = \mu(dV/dy)$ , or the shear stress is proportional to the velocity gradient or rate of shearing strain. Thus for these fluids the plotting of shear stress against velocity gradient is a straight line passing through the origin. The slope of the line determines the viscosity.
- For the "ideal" fluid, the resistance to shearing deformation is zero, and hence the plotting coincides with the  $x$ -axis. While no ideal fluids exist, in certain analyses the assumption of an ideal fluid is useful and justified.
- For the "ideal" or elastic solid, no deformation will occur under any loading condition, and the plotting coincides with the  $y$ -axis. Real solids have some deformation and, within the proportional limit (Hooke's law), the plotting is a straight line which is almost vertical.
- Non-Newtonian fluids deform in such a way that shear stress is not proportional to rate of shearing deformation, except perhaps at very low shear stresses. The deformation of these fluids might be classified as plastic.
- The "ideal" plastic material could sustain a certain amount of shearing stress without deformation, and thereafter it would deform in proportion to the shearing stress.

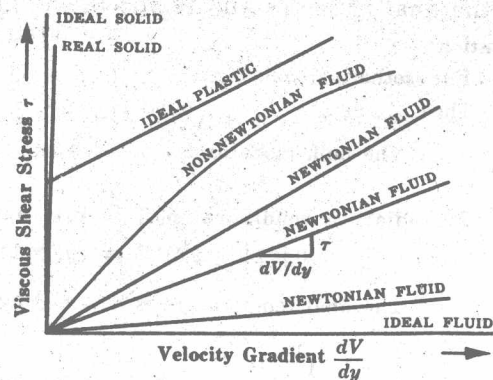


Fig. 1-2