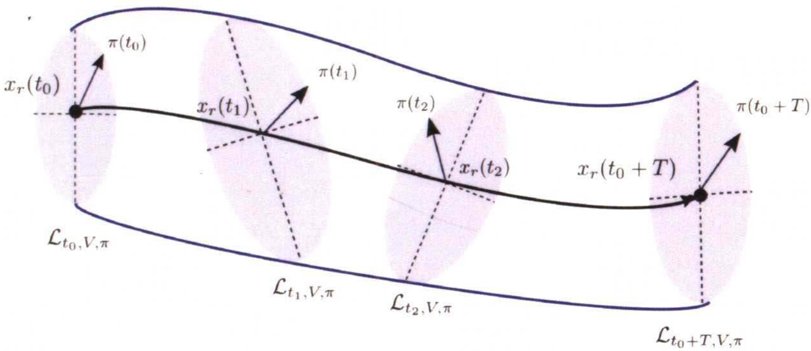
OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG

CONTRIBUTIONS IN SYSTEMS THEORY AND AUTOMATIC CONTROL

Timm Faulwasser

Optimization-based Solutions to Constrained Trajectory-tracking and Path-following Problems



Contributions in Systems Theory and Automatic Control
Otto-von-Guericke-Universität Magdeburg

Band 3

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Shaker Verlag
Aachen 2013

Bibliographic information published by the Deutsche Nationalbibliothek
The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at http://dnb.d-nb.de.

Zugl.: Magdeburg, Univ., Diss., 2012

Printed in Germany.

ISBN 978-3-8440-1594-2

Shaker Verlag GmbH • P.O. BOX 101818 • D-52018 Aachen
Phone: 0049/2407/9596-0 • Telefax: 0049/2407/9596-9
Internet: www.shaker.de • e-mail: info@shaker.de

Optimization-based Solutions to Constrained Trajectory-tracking and Path-following Problems

Dissertation
zur Erlangung des akademischen Grades
Doktoringenieur (Dr.-Ing.)

von

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geboren am 5. Mai 1981 in Erfurt

genehmigt durch die Fakultät für Elektrotechnik und Informationstechnik der
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eingereicht am 23. März 2012
Promotionskolloquium am 5. Oktober 2012

Acknowledgments

This thesis is the result of my PhD studies in the Systems and Control Group of Rolf Findeisen at the Institute for Automation Engineering, Faculty of Electrical Engineering and Information Technology at the Otto-von-Guericke-Universität Magdeburg, Germany. I would like to express my gratitude to the people who have made this thesis possible.

I am indebted to my supervisor Prof. Dr.-Ing. Rolf Findeisen for leaving large scientific freedom, the opportunity to learn about the administrative aspects of academia, and many lively discussions about scientific and non-scientific topics. Furthermore, I would like to thank Prof. Dr. techn. Andreas Kugi from the Technische Universität Wien, Austria, and Prof. Colin Jones, PhD from the École Polytechnique Fédérale de Lausanne, Switzerland for their willingness to act as external reviewers of this thesis.

In the first months of my time in Magdeburg, when the lab had to be filled with life, and many administrative issues had to be solved, scientific progress would have been impossible without the excellent collaboration in the team of the research assistants in the lab. In particular, I would like to thank Philipp Rumschinski and Paolo Varutti for their excellent cooperation and teamwork not only in these early days. Later on, as the lab grew in terms of team members, I enjoyed working with Jürgen Ihlow, Friedrich von Haeseler, and Stefan Streif as well as with the fellow PhD students: Benjamin Kern, Steffen Borchers, Juan Pablo Zometa, Markus Kögel, Anton Shavchenko, Solvey Maldonado, Petar Andonov, Tobias Bäthge, Nadine Strobel, and Michael Maiworm. The support of Ulrike Thürmer with respect to all non-scientific issues was extremely valuable.

Aside from the persons mentioned I would like to acknowledge the fruitful discussions with Veit Hagenmeyer (BASF), and Friedrich von Haeseler. Their willingness to discuss scientific ideas was a great help. Furthermore, I had the pleasure to learn a lot from the cooperations with Denise Lam (University of Melbourne, Australia), as well as Johann Dauer and Sven Lorenz (Institute for Flight Systems, DLR Braunschweig, Germany).

It is much easier to focus on scientific problems if you can rely on your family and friends. Thus I am indebted to my parents, to my brothers and their families, and to Sophia Hengst for their great support and patience.

December 2012 Timm Faulwasser

Abstract

Systems and control theory is of great importance for the analysis, the design, and the operation of complex dynamical systems. The prototypical problem in control theory is the *stabilization of a set point*. This is the problem of designing a feedback such that the closed-loop solutions stay in a neighborhood of the set point and converge to it. When, instead of a set point, a time-varying reference needs to be stabilized—i.e., the closed-loop solutions shall converge to a time-varying reference trajectory—then the problem is called *trajectory tracking*. Typical examples of trajectory-tracking problems are set point changes along precomputed references, synchronization tasks or startup of processes. While stabilization and trajectory tracking are well-understood for a wide range of systems, not all control tasks arising in practise belong to these categories.

For example, consider the case of steering a car automatically along a road. Usually, the driving velocity is not predetermined. The only requirements are to keep the car on the road while driving sufficiently fast. Formally, we can reformulate this task as follows: the system state or output should converge to a known geometric curve and follow it along in a specific direction. These kinds of control tasks are known as *path-following problems*. The major difference to trajectory tracking is that the speed along the path along is not fixed a priori. Instead it is an additional degree of freedom which can be tuned to achieve fast convergence, and small deviations of the system state or output to the path. Path-following problems typically arise in robotics and vehicle-like applications, such as unmanned aerial vehicles, robots, or milling machines. Moreover, some process control problems can be formulated as path following, for example, the task of steering a batch reactor along a temperature profile while maximizing the process productivity.

This thesis deals with the design of optimization-based control schemes for trajectory-tracking and path-following problems in the presence of constraints. We present novel results on the design of nonlinear model predictive control (NMPC) schemes for both problem classes.

Sufficient conditions for the stability of NMPC for trajectory tracking based on time-varying terminal regions are derived. The key feature of our approach is the introduction of positive invariant time-varying level sets of Lyapunov functions as terminal regions. These sets allow enlarging the region of attraction of the proposed control schemes and thereby improve the control performance.

Furthermore, we discuss path-following problems for constrained nonlinear systems. We propose using a nonlinear normal form of an augmented system to analyze these

problems. We investigate sufficient conditions for exact path followability of unconstrained and constrained nonlinear systems. Based on these results we present tailored predictive control schemes for path following. We derive sufficient conditions which guarantee the convergence of the system output to the path. In contrast to previous works—for example, in the field of robotics—our schemes can handle constraints on states and inputs as well as situations where the system does not start on the path. In other words, using the proposed control schemes makes it possible to stabilize the motion of a system in the presence of constraints with respect to a path defined in an output space. Examples from robotics and chemical engineering are drawn upon to support our results.
The main intention of this thesis is twofold: Firstly, we show that nonlinear model predictive control is very well applicable to problems beyond set point stabilization. Secondly, we demonstrate that path-following concepts provide a suitable framework for many challenging control problems ranging from robotics to chemical engineering.

Deutsche Kurzfassung

Die Bedeutung der Regelungstechnik und Systemtheorie für die Analyse, den Entwurf und den Betrieb komplexer dynamischer Systeme ist immens. Das prototypische Problem des Reglerentwurfs ist die *Stabilisierung eines Arbeitspunktes*. Dabei ist es das Ziel, durch Wahl einer geeigneten Rückführung, die Trajektorien eines dynamischen Systems in der Umgebung eines festen Arbeitspunktes zu halten und die Konvergenz der Lösungen des geregelten Systems gegen diesen Arbeitspunkt sicherzustellen. Wenn anstelle eines festen Arbeitspunktes eine zeitveränderliche Referenztrajektorie stabilisiert werden soll, so spricht man von einem *Trajektorienfolgeproblem*. Typischerweise werden Anfahrprozesse von Reaktoren, Synchronisationsaufgaben oder Wechsel zwischen Arbeitspunkten entlang vorab berechneter Verläufe als Trajektorienfolgeprobleme formuliert. Für den Entwurf stabilisierender Rückführungen und Folgeregelungen sind verschiedene Methoden und Lösungsansätze bekannt. Jedoch lassen sich bei weitem nicht alle in der Praxis auftretenden Fragestellungen als Probleme der Stabilisierung von Arbeitspunkten oder der Trajektorienfolge auffassen.

Falls beispielsweise ein autonomes Fahrzeug einer Straße folgen soll, so ist nicht notwendigerweise auch die Geschwindigkeit vorgegeben. Stattdessen reicht es aus, das Fahrzeug auf der Straße zu halten und hinreichend schnell die Strasse entlang zu führen. Abstrakter gesprochen: Der Zustand oder der Ausgang eines Systems soll gegen eine geometrische Kurve konvergieren und dieser Kurve in vorgegebener Richtung folgen. Solche Fragestellungen bezeichnet man als *Pfadverfolgungsprobleme*. Der maßgebliche Unterschied zur Trajektorienfolge ist, dass die Geschwindigkeit der Bewegung entlang der Kurve, beziehungsweise entlang des Pfades, nicht fest vorgegeben ist. Vielmehr kann dieser Freiheitsgrad genutzt werden, um eine schnellere Konvergenz gegen den Pfad und eine geringere Abweichung zu erzielen. Typische Anwendungsbeispiele für Pfadverfolgungsprobleme sind die angesprochenen autonomen Fahrzeuge, unbemannte Fluggeräte, Roboter oder Fräs- und Werkzeugmaschinen. Weiterhin kann das Abfahren von Temperatur- oder Konzentrationsprofilen in verfahrenstechnischen Prozessen als Pfadverfolgungsproblem aufgefasst werden.

Die vorliegende Arbeit beschäftigt sich mit dem Entwurf von optimierungsbasierten Regelungsverfahren für Trajektorien- und Pfadverfolgungsprobleme unter Berücksichtigung von Beschränkungen der Stell- und Zustandsgrößen. Es werden neue, nichtlineare, modell-basierte, prädiktive Regelungsverfahren für beide Problemklassen vorgestellt. Unter anderem werden hinreichende Bedingungen für die Stabilität, beziehungsweise die Konvergenz, der vorgeschlagenen Verfahren entwickelt.

Als Hilfsmittel für den Entwurf prädiktiver Regler für Probleme der Trajektorienfolge werden dabei vorwärtsinvariante, zeitveränderliche Niveaumengen von Lyapunovfunktionen eingeführt. Dies erlaubt es, den Einzugsbereich der Regler deutlich zu vergrößern und dadurch die Regelgüte zu verbessern.

Außerdem werden Beiträge zur Beschreibung, Analyse und Lösung von Pfadverfolgungsproblemen mit Hilfe erweiterter Systemdynamiken und geeigneter Normalformkoordinaten geleistet. Es werden hinreichende Bedingungen für die exakte Verfolgbarkeit von Pfaden in Ausgangsräumen vorgestellt. Insbesondere wird auf Fragestellungen der Verfolgbarkeit von Pfaden unter Beschränkungen der Stell- und Zustandsgrößen eingegangen.

Aufbauend auf diese Ergebnisse zur Analyse werden neue, modell-basierte, prädiktive Regelungsansätze für Pfadverfolgungsprobleme unter Berücksichtigung von Beschränkungen entwickelt. Im Gegensatz zu existierenden Ansätzen für diese Probleme, zum Beispiel aus dem Bereich der Robotik, können hierbei Fälle berücksichtigt werden, bei denen das System nicht direkt auf dem Pfad startet. Es werden rigorose Bedingungen vorgestellt, welche garantieren, dass die Anwendung der vorgeschlagenen prädiktiven Regelungsschemata zur Konvergenz des Ausgangs gegen den zu verfolgenden Pfad führt. Die Ergebnisse der Arbeit werden anhand verschiedener Beispiele aus der Robotik und der Verfahrenstechnik illustriert.

Die Anliegen dieser Arbeit können in zwei Punkten zusammengefasst werden: Zum einen ist es das Ziel, die Anwendbarkeit der nichtlinearen modell-prädiktiven Regelung auf Probleme jenseits typischer Arbeitspunktstabilisierung aufzuzeigen. Zum anderen soll verdeutlicht werden, dass die Pfadverfolgung einen geeigneten Rahmen für die Formulierung und Lösung herausfordernder Fragestellungen aus verschiedenen Anwendungsgebieten – von der Robotik bis hin zur Verfahrenstechnik – bietet.

Index of Notation

Abbreviations and Acronyms

This list serves as a reference for abbreviations and acronyms.

CSTR	continuously stirred tank reactor
DAE	differential algebraic equation
LTI	linear time invariant
LTV	linear time-varying
LP	linear program
LQR	linear quadratic regulator
MIMO	multiple input multiple output
MPC	model predictive control
MPFC	model predictive path-following control
NMPC	nonlinear model predictive control
OCP	optimal control problem
ODE	ordinary differential equation
RDE	Riccati differential equation
QP	quadratic program
SISO	single input single output
w.l.o.g.	without loss of generality

Mathematical Notation

The following symbols are used throughout the thesis.

t	time variable
θ	scalar path parameter
x	state vector $x \in \mathbb{R}^{n_x}$
y	output vector $y \in \mathbb{R}^{n_y}$
u	input vector $u \in \mathbb{R}^{n_u}$
n_x	dimension of the real valued state vector x
n_u	dimension of the real valued input vector u
n_y	dimension of the real valued output vector y
f	vector field describing the system dynamics
x_0	initial condition of the system dynamics

$x(\cdot, t_0, x_0 \vert u(\cdot))$	state trajectory starting at t_0 at x_0 driven by an signal $u(\cdot)$; if an autonomous system is considered, we drop t_0 and write $x(\cdot, x_0 \vert u(\cdot))$
$\frac{\partial f_j(x)}{\partial x_i}$	partial derivative of the j-th component of f with respect to the i-th component of $x = (x_1, \ldots, x_i, \ldots, x_{n_x})^T$
$\nabla E(x)$	gradient of $E : \mathbb{R}^k \to \mathbb{R}$. $\nabla E = \left(\frac{\partial E}{\partial x_1}, \ldots, \frac{\partial E}{\partial x_k}\right)^T$
$\frac{\partial E(x)}{\partial x}$	transposed gradient $\frac{\partial E(x)}{\partial x} = (\nabla E(x))^T$
$\mathrm{L}_f\, h_i$	Lie derivative of h_i along f, i.e. $\mathrm{L}_f\, h_i = \frac{\partial h_i}{\partial x} f(x, u)$ whereby $h_i : \mathbb{R}^{n_x} \to \mathbb{R}$ and $f : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \to \mathbb{R}^{n_x}$
$\mathbf{I}^{n \times n}$	identity matrix of $\mathbb{R}^n$
$\mathbf{0}^{n \times m}$	zero matrix of $\mathbb{R}^{n \times m}$
$\mathrm{diag}(q_1, \ldots, q_n)$	diagonal matrix with entries $q_1, \ldots, q_n$
$Q \geq 0$	positive semi-definite matrix Q
$R > 0$	positive definite matrix R
$\Vert x \Vert$	2-norm of a vector $x \in \mathbb{R}^n$
$\Vert x \Vert_Q^2$	brief notation for $x^T Q x$, $Q \geq 0$
$\mathcal{X}$	set describing the state constraints $\mathcal{X} \subseteq \mathbb{R}^{n_x}$
$\mathcal{U}$	set describing the input constraints $\mathcal{U} \subseteq \mathbb{R}^{n_u}$
$\mathrm{int}\, \mathcal{A}$	interior of a set $\mathcal{A}$
$\partial \mathcal{A}$	boundary of a set $\mathcal{A}$
$\mathcal{K}$	set of class $\mathcal{K}$ functions, cf. Definition B.1, Appendix B
$\mathcal{KL}$	set of class $\mathcal{KL}$ functions, cf. Definition B.2, Appendix B
$\mathcal{C}^k$	set of k-times continuously differentiable functions mapping from $[t_0, t_1] \subseteq \mathbb{R}$ to $\mathbb{R}^n$ [1]
$\mathcal{PC}(\mathcal{A})$	set of piecewise continuous and right continuous functions mapping from $[t_0, t_1] \subseteq \mathbb{R}$ to $\mathcal{A} \subseteq \mathbb{R}^n$ [1]
$\mathcal{L}^p$	set of p-integrable functions—with $p \in [1, \infty)$—mapping from $[t_0, t_1] \subseteq \mathbb{R}$ to $\mathbb{R}^n$ [1]
$\mathcal{BC}(\mathbb{R}^{n \times n})$	set of elementwise bounded and elementwise continuous time-varying matrices on $\mathbb{R}^{n \times n}$
$\mathcal{BC}^+(\mathbb{R}^{n \times n})$	set of elementwise bounded and elementwise continuous time-varying matrices on $\mathbb{R}^{n \times n}$ which are symmetric and strictly positive definite
$\mathcal{BC}_0^+(\mathbb{R}^{n \times n})$	set of elementwise bounded and elementwise continuous time-varying matrices on $\mathbb{R}^{n \times n}$ which are symmetric and positive semi-definite

[1] The dimension $n < \infty$ and the size of the domain $[t_0, t_1]$ follow from the context.

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Part I

Basics and Introduction

1 Set Point Stabilization, Trajectory Tracking, and Path Following

Feedback is a fundamental concept, which is present in many technical, as well as non-technical systems. It is of great importance for the solution to manifold automation and control problems. An omnipresent and prototypical problem in control is *set point stabilization*. Consider a dynamical system with control

$$\dot{x}(t) = f(x(t), u(t)), \quad x(t_0) = x_0 \tag{1.1}$$

where $t \in \mathbb{R}$ is the time, $x \in \mathbb{R}^{n_x}$ is the state, $u \in \mathbb{R}^{n_u}$ is the input, and x_s is a set point to be stabilized. The stabilization problem can be stated as follows: Design a feedback $k : x \mapsto u$ such that the solutions $x(t, x_0|k(x))$—i.e., the solutions of (1.1) starting at x_0 driven by the feedback $u = k(x)$—stay close to the desired set point x_s and converge

$$\lim_{t\to\infty} \|x(t) - x_s\| = 0. \tag{1.2}$$

The stabilization problem is well understood for a wide range of systems: linear and nonlinear, continuous and discrete time, finite and infinite dimensional. Any many control tasks belong to this class. Typical examples are temperature control in building automation or the task to keep an unstable vehicle like a *SEGWAY*® in an upright position. Yet, not all control problems arising in applications are set point stabilization problems.

For example, consider the task of driving a car automatically along a road. From the driver's point of view two main objectives are evident. Keep the car on the road, *and* ensure that the car moves forward sufficiently fast. One can break this task down into two subproblems: control of the lateral position of the car on the road, and assignment of an admissible speed. Often both subproblems are solved as decoupled stabilization problems: One controller stabilizes the lateral position by using the steering angle as input. Another controller stabilizes the vehicle speed via engine thrust. Although quite simplistic this approach is successfully employed in practice. Modern drivers assistance systems such as cruise control, and lane assistance systems are instances of this decomposition approach. The achievable performance is, however, limited due to the problem reformulation.

Not necessarily it is required to keep the car in the middle of the road. It might suffice to stay on the road. If this is the case, one can take another point of view to formulate the problem. Firstly, compute *offline* a reference motion $x_r : [t_0, \infty) \to \mathbb{R}^{n_x}$ for the car along the road. And secondly, design a controller which tracks online this reference motion such that

$$\lim_{t\to\infty} \|x(t) - x_r(t)\| = 0. \tag{1.3}$$

This means that we determine the velocity along the road offline and design a controller to track this reference motion online. Literally speaking the trajectory $x_r : [t_0, \infty) \to \mathbb{R}^{n_x}$ is an explicit requirement *when to be where* on the road. Following this concept automatic car control is considered as a *trajectory-tracking problem.*[1]

In the control literature the aforementioned two problems of stabilization and tracking are predominant. Nevertheless, one can take a third point of view on the problem of driving a car automatically. Instead of breaking the problem down—either into two control tasks or into reference trajectory generation and tracking—we can consider it in the original form. This means that we regard the road as a geometric reference curve without any preassigned timing information. Let us assume we are given a description of the road as a geometric curve

$$\mathcal{P} = \{x \in \mathbb{R}^{n_x} \mid \theta \mapsto p(\theta)\}$$

whereby $p : \mathbb{R} \to \mathbb{R}^{n_x}$ is a parametrization of $\mathcal{P}$ and increasing values of θ denote forward movement on the road. Then the task can be formulated as

$$\lim_{t\to\infty} \|x(t) - p(\theta(t))\| = 0 \quad \text{and} \quad \dot{\theta} \geq 0. \tag{1.4}$$

Here, the first part reflects the objective of staying on the road. And the second part expresses the objective of moving forward. Clearly, the challenge of this problem formulation is that the timing along the track or specifically the map $t \mapsto \theta(t)$ is not given a priori. Rather it needs to be determined online in the controller. In other words, we have to design a controller which keeps the car on track and assigns a reference velocity online. Taking this point of view, automatic car control is considered as a *path-following problem.* While literally being more complicated, this formulation introduces additional freedom to the controller design—the speed to move along the curve—and does not limit the performance from the beginning.

[1]The terminology with respect to trajectory-tracking problems is not unified. We follow along the classic lines of [Athans and Falb 1966]. If the task is to track a trajectory defined in an output space and this reference trajectory is generated by an exogenous system—a so-called exo-system—then one denotes the problem either as *model-following problem, servo problem*, or as *output regulation problem.* The first two denominations stem from optimal control approaches towards the problem [Anderson and Moore 1990]. The latter is common if one refers to geometric approaches to tracking problems [Isidori 1995].