Andrew J. Majda, Xiaoming Wang

Nonlinear Dynamics and Statistical Theories for Basic Geophysical Flows

非线性动力学和统计理论在地球物理流动中的应用

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Preface

This book is an introduction to the fascinating and important interplay between non-linear dynamics and statistical theories for geophysical flows. The book is designed for a multi-disciplinary audience ranging from beginning graduate students to senior researchers in applied mathematics as well as theoretically inclined graduate students and researchers in atmosphere/ocean science. The approach in this book emphasizes the serendipity between physical phenomena and modern applied mathematics, including rigorous mathematical analysis, qualitative models, and numerical simulations. The book includes more conventional topics for non-linear dynamics applied to geophysical flows, such as long time selective decay, the effect of large-scale forcing, non-linear stability and fluid flow on the sphere, as well as emerging contemporary research topics involving applications of chaotic dynamics, equilibrium statistical mechanics, and information theory. The various competing approaches for equilibrium statistical theories for geophysical flows are compared and contrasted systematically from the viewpoint of modern applied mathematics, including an application for predicting the Great Red Spot of Jupiter in a fashion consistent with the observational record. Novel applications of information theory are utilized to simplify, unify, and compare the equilibrium statistical theories and also to quantify aspects of predictability in non-linear dynamical systems with many degrees of freedom. No previous background in geophysical flows, probability theory, information theory, or equilibrium statistical mechanics is needed to read the text. These topics and related background concepts are all introduced and developed through elementary examples and discussion throughout the text as they arise. The book is also of wider interest to applied mathematicians and other scientists to illustrate how ideas from statistical physics can be applied in novel ways to inhomogeneous large-scale complex non-linear systems.

The material in the book is based on lectures of the first author given at the Courant Institute in 1995, 1997, 2001, and 2004. The first author thanks Professor

Pedro Embid as well as his former Ph.D. students Professor Pete Kramer and Seuyung Shim for their help with early versions of Chapters 1, 2, 3, 4, and 6 of the present book. Joint research work with Professors Richard Kleeman and Bruce Turkington as well as Majdas former Courant post docs, Professors Marcus Grote, Ilya Timofeyev, Rafail Abramov, and Mark DeBattista have been incorporated into the book; their explicit and implicit contributions are acknowledged warmly. The authors acknowledge generous support of the National Science Foundation and the Office of Naval Research during the development of this book, including partial salary support for Xiaoming Wangs visit to Courant in the spring semester of 2001.

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Barotropic geophysical flows and two-dimensional fluid flows: elementary introduction

1.1 Introduction

The atmosphere and the ocean are the two most important fluid systems of our planet. The bulk of the atmosphere is a thin layer of air 10 km thick that engulfs the earth, and the oceans cover about 70% of the surface of our planet. Both the atmosphere and the ocean are in states of constant motion where the main source of energy is supplied by the radiation of the sun. The large-scale motions of the atmosphere and the ocean constitute geophysical flows and the science that studies them is geophysical fluid dynamics. The motions of the atmosphere and the ocean become powerful mechanisms for the transport and redistribution of energy and matter. For example, the motion of cold and warm atmospheric fronts determine the local weather conditions; the warm waters of the Gulf Stream are responsible for the temperate climate in northern Europe; the winds and the currents transport the pollutants produced by industries. It is clear that the motions of the atmosphere and the ocean play a fundamental role in the dynamics of our planet and greatly affect the activities of mankind.

It is apparent that the dynamical processes involved in the description of geophysical flows in the atmosphere and the ocean are extremely complex. This is due to the large number of physical variables needed to describe the state of the system and the wide range of space and time scales involved in these processes. The physical variables may include the velocity, the pressure, the density, and, in addition, the humidity in the case of atmospheric motions or the salinity in the case of oceanic motions. The physical processes that determine the evolution of the geophysical flows are also numerous. They may include the Coriolis force due to the earth's rotation; the sun's radiation; the presence of topographical barriers, as represented by mountain ranges in the case of atmospheric flows and the ocean floor and the continental masses in the case of oceanic flows. There may be also dissipative energy mechanisms, for example due to eddy diffusivity or Ekman drag. The ranges of spatial and temporal scales involved in the description of geophysical flows is also very large. The space scales may vary from a few hundred meters to thousands of kilometers. Similarly, the time scales maybe as short as minutes and as long as days, months, or even years.

The above remarks make evident the need for simplifying assumptions regarding the relevant physical mechanisms involved in a given geophysical flow process, as well as the relevant range of space and time scales needed to describe the process. The treatises of Pedlosky (1987) and Gill (1982) are two excellent references to consult regarding the physical foundations of geophysical flows and different simplifying approximations utilized in the study of the various aspects of geophysical fluids. Here we concentrate on large-scale flows for the atmosphere or *mesoscale* flows in the oceans. The simplest set of equations that meaningfully describes the motion of geophysical flows under these circumstances is given by the:

Barotropic quasi-geostrophic equations

$$\begin{aligned} &\frac{Dq}{Dt} = \mathcal{D}(\Delta)\psi + \mathcal{F}(\vec{x}, t) \\ &q = \omega + \beta y + h(x, y), \ \text{where } \omega = \Delta\psi \\ &\vec{v} = \nabla^{\perp}\psi = \begin{pmatrix} -\dfrac{\partial\psi}{\partial y} \\ \dfrac{\partial\psi}{\partial x} \end{pmatrix}, \end{aligned} \tag{1.1}$$

where $\frac{D}{Dt}$ stands for the advective (or material) derivative

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + v_1\frac{\partial}{\partial x} + v_2\frac{\partial}{\partial y}$$

and Δ denotes the Laplacian operator

$$\Delta = \operatorname{div}\nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.$$

In equation (1.1), q is the potential vorticity, $\vec{v}$ is the horizontal velocity field, ω, is the relative vorticity, and ψ is the stream function. The horizontal space variables are given by $\vec{x} = (x, y)$ and t denotes time. The term βy is called the beta-plane effect from the Coriolis force and its significance will be explained later. The term $h = h(x, y)$ represents the bottom floor topography. The term $\mathcal{D}(\Delta)\psi$ represents various possible dissipation mechanisms. Finally, the term $\mathcal{F}(\vec{x}, t)$ accounts for additional external forcing. The fluid density is set to 1.

Before continuing, we would like to explain briefly, in physical terms and without going into any technical details, the origin of the barotropic quasi-geostrophic equations. The barotropic rotational equations, also called rotating shallow water equations (Pedlosky, 1987), admit two different modes of propagation, slow and