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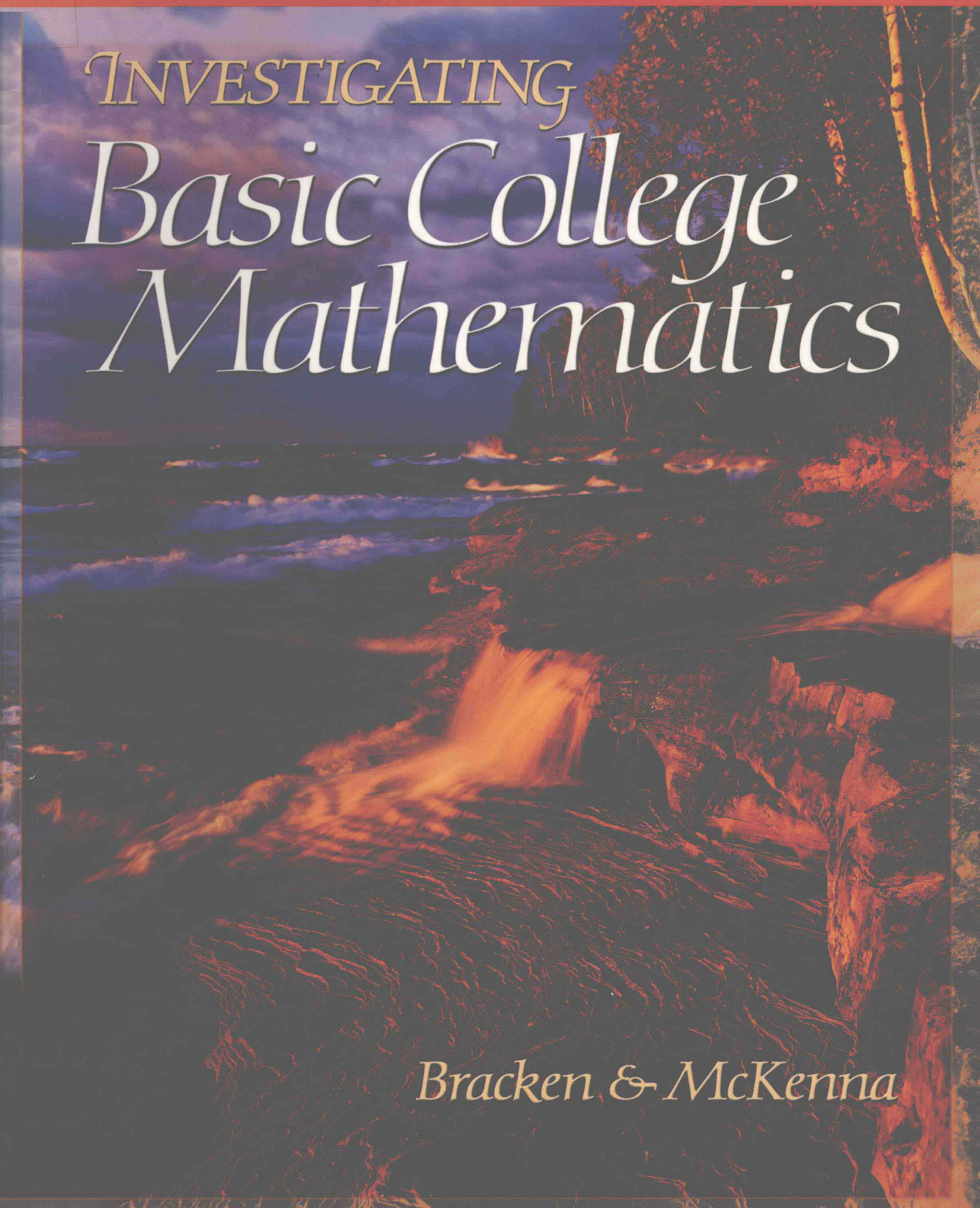

Investigating Basic College Mathematics

Bracken & McKenna

ANNOTATED INSTRUCTOR'S EDITION

INVESTIGATING Basic College Mathematics

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In honor of my Dad, a kind and encouraging father, and a good man.

— Laura Bracken

For Kirsty and Emily—
the next generation.

— Hazel McKenna

Index of Applications

This index organizes the context or setting of application problems by section (for example, in the entry "Sec. 1.4: 25," we are referring to problem number 25 in Section 1.4). To make concepts more accessible to students, many skill practice and word problems are set in familiar contexts or in an academic discipline. Even though students may not be ready to do truly authentic problem solving, they can begin to see the importance of mathematics in their disciplines and in their daily lives.

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To the Instructor

Investigating Basic College Mathematics is a flexible worktext that supports a variety of teaching methods and prepares students for their first course in algebra. The book engages students in active learning, allowing them to describe their understandings. It provides a comprehensive foundation in prealgebra concepts. This is accomplished through a blend of lecture-type *Discussions* and *Investigations* that help students take ownership of their learning. We wrote this text using activities and problems that we knew motivated our students and which were developed in our classrooms. We see our students changing from passive note-takers into energized and involved learners. In the spirit of the *Standards* of the American Mathematical Association of Two-Year Colleges (AMATYC), we use authentic applications and student-centered activities to teach the fundamental understandings needed for success in algebra.

CONTENT

The text is organized around the topics of whole numbers, integers, fractions, and decimals. Students learn to evaluate, simplify, graph, and solve. Problem solving and plane geometry are integrated throughout the text. Basic mathematics courses at different institutions range from three to five credits. We have included some topics, for example unit analysis, in such a way that they can be omitted without disturbing the rest of the course. Work with measurement systems is stressed throughout. If students are to do relevant work with applications, we believe they must have a basic understanding of units of measurement.

Chapter 1: Whole Numbers

Students work with basic concepts of number sense and organization of numbers and data including sets, number lines, place value, relative order, tables, and bar graphs. The axioms and algorithms of addition, subtraction, multiplication, and division of whole numbers are thoroughly reviewed, exponents are introduced, and the order of operations is used to evaluate expressions. Within the context of measurement, rounding and estimation is taught. The fundamental geometric concepts of angle, circle, degree, area, and volume are integrated. A five-step problem-solving strategy is used to help students develop their skills and confidence in solving word problems. Here the concepts of variable and equation have a just-in-time introduction. However, variables are limited to a single variable isolated on the left side of an equation.

Chapter 2: Integers

The work done with whole numbers is extended to the integers, including addition, subtraction, multiplication, division, exponents, and the order of operations. The concepts of opposite and absolute value are introduced. Students continue to become familiar with the concept of variable by solving problems using formulas. An optional section on line graphs (2.7) is also included.

Chapter 3: Fractions

The chapter begins with foundation work in multiples, divisors, prime and composite numbers, and factorization. Using concrete models, students investigate arithmetic operations with fractions and mixed numbers.

Chapter 4: Decimals

With a conceptual understanding of fractions, students are ready to study decimal notation. Using the concrete model of money, students extend the algorithms for addition, subtraction, multiplication, and division of integers to decimal numbers. They begin work with irrational numbers, including pi, allowing for extensive investigation of geometric formulas. Optional sections on unit analysis and scientific notation are also found in this chapter.

Chapter 5: Expressions and Equations

Students now use their skills with arithmetic to evaluate and simplify algebraic expressions. The properties of equality are used to solve equations in one variable. Equations are restricted to those with variables on one side of the equation. The final section integrates solving of equations and work with applications.

Chapter 6: Rates, Ratios, and Percents

With a conceptual understanding of fractions and decimals and the skills to solve simple equations, students are ready to study rates, ratios, and percents and their applications. An optional section on circle graphs is also included.

PEDAGOGICAL APPROACH

Developmental students and their instructors are a diverse group. Features in this text support the wide range of approaches used to teach basic mathematics and prealgebra to college students. Content is presented to students in *Investigations* and *Discussions.*

Investigations

The *Investigations* can be completed in cooperative groups of students, as teacher-led whole-class activities, or by a student working individually at home or in a self-paced lab setting. The purpose of the *Investigations* is to engage students in active learning, moving away from the passive approach that might have contributed to past failure. Unlike pure discovery learning, these are highly structured activities that often require the student to look for patterns and develop algorithms. The process of analyzing and describing helps students understand and retain important concepts and procedures. The *Investigations* are written in a worktext format to permit quick assessment of work. Each chapter of the text includes an *Investigation.* Additional *Investigations* are found in the *Instructor's Resource Manual* and may be reproduced for student use.

Discussions

The *Discussions* are a more traditional presentation of definitions and algorithms and include detailed examples. The *Discussions* can support lecture or video presentations, assist students in a self-paced setting, and help students complete exercises. *Investigations* and *Discussions* offer different perspectives for students of fundamental concepts.

Problem Solving

Integrated in both *Investigations* and *Discussions* is the five-step problem-solving strategy. Following the lead of AMATYC and the National Council of Teachers of Mathematics (NCTM), we ask students to apply concepts to the process of problem solving in almost every section. Extensive research and class testing was used to develop a large selection of relevant word problems. In our experience, students overcome their fear and loathing of word problems through extensive practice and success. Using the five-step strategy, students organize the information in the problem and express the relationships in both words and algebraic equations. Since this is difficult at first for most students,

we begin with straightforward, even obvious, problems. Once students master the approach, they are empowered to solve more difficult problems.

How to Use This Text

If you choose to teach predominantly with lecture, this book can be used the same way as any textbook. Assigning reading to be completed before class allows the student to be better prepared to ask questions on more difficult concepts. Such an assignment might include an *Investigation* to be done at home. Lecturing can include discussion of concepts and, perhaps, leading the students as a class through an *Investigation.* If you choose to include cooperative group activities, a class period will be a blend of *Investigations,* coaching, and lecturing over certain topics. As the class works, you will walk about the room, answering questions and checking work. Convenient grading rubrics are included in the *Instructor's Resource Manual.* You may discover that the class shares a common struggle or misunderstanding and choose to interrupt them during the activity for a few minutes of explanation or to complete an additional example. When the activity concludes, you may move the students directly into another activity or use a summary of the findings to move into a short lecture on another topic.

FEATURES

Text

Investigating Basic College Mathematics contains several important and unique features. Included in the text narrative are *Sneak Previews, Examples,* and *Check Your Understanding* problems. The objectives from the *Sneak Preview* are included in the student text to support a brief "this is where we are going" and "this is where we've been" summary at the beginning and end of class. As a summary of what students are expected to do at the completion of the section, these also can serve as a study aid. The problems in the *Examples* provide detailed step-by-step explanations. We were careful to include every step to help students working on their own. The *Check Your Understanding* problems provide student practice with immediate feedback. These can also be used for additional in-class practice following an *Investigation* or lecture.

Exercises

The *Exercises* are a blend of skill practice and application problems set in relevant contexts. Except for the word problems, the exercises are arranged in a worktext format to facilitate efficient grading. In the skill practice problems, the same numbers are often repeated, accompanied by a slight change in operations or grouping. When the numbers are the same but the result is different, students have an opportunity to see the effect of such changes. The word problems are factual and authentic, drawn from the media, other disciplines, and reference resources. These real-life applications are often motivating for developmental students. However, we believe that a balance of problems is essential. Students should not be misled into thinking that the only valuable mathematics is applied mathematics.

At the end of each set of problems, we include a unique feature called *Error Analysis.* Students are asked to describe the error made in a problem and then redo the problem correctly. The errors are representative of common student mistakes at this level. Our students often comment on the insight they gain from completing these problems. Also included are a limited number of *Review Questions.* These questions reinforce concepts and skills previously learned, particularly those prerequisite to the next section.

End-of-Chapter Materials

Included in the end-of-chapter materials are *Read, Reflect, and Respond* selections, a *Glossary and Procedures* list, a *Chapter Review,* and a chapter *Test.* The *Read, Reflect, and Respond* selections ask

students to thoughtfully consider ideas about learning and mathematics and to reflect on their own practices. Often developmental math students often have barriers to learning mathematics that have little to do with mathematical processes. These barriers include ineffective study skills and negative attitudes about mathematics. However, there is often little time to discuss these barriers in the classroom. The *Read, Reflect, and Respond* selections give students additional information about learning and studying, mathematics history, and the usefulness of mathematics. Students have the opportunity to think about how they might change their previous behaviors and attitudes. If you assign and assess these selections, you can privately connect with your students using brief written comments. If you have a writing-across-the-curriculum initiative, this allows you to integrate writing in a nonthreatening and interesting way.

Student Study Aids

New terms in both the *Investigations* and the *Discussions* are in bold type. Students are asked to keep a list of algorithms and concepts in their own words in the *Glossary and Procedures* section, which can be useful for learning and reviewing. You may wish to require students to prepare note cards or other study aids for these terms. The *Chapter Review* may be assigned as a study aid for the chapter test. Students may also benefit from using the chapter *Test* as a timed practice test before the class exam. All of the answers to the odd-numbered problems in the exercises, chapter reviews, and chapter tests are listed as an appendix.

Calculators

We do not mark exercises or examples as intended for completion with a calculator. Although AMATYC strongly supports the appropriate use of calculators at this level of mathematics, all departments and instructors do not share this position. Some departments also are constrained by the restriction of calculators on state competency exams. Instructors teaching under these constraints often prefer that calculator use not be specified by the text for particular problems. We leave the decision of appropriate technology use at this level to you and your department. On a personal level, we both require students to be able to perform the basic arithmetic algorithms without calculators on "reasonable problems." However, our views of "reasonable problems" differ considerably. We both agree that using calculator support in word problems allows students to concentrate on the process of problem solving instead of being mired in arithmetic errors.

SUPPLEMENTS

For the Instructor

Annotated Instructor's Edition (0-534-39307-1) This special version of the complete student text contains a Resource Integration Guide as well as answers printed next to the respective exercises.

Test Bank (0-534-40503-7) The test bank includes 8 tests per chapter as well as 3 final exams. The tests are made up of a combination of multiple-choice, free-response, true/false, and fill-in-the-blank questions.

Instructor's Resource Manual (0-534-40501-0) The Instructor's Resource Manual provides additional comments, suggestions, and materials for teaching with the text. It also contains the complete worked out solutions to all of the problems in the text.

BCA Testing (0-534-40505-3) With a balance of efficiency and high performance, simplicity, and versatility, *Brooks/Cole Assessment* gives you the power to transform the learning and teaching experience. This revolutionary, internet-ready testing suite is text-specific and allows instructors to customize exams and track student progress in an accessible, browser-based format. BCA offers full algorithmic generation of problems and free-response mathematics. No longer are you limited to multiple-choice or true/false test questions. The complete integration of the testing and course management components simplifies your routine tasks. Test results flow automatically to your gradebook and you can easily communicate to individuals, sections, or entire courses.

Text-Specific Videotapes (0-534-40506-1) This set of videotapes is available free upon adoption of the text. Each tape offers one chapter of the text and is broken down into 10 to 20 minute problem-solving lessons that cover topics and concepts in each section of the chapter.

For the Student

Student Solutions Manual (0-534-40500-2) The student solutions manual provides worked out solutions to the odd-numbered problems in the text.

Investigations Manual (0-534-40502-9) The Investigations Manual contains additional exercises related to the text that can be worked in a group setting or in a self-paced environment. This manual helps the student to further learn about and understand the concepts and skills presented in the text.

Website **www.brookscole.com/mathematics**
The Brooks/Cole Mathematics Resource Center offers book-specific student and instructor resources, discipline specific links, and a complete catalog of Brooks/Cole mathematics products.

BCA Tutorial Instructor and Student Versions This text-specific, interactive tutorial software is delivered via the web (at http://bca.brookscole.com) and is offered in both student and instructor versions. Like *BCA Testing,* it is browser based, making it an intuitive mathematical guide even for students with little technological proficiency. So sophisticated, it's simple, *BCA Tutorial* allows students to work with real math notation in real time, providing instant analysis and feedback. The tracking program built into the instructor version of the software enables instructors to carefully monitor student progress.

Interactive Video Skillbuilder CD (0-534-40504-5) Think of it as portable office hours! The Interactive Video Skillbuilder CD-ROM contains more than 8 hours of video instruction. The problems worked during each video lesson are shown next to the viewing screen so that student can try working them before watching the solution. To help students evaluate their progress, each section contains a 10-question Web quiz (the results of which can be emailed to the instructor) and each chapter contains a chapter test, with answers to each problem on each test. The CD also includes MathCue Tutorial software. This dual-platform software presents and scores problems and tutor students by displaying annotated, step-by-step solutions. Problem sets may be customized as desired.

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Investigating Basic College Mathematics is a text that is flexible enough to meet the needs of instructors with a variety of teaching methods and student populations. We know this because of our own experiences: Our students and our teaching styles are very different and we both teach successfully with this text. We welcome your comments and suggestions.

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To the Student

You are beginning an important task, learning the math you need for your future. It is not enough to simply want to succeed. You must put in the time and effort that is necessary for you to learn. Learning differs from person to person, and it is important for you to discover how you learn best. Attending class, doing homework, asking questions, preparing for tests, and asking for extra help when you do not understand are all part of this required effort. Learning math does not have to be done by yourself. In fact, the support and insights of other students can really help you learn math. If possible, complete the following activity with four of your fellow students.

INVESTIGATION Positive and Negative Experiences

In this first investigation we will explore how a group can work together to learn mathematics. When investigations are done in a group, we should take turns reading the information aloud. This helps students who learn best when they hear information. *In general, a new paragraph means a new reader.*

Some topics will be review for some group members but will be new topics for others. Those of us who are learning a concept for the first time should not compare our speed and progress to others. This is not a race. We do not get extra points for being done first. We can take the time to help each other.

One way to learn mathematics is to work in cooperative groups. Such groups are most effective when everyone participates. Desks or chairs should be arranged to allow for convenient interaction. We should speak in voices that are clear and easy to hear, but not so loud that other groups are distracted. We must work together on the same problem. Our instructor will monitor our progress to make sure we are going fast enough. We may ask the instructor a question only if the entire group has discussed the question and cannot agree on an answer. This ensures that we work together and that we learn to use each other as resources. We need to be ready to interrupt our group work to listen to our instructor provide extra examples or clarification. When we reach a "stop sign," we need to have our work checked by our instructor.

Each individual member brings different experiences in mathematics to this class. Our goal is to learn with each other, drawing on each other's strengths and helping each other. To begin this process, we will break our group into pairs. Each of us will describe to our partner *the most positive experience* we ever had in learning mathematics and *the most negative experience* we ever had in learning mathematics. Our partner will take notes about our experiences.

We will then rejoin our other group members. Each person will tell the group about the positive and negative experiences of his or her partner. Finally, we will decide on one positive and one negative experience to report to the whole class. One person should volunteer to be the "reporter."

1. Names of your group members (first and last):

2. Positive experiences of your partner:

3. Negative experiences of your partner:

4. Positive experiences of your group to report:

5. Negative experiences of your group to report:

Your personal attitudes about math can affect your success in learning math. Although you do not have to like math to learn it, it is important to think about your personal attitudes and their impact on your actions. Before you go any further in this course, we strongly recommend that you complete the following reading and answer the questions.

READ, REFLECT, AND RESPOND Assessing Your Attitudes

READ AND REFLECT

Thinking about your attitudes toward learning mathematics can help you discover barriers that may be holding back your progress. Begin by filling out the following questionnaire.

If you Strongly Agree, score 5.
If you Agree Most of the Time, score 4.
If you Agree Sometimes, score 3.
If you Disagree Most of the Time, score 2.
If you Strongly Disagree, score 1.

___ **1.** I feel sick when I go to math class.
___ **2.** When math teachers talk, I find myself unable to concentrate.
___ **3.** Although I attend my other classes regularly, I often miss math class.
___ **4.** I know I am probably going to have to take this class over.
___ **5.** I can do all the homework fine but when it comes to a test I just go blank.
___ **6.** I never get math the first time anyone explains it to me.
___ **7.** I often leave my math homework until the very last.
___ **8.** I've never been good at math.
___ **9.** I just dread going to class, I always feel so stupid.
___ **10.** I really worry that the teacher will call on me for an answer.
___ **TOTAL**

If your score is below 30, your attitudes toward mathematics are probably not affecting your success in math classes. The closer your score approaches 50, the more your feelings about math are preventing you from learning math.

Math anxiety is a term educators use for the collection of feelings described in the questionnaire statements. Most people experience some anxiety when learning something new and difficult, a normal feeling that actually increases concentration and motivation to study. However, too much worry can prevent learning. High levels of math anxiety may cause students to do things that stop learning such as skipping class or avoiding homework. Before or during class, anxious students may experience an uncomfortable increase in heart rate, get a pounding headache, feel sweaty or chilled, or become nauseous.

People with math anxiety can learn to control and diminish their fear enough to do well in math classes. However, good intentions are only the first step. Changing patterns of behavior and attitudes requires thoughtful practice. Some suggestions follow. Sometimes only one change in behavior can result in a huge reduction in math anxiety. If these suggestions do not help, seek the advice of a trained counselor at your school. These professionals are trained to help identify the causes of your fears and help you control them.

SUGGESTION 1 Discover the sources of your feelings about math.

As described in the following true stories, math anxiety may begin with a particular experience.

When Randi was in third grade, she moved to a school where her teacher seated the kids depending on their score on their last math test. Randi had to take a test the second day she was at her new school. She did very poorly because she hadn't learned double digit multiplication yet. She was moved to the end of the very last row. She stayed near that seat the rest of the year, embarrassed and frightened that she would never catch up. Now she is an intensive care nurse. She still worries that she will make a mistake on a dosage calculation and hurt a patient.

Mark's favorite subject was math until he had to learn his multiplication facts. He tried to remember them on the timed tests but the harder he tried, the worse he did. His parents made him work on them every night. The other kids got to do fun stuff but he had to keep doing flash cards to learn his tables. He never did get them. Math just got worse then because multiplication and division depended on knowing the facts. Mark now hates math and avoids it whenever possible.

Hiro has struggled with math all his life. People seem to expect him to be good at math just because he is Japanese. This really bothers him. He is worried that his teacher will expect him to learn new procedures with only one explanation.

When Nicole started algebra in ninth grade, she just did not get it. There were so many steps that she couldn't remember. When she asked questions, her teacher acted like she was wasting his time. Her dad tried to help her but he got impatient with her, too. At the end of the year, Mr. Anderson gave her a "D" instead of the "F" she should have received. He told her he did not want to put up with her for another year and she would probably just get married anyway. Nicole got a D in her next math class, too.

Laurie was always good at writing but never very good at math. Her brother was really good at math. Her mom always told her that it was the same way in her family. Her dad told her not to worry, just to try her best. Laurie avoided all classes that required math like chemistry. She figured there was no point in working hard at something that wasn't her thing anyway.

John quit school when he was seventeen. He wanted to move out and he needed a full time job to do that. After a while, his girlfriend talked him into getting his GED. He did the math for it with the help of a tutor. Now John wants to become a police officer which requires completion of a college math course. He is very scared of failing; he doesn't remember how to do much.

Many students in this class fear or even hate mathematics. These feelings often began with a negative experience like these. The first step in controlling fear of mathematics may be to identify where and when it started. Fortunately, attitudes can change. Past experiences can be left behind.

SUGGESTION 2 Encourage yourself; tell yourself that you can succeed.

Start to believe you can succeed by telling yourself that you can succeed. Talk to yourself before every class, before every homework assignment, and before every quiz or test. Try saying:

- I am a person who does many things well. Math may be difficult for me but I can succeed at it, too.
- I will judge myself by my own standards. I decide if I have succeeded or failed. I may learn slower than others do but the important thing is that I can learn.
- I am going to do the work I need to do to succeed, even if I do not enjoy it.
- I am willing to ask for help because I want to succeed. I want to succeed more than I am worried about looking stupid.
- I do not have to understand everything the first time it is explained to me.

SUGGESTION 3 Visualize yourself succeeding.

Identify those people in your math class that you think or know are successful. How do they act? How do they interact with other students? How do they interact with the instructor? Where do they sit? What did they bring with them to class? Now zero in on such a person who seems to be somewhat like you. Perhaps they are the same age or in the same situation. Perhaps they are from your culture. This is a successful person who shares some of your experiences and values. Visualize yourself as a successful person in math class. See yourself doing the things needed to succeed. Do this before class, before you do homework, when you feel fear settling in your stomach, whenever you have the urge to procrastinate preparing for class.

SUGGESTION 4 Evaluate the way you study and prepare for math class and tests.

Different people need to use different strategies for preparing for tests. When students find an effective way to study, class is not so confusing; homework is possible to complete; tests are less frightening. The more success you have, the less you fear; the anxiety goes down more. Do not depend on luck to do well; prepare carefully and thoughtfully. Ask your instructor for help and guidance.

Think about your own attitudes toward math. Are you afraid of math? If so, can you think of any event or events in your life where this attitude began? To build new patterns of success in mathematics that will help dissipate your fear, start encouraging yourself. Get into the habit of talking and thinking positively. Visualize yourself being successful in mathematics and then start acting like that person. Talking and thinking positively are important but not sufficient. You need to study effectively for math tests. This may require you to make some changes in the way you study.

RESPOND

1. What is your total score on the survey in this reading?

2. Describe your feelings/attitudes about math and learning math.