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PHILOSOPHY  
*and the*  
LABYRINTHS  
OF THE MIND

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ROY  
SORENSEN



A BRIEF HISTORY  
*of the*  
**Paradox**

PHILOSOPHY AND THE  
LABYRINTHS OF THE MIND

*Roy Sorensen*

**OXFORD**  
UNIVERSITY PRESS

2003

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UNIVERSITY PRESS

Oxford New York  
Auckland Bangkok Buenos Aires Cape Town Chennai  
Dar es Salaam Delhi Hong Kong Istanbul Karachi Kolkata  
Kuala Lumpur Madrid Melbourne Mexico City Mumbai Nairobi  
São Paulo Shanghai Taipei Tokyo Toronto

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Published by Oxford University Press, Inc.  
198 Madison Avenue, New York, New York 10016  
www.oup.com

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Library of Congress Cataloging-in-Publication Data  
Sorensen, Roy. A.

A brief history of the paradox: philosophy and the labyrinths  
of the mind/ Roy Sorensen.

p. cm.

Includes bibliographical references and index.

ISBN 0-19-515903-9

1. Paradox. 2. Paradoxes. I. Title.

BC199.P2S67 2003

165—dc21 2003048631

Permission to print V. Alan White's "Antimony" kindly granted by the author.

Book design by planettheo.com

9 8 7 6 5 4 3 2 1

Printed in the United States of America  
on acid-free paper

A BRIEF HISTORY  
*of the*  
Paradox

To  
those who never  
have a book dedicated to them.

There are two famous labyrinths where our reason very often goes astray: one concerns the great question of the Free and the Necessary, above all in the production and the origin of Evil; the other consists in the discussion of continuity and of the indivisibles which appear to be the elements thereof, and where the consideration of the infinite must enter in. The first perplexes almost all the human race, the other exercises philosophers only.

—Gottfried Leibniz, *Theodicy*

Here and elsewhere we shall not obtain the best insight into things until we actually see them growing from the beginning . . .

—Aristotle, *Politics*

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## Preface

Mathematicians characterize prime numbers as their atoms because all numbers can be analyzed as products of the primes. I regard paradoxes as the atoms of philosophy because they constitute the basic points of departure for disciplined speculation.

Philosophy is held together by its questions rather than by its answers. The basic philosophical questions come from troubles within our ordinary conceptual scheme. These paradoxes bind generations together with common problems and an accumulating reservoir of responses.

Philosophy is generally presented in terms of its issues or in terms of its history. A study of the history of paradoxes provides an opportunity to practice both approaches simultaneously.

This book is guided by an anthropological hypothesis: paradoxes developed from the riddles of Greek folklore (as did the oracles of Delphi, Christian catechisms, and the game of charades). Accordingly, I begin classically with the Greek philosophers. They refined informal verbal dueling into "dialectic," the procedure best known through Plato's dialogues. The efforts of the Greeks were improved in turn, yielding contemporary logic and dialectical conceptions of history and science.

Paradoxes are questions (or in some cases, pseudoquestions) that suspend us between *too many* good answers. When an amoeba divides in two, does it go out of existence? On the one hand, organisms can survive the loss of half of their bodies. The only problem with the mother amoeba is that she has been too successful; instead of losing half her body as a dead tissue, she has created a second healthy amoeba. On the other hand, amoeba reproduction seems like suicide because there is nothing to survive *as*. It would be arbitrary to identify the mother amoeba with just one of her daughters. And to say that the mother amoeba continues as the pair of daughters conflicts with the idea that organisms are unified individuals.

Typically, the case for one solution to a paradox looks compelling in isolation. The question is kept alive by the tug of war between evenly matched contestants. The Greeks were intrigued by surprising, enduring oppositions such as these.

Common sense may seem like a seamless, timeless whole. But it really resembles the earth's surface; a jigsaw puzzle of giant plates that slowly collide and rub against each other. The stability of terra firma is the result of great forces and counterforces. The equilibrium is imperfect; there is constant underlying tension and, occasionally, sudden slippage. Paradoxes mark fault lines in our common-sense world.

Do these fissures reach into reason itself? Many philosophers urge us to follow the argument wherever it leads; in the case of Socrates, even to death. But what do we do when compelling arguments lead us in *conflicting* directions?

One radical response, pioneered by Heraclitus, is to accept the reality of contradictions. He thinks the paradoxes are out there. This line of thought has been extended by



Hegel, Marx, and nowadays, by the dialethic logicians of Australia.

At the other extreme are those who trace our inconsistency to reliance on our senses. Parmenides dismisses the appearance of there being many things that are changing and moving. He conceived of reality as a single, unified whole. Zeno's paradoxes were intended to reinforce Parmenides's conclusion by extracting absurdities from common sense.

Most philosophers are moderates who try to reconcile perception with reason. Democritus's compromise was a changing universe of complex objects built up from unchanging, indivisible atoms moving about in the void. Rationalists pitch the negotiation in reason's favor. They trace paradoxes to shortages of a priori insights. With the rise of science, empiricists have driven a hard bargain in the opposite direction. They trace paradoxes to a glut of misinformation. If we could cleanse ourselves of superstition and subtler contaminants, we would gain the patience needed to answer what riddles can be answered and the maturity to admit ignorance when at the outer range of our senses. Paradoxes have both shaped and been shaped by the classic debate between rationalists and empiricists. A faithful portrayal of paradoxes situates them in their natural intellectual environments. Without this background, they take on the appearance of circus animals.

I concede that paradoxes *sometimes* ought to be studied in isolation. Logicians and mathematicians routinely assemble paradoxes in a clinical setting. Antinomies, paralogisms, and sophisms are stood before the reader like draftees at a mass medical screening. Much has been learned by analytical methods that ignore the bigger picture. But why *always* ignore the bigger picture?

In any case, I am interested in the developmental and antiquarian aspects of paradoxes. Consequently, my approach is more leisurely. Although I have my own theory of paradoxes, my general intent is to have the paradoxes enter at their own initiative and in their original order.

The deepest paradoxes are extroverts, naturally good at introducing themselves. These challenges to compulsory, universal beliefs are self-illuminating; they stimulate us to draw distinctions and formulate hypotheses that bear on the issue of how we ought to react to paradoxes. Is common sense ever mistaken? Are paradoxes symptoms of the frailties of human reason? Do they point to ineffable truths? When is it rational to ignore arguments?

When Aristotle's nephew Callisthenes volunteered to record the expedition of Alexander the Great, he had to follow the impetuous Alexander into situations that invited miscalculation. The discoverers of paradoxes expose their historians to a parallel danger. From what appears to be a safe distance, I see the inquirer crane his head for a better look, eventually placing one foot on one solid-looking principle and the other foot on a second principle that is actually incompatible with the first. In my eagerness to document his insecure footing, I risk misstep myself. In the following pages, I take this chance over and over, across two millennia. Sooner or later, I must share the fate of those I chronicle. I apologize for these errors but am grateful to those who led me up to a position to make them.

I also have more specific acknowledgments. I thank the editor of *Mind* for permission to reprint, in chapter one, a portion of "The Egg Came Before the Chicken," *Mind* 101/403 (July 1992): 541-42. I am grateful to V. Alan White for

permission to quote “Antinomy” from his website devoted to philosophy songs at [www.manitowoc.uwc.edu/staff/awhite/phisong.htm](http://www.manitowoc.uwc.edu/staff/awhite/phisong.htm). Finally, I thank colleagues and my students at Dartmouth College for their comments and suggestions on earlier drafts of this book.

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# Anaximander and the Riddle of Origin

“... 5, 1, 4, 1, 3—*Done!*” exclaims a haggard old man.

“You look exhausted, what have you been doing?”

*“Reciting the complete decimal expansion of  $\pi$  backwards.”*

So goes one of Ludwig Wittgenstein’s philosophical jokes. A beginningless individual borders on contradiction. Yet philosophy itself may have begun by embracing this absurdity. For this is Anaximander’s (ca. 610 B.C.–585 B.C.) solution to the first paradox in recorded history.

## WHERE DO WE COME FROM?

People are interested in tracing their ancestral lines. Anaximander generalized this curiosity. He notes that each human being begins as a baby who survives only if nurtured. Anaximander infers that the first human beings were cared for by

animals. The Greeks knew of sharks that gave birth to live, autonomous young. Anaximander conjectured that the first human beings were born from aquatic creatures who then reared them.

But where did our animal ancestors come from? Here again, Anaximander seems ahead of his time. He infers that these creatures had inanimate precursors.

What were the precursors of *those* precursors? However long we continue the series, it makes sense to ask, what happened before that? Yet it seems impossible for history to be without a beginning. Isn't that the point of Wittgenstein's joke?

Perhaps some of Anaximander's contemporaries tried to precisely formulate the absurdity as an impossible wait: If there is an infinite past, then an infinite amount of time would have had to elapse to reach the present moment. An infinite wait is endless. But here we are at the present moment! Therefore, the past must have a beginning.

Unlike Anaximander, readers of this book are at home with negative numbers. We can model an infinite past by letting 0 represent the present moment, -1 represent yesterday, -2 the day before yesterday, and so on. For us, the fact that there are infinitely many numbers before 0 does not raise a mystery about how 0 can be reached. Why should an infinite past be any more puzzling than the infinite sequence of negative integers?

This mathematical model seems apt for an infinite future. +1 could be tomorrow, +2 could represent the day after tomorrow, and so on. You can imagine encountering an immortal destined to count forever. Each positive integer will be counted by this number god.

But negative numbers are not enough to solve the paradox of origin. There is a “something from nothing” feel about the claim to have recited infinitely many digits.

### WHAT IS A PARADOX?

When discussing whether the barbarians originated philosophy, Diogenes Laertius reports, “As to the Gymnosophists and Druids we are told that they uttered their philosophy in riddles . . . “ I take paradoxes to be a species of riddle. The oldest philosophical questions evolved from folklore and show vestiges of the verbal games that generated them.

Seduction riddles are constructed to make a bad answer appear as a good answer. How much dirt is in a hole two meters wide, two meters long, and two meters deep? This question entices us to answer, eight cubic meters of dirt. The riddler then reminds us that no dirt is in a *hole*.

Mystery riddles, in contrast, appear to have no answer. One way to achieve this aura of insolubility is by describing an object in an apparently contradictory way. As a boy, Anaximander must have been asked the ancient Greek riddle, “What has a mouth but never eats, a bed but never sleeps?” (Answer: A river.) Literary riddles elaborate the genres found in folklore. Anaximander probably learned of the riddle of the Sphinx from Hesiod’s *Theogony*. We know it best from Sophocles’ play *Oedipus the King*. The Sphinx is a monster who challenges travelers with a riddle she learned from the Muses: “What goes on four legs in the morning, two legs in the afternoon, three legs in the evening?” She wants her victims to remain ignorant of the underlying metaphors.



Oedipus answers by *decoding* the question: At the dawn of life, a baby begins life on all fours, then learns to walk upright on two legs, and finally spends his twilight years hobbling around with a cane. Tragically, Oedipus fails to solve deeper question of his own origin (continuously posed by the blind prophet Tiresias in his “riddling speech”).

With most mystery riddles, there is little hope of understanding the question until after the answer is revealed. Two weeks before flying a plane into one of the World Trade Center’s towers, Mohammed Atta phoned Ramzi Binalshibh asking help with a riddle: Two sticks, a dash and a cake with a stick down—what is it? Binalshibh was baffled. After the attack on September 11, he realized that two sticks stand for 11, a dash is a dash and a cake with a stick down signifies 9.

Sometimes the riddler himself is in the dark. When the Mad Hatter asks Alice, “Why is a raven like a writing desk?” he has no idea of what the answer is. Neither did the creator of the Mad Hatter, the logician Lewis Carroll.

The poser of a paradox need not drape its meaning behind ambiguities and metaphor. He can afford to be open because the riddle works by overburdening the audience with too many good answers. Consider the folk paradox, “Which came first, the chicken or the egg?” The egg answer is backed by an apparently compelling principle: Every chicken comes from an egg. The trouble is that there is an equally compelling principle supporting the opposite answer: Every egg comes from a chicken.

Bodies of conflicting evidence are usually unstable. Our ambivalence gets washed away by further witnesses, new measurements, and recalculations. In contrast, paradoxes are exceptionally bouyant. Whenever one side seems to prevail, balance is restored by a counterdevelopment. From engineer-