

N.M. Belyaev

**Strength of
Materials**



Н. М. БЕЛЯЕВ

СОПРОТИВЛЕНИЕ МАТЕРИАЛОВ

ИЗДАТЕЛЬСТВО «НАУКА» МОСКВА

N. M. BELYAEV

Strength of Materials

*translated from the Russian
by N. K. Mehta*

MIR PUBLISHERS MOSCOW

First published 1979

Revised from the 1976 Russian edition

На английском языке

© Издательство «Наука», 1976

© English translation, Mir Publishers, 1979

Printed in the Union of Soviet Socialist Republics

Nikolai Mikhailovich Belyaev

(1890—1944)

Nikolai Mikhailovich Belyaev occupied a leading position among eminent Soviet scientists who worked on the technical application of theory of elasticity and strength of materials and structures.

After graduating from the St. Petersburg Institute of Railway Engineering in 1916, Nikolai Mikhailovich Belyaev was invited to stay at the Strength of Materials Department, where he worked under S. P. Timoshenko.

Nikolai Mikhailovich Belyaev was associated with this institute (now the Leningrad Institute of Railway Engineering) throughout his life. At the institute he taught subjects like engineering structures, bridges, theoretical mechanics, strength of materials and theory of elasticity, and from 1924 to the end of his life was Head of the Strength of Materials Department.

All his life Nikolai Mikhailovich Belyaev was a leading engineer and research worker. He was the first to formulate and solve the problem of stability of prismatic bars under variable axial loading—a problem interesting from the theoretical aspect and important from the point of view of applications. Simultaneously, Nikolai Mikhailovich Belyaev worked on the problem of local stresses in bodies in contact under compression. Here he considerably developed the works of Hertz. The work first published by Nikolai Mikhailovich Belyaev in 1924 has completely retained its value to this day.

In the Soviet Union Belyaev was one of the first to undertake the study of the theory of plastic deformation, and he contributed a lot towards the development of this field.

Nikolai Mikhailovich Belyaev spent the last years of his life in fruitful research on problems of creep and relaxation of metals under high temperatures.

Nikolai Mikhailovich Belyaev was a rare talent who successfully combined theory with experimental research. In 1924 he took over as Head of the mechanical engineering laboratory of the Leningrad Institute of Railway Engineering, and in the course of 16 years of administration changed the laboratory into a leading scientific research centre.

New technical specifications ensuring long and reliable performance of rails were compiled as a result of the research conducted at the laboratory under the guidance and with direct participation of Nikolai Mikhailovich Belyaev. These specifications with minor additions are in force to this day.

Research done by Nikolai Mikhailovich Belyaev in the field of technology of concrete won wide acclaim all over the Soviet Union.

The pedagogical activity of Nikolai Mikhailovich Belyaev was not restricted to the Leningrad Institute of Railway Engineering. He worked at the Leningrad Technological Institute (1919-1926), Leningrad Institute of Civil Aviation (1931-1934), and from 1934 onwards was Head of the Strength of Materials Department at the Leningrad Polytechnical Institute—the biggest institute in the country.

In 1939 Nikolai Mikhailovich Belyaev was elected Corresponding Member of the USSR Academy of Sciences, and from 1942 occupied the post of Deputy Director of the Institute of Mechanics of the Academy of Sciences of the USSR.

His book *Strength of Materials* has won wide recognition in the USSR.

Preface to the Fifteenth Russian Edition

The new edition of *Strength of Materials* by N. M. Belyaev has been published after 11 years. In 33 years that lapsed between the publication by N. M. Belyaev of the first edition in 1932 and the last fourteenth edition in 1965 a total of 675 000 copies of the book were sold, testifying to its wide popularity. During this period the book was periodically enlarged and revised by N. M. Belyaev and, after his death in 1944, by a group of four of his co-workers. This group, which prepared from the fifth to the fourteenth editions for publication, did not consider it proper to make substantial changes in the original work of N. M. Belyaev. Additions were done at one time or another only when they became absolutely necessary due to changes in standards and technical specifications and in the light of recent research.

In the present edition, prepared by the same group, a number of topics have been dropped either owing to their irrelevance to strength of materials or because they are rarely taught in the main course. The topics that have been dropped include Contact Stresses, Riveted Beams, Reinforced Concrete Beams, Approximate Methods for Calculating Deflection of Beams, Beams on Elastic Foundation, Design of Thin-walled Bars, all graphical methods, and a part of Complicated Problems of Stability Analysis, the other part of the last topic being presented in an abbreviated version. The reader may refer to the earlier editions of this book or special monographs in case information is required on these topics.

Considering the availability of a large number of problem books (see, for instance, *Problems on Strength of Materials* edited by V. K. Kachurin) on the market, most of the examples have been dropped from the present edition. Only examples that are essential for the explanation of theoretical part have been retained.

For greater compactness the problem of design for safe loads has now been included in Chapter 26. For the first time the chapter includes the principles of design for limiting states, which though beyond the limits of the basic course of strength of materials are important enough to require an exposition of the basic concepts even at this stage of teaching.

The problems of strength, which in the previous editions occupied two chapters, have been grouped into one. The part dealing with actual stresses has been transferred to Chapter 2, where it has been presented in a sufficiently detailed manner.

The tables containing data on materials have been dropped from the appendices. A part of the data on materials has been transferred to

corresponding sections. The obsolete steel profiles grading has been replaced by new ones.

As in the previous editions it was our endeavour to preserve Belyaev's style and method of presentation of material. Therefore the author's text has in general been preserved. If Nikolai Mikhailovich Belyaev were alive today he would possibly write many things in a different way. However, since the book won wide popularity as written by N. M. Belyaev, we tried to preserve the original text as far as possible.

The work involved in preparing the fifteenth edition for publication was distributed among the group as follows: Chapter 13, § 80 of Chapter 14, Chapters 15-19, 24-25—L. A. Belyavskii; Chapters 6, 8-12, 27-28—Ya. I. Kipnis; Chapters 1-5, 26 and appendices—N. Yu. Kushelev; Chapter 7, § 79 of Chapter 14, Chapters 20-23, 29-32—A. K. Sinitskii.

A. K. Sinitskii

March 1976

Contents

Nikolai Mikhailovich Belyaev 5
Preface to the Fifteenth Russian Edition 7

PART I. Introduction. Tension and Compression

Chapter 1. Introduction 17

- § 1. The science of strength of materials 17
- § 2. Classification of forces acting on elements of structures 18
- § 3. Deformations and stresses 21
- § 4. Scheme of a solution of the fundamental problem of strength of materials 23
- § 5. Types of deformations 27

Chapter 2. Stress and Strain in Tension and Compression Within the Elastic Limit. Selection of Cross-sectional Area 27

- § 6. Determining the stresses in planes perpendicular to the axis of the bar 27
- § 7. Permissible stresses. Selecting the cross-sectional area 30
- § 8. Deformations under tension and compression. Hooke's law 32
- § 9. Lateral deformation coefficient. Poisson's ratio 36

Chapter 3. Experimental Study of Tension and Compression in Various Materials and the Basis of Selecting the Permissible Stresses 40

- § 10. Tension test diagram. Mechanical properties of materials 40
- § 11. Stress-strain diagram 47
- § 12. True stress-strain diagram 48
- § 13. Stress-strain diagram for ductile and brittle materials 52
- § 14. Rupture in compression of brittle and ductile materials. Compression test diagram 54
- § 15. Comparative study of the mechanical properties of ductile and brittle materials 57
- § 16. Considerations in selection of safety factor, 59
- § 17. Permissible stresses under tension and compression for various materials 64

PART II. Complicated Cases of Tension and Compression

Chapter 4. Design of Statically Indeterminate Systems for Permissible Stresses 66

- § 18. Statically indeterminate systems 66
- § 19. The effect of manufacturing inaccuracies on the forces acting in the elements of statically indeterminate structures 73
- § 20. Tension and compression in bars made of heterogeneous materials 77
- § 21. Stresses due to temperature change 79
- § 22. Simultaneous account for various factors 82
- § 23. More complicated cases of statically indeterminate structures 85

Chapter 5. Account for Dead Weight in Tension and Compression. Design of Flexible Strings 86

- § 24. Selecting the cross-sectional area with the account for the dead weight (in tension and compression) 86
- § 25. Deformations due to dead weight 91
- § 26. Flexible cables 92

Chapter 6. Compound Stressed State. Stress and Strain 99

- § 27. Stresses along inclined sections under axial tension or compression (uniaxial stress) 99
- § 28. Concept of principal stresses. Types of stresses of materials 101
- § 29. Examples of biaxial and triaxial stresses. Design of a cylindrical reservoir 103
- § 30. Stresses in a biaxial stressed state 107
- § 31. Graphic determination of stresses (Mohr's circle) 110
- § 32. Determination of the principal stresses with the help of the stress circle 114
- § 33. Stresses in triaxial stressed state 117
- § 34. Deformations in the compound stress 121
- § 35. Potential energy of elastic deformation in compound stress 124
- § 36. Pure shear. Stresses and strains. Hooke's law. Potential energy 127

Chapter 7. Strength of Materials in Compound Stress 132

- § 37. Resistance to failure. Rupture and shear 132
- § 38. Strength theories 136
- § 39. Theories of brittle failure (theories of rupture) 138
- § 40. Theories of ductile failure (theories of shear) 140
- § 41. Reduced stresses according to different strength theories 147
- § 42. Permissible stresses in pure shear 149

PART III. Shear and Torsion**Chapter 8. Practical Methods of Design on Shear 151**

- § 43. Design of riveted and bolted joints 151
- § 44. Design of welded joints 158

Chapter 9. Torsion. Strength and Rigidity of Twisted Bars 164

- § 45. Torque 164
- § 46. Calculation of torques transmitted to the shaft 167
- § 47. Determining stresses in a round shaft under torsion 168
- § 48. Determination of polar moments of inertia and section moduli of a shaft section 174
- § 49. Strength condition in torsion 176
- § 50. Deformations in torsion. Rigidity condition 176
- § 51. Stresses under torsion in a section inclined to the shaft axis 178
- § 52. Potential energy of torsion 180
- § 53. Stress and strain in close-coiled helical springs 181
- § 54. Torsion in rods of non-circular section 187

PART IV. Bending. Strength of Beams**Chapter 10. Internal Forces in Bending. Shearing-force and Bending-moment Diagrams 195**

- § 55. Fundamental concepts of deformation in bending. Construction of beam supports 195
- § 56. Nature of stresses in a beam. Bending moment and shearing force 200
- § 57. Differential relation between the intensity of a continuous load, shearing force and bending moment 205
- § 58. Plotting bending-moment and shearing-force diagrams 207
- § 59. Plotting bending-moment and shearing-force diagrams for more complicated loads 214
- § 60. The check of proper plotting of Q - and M -diagrams 221
- § 61. Application of the principle of superposition of forces in plotting shearing-force and bending-moment diagrams 223

Chapter 11. Determination of Normal Stresses in Bending and Strength of Beams 225

- § 62. Experimental investigation of the working of materials in pure bending 225
- § 63. Determination of normal stresses in bending. Hooke's law and potential energy of bending 228
- § 64. Application of the results derived above in checking the strength of beams 235

Chapter 12. Determination of Moments of Inertia of Plane Figures 239

- § 65. Determination of moments of inertia and section moduli for simple sections 239
- § 66. General method of calculating the moments of inertia of complex sections 244
- § 67. Relation between moments of inertia about two parallel axes one of which is the central axis 246
- § 68. Relation between the moments of inertia under rotation of axes 247
- § 69. Principal axes of inertia and principal moments of inertia 250
- § 70. The maximum and minimum values of the central moments of inertia 254
- § 71. Application of the formula for determining normal stresses to beams of non-symmetrical sections 254
- § 72. Radii of inertia. Concept of the momental ellipse 256
- § 73. Strength check, choice of section and determination of permissible load in bending 258

Chapter 13. Shearing and Principal Stresses in Beams 263

- § 74. Shearing stresses in a beam of rectangular section 263
- § 75. Shearing stresses in I-beams 270
- § 76. Shearing stresses in beams of circular and ring sections 272
- § 77. Strength check for principal stresses 275
- § 78. Directions of the principal stresses 280

Chapter 14. Shear Centre. Composite Beams 283

- § 79. Shearing stresses parallel to the neutral axis. Concept of shear centre 283
- § 80. Riveted and welded beams 289

PART V. Deformation of Beams due to Bending

Chapter 15. Analytical Method of Determining Deformations 292

- § 81. Deflection and rotation of beam sections 292
- § 82. Differential equation of the deflected axis 294
- § 83. Integration of the differential equation of the deflected axis of a beam fixed at one end 296
- 84. Integrating the differential equation of the deflected axis of a simply supported beam 299
- § 85. Method of equating the constants of integration of differential equations when the beam has a number of differently loaded portions 301
- § 86. Method of initial parameters for determining displacements in beams 304
- § 87. Simply supported beam unsymmetrically loaded by a force 305
- § 88. Integrating the differential equation for a hinged beam 307
- § 89. Superposition of forces 310
- § 90. Differential relations in bending 312

Chapter 16. Graph-analytic Method of Calculating Displacement in Bending 313

- § 91. Graph-analytic method 313
- § 92. Examples of determining deformations by the graph-analytic method 317
- § 93. The graph-analytic method applied to curvilinear bending-moment diagrams 320

Chapter 17. Non-uniform Beams 324

- § 94. Selecting the section in beams of uniform strength 324
- § 95. Practical examples of beams of uniform strength 325
- § 96. Displacements in non-uniform beams 326

PART VI. Potential Energy. Statically Indeterminate Beams

Chapter 18. Application of the Concept of Potential Energy in Determining Displacements 331

- § 97. Statement of the problem 331
- § 98. Potential energy in the simplest cases of loading 333
- § 99. Potential energy for the case of several forces 334
- § 100. Calculating bending energy using internal forces 336
- § 101. Castigliano's theorem 337
- § 102. Examples of application of Castigliano's theorem 341
- § 103. Method of introducing an external force 344
- § 104. Theorem of reciprocity of works 346
- § 105. The theorem of Maxwell and Mohr 347
- § 106. Vereshchagin's method 349
- § 107. Displacements in frames 351
- § 108. Deflection of beams due to shearing force 353

Chapter 19. Statically Indeterminate Beams 356

- § 109. Fundamental concepts 356
- § 110. Removing static indeterminacy via the differential equation of the deflected beam axis 357

- § 111. Concepts of redundant unknown and base beam 359
- § 112. Method of comparison of displacements 360
- § 113. Application of the theorems of Castigliano and Mohr and Vereshchagin's method 362
- § 114. Solution of a simple statically indeterminate frame 364
- § 115. Analysis of continuous beams 366
- § 116. The theorem of three moments 368
- § 117. An example on application of the theorem of three moments 372
- § 118. Continuous beams with cantilevers. Beams with rigidly fixed ends 375

PART VII. Resistance Under Compound Loading

Chapter 20. Unsymmetric Bending 378

- § 119. Fundamental concepts 378
- § 120. Unsymmetric bending. Determination of stresses 379
- § 121. Determining displacements in unsymmetric bending 385

Chapter 21. Combined Bending and Tension or Compression 389

- § 122. Deflection of a beam subjected to axial and lateral forces 389
- § 123. Eccentric tension or compression 392
- § 124. Core of section 396

Chapter 22. Combined bending and torsion 401

- § 125. Determination of twisting and bending moments 401
- § 126. Determination of stresses and strength check in combined bending and torsion 404

Chapter 23. General Compound Loading 408

- § 127. Stresses in a bar section subjected to general compound loading 408
- § 128. Determination of normal stresses 410
- § 129. Determination of shearing stresses 413
- § 130. Determination of displacements 414
- § 131. Design of a simple crank rod 417

Chapter 24. Curved Bars 423

- § 132. General concepts 423
- § 133. Determination of bending moments and normal and shearing forces 424
- § 134. Determination of stresses due to normal and shearing forces 426
- § 135. Determination of stresses due to bending moment 427
- § 136. Computation of the radius of curvature of the neutral layer in a rectangular section 433
- § 137. Determination of the radius of curvature of the neutral layer for circle and trapezoid 434
- § 138. Determining the location of neutral layer from tables 436
- § 139. Analysis of the formula for normal stresses in a curved bar 436
- § 140. Additional remarks on the formula for normal stresses 439
- § 141. An example on determining stresses in a curved bar 441
- § 142. Determination of displacements in curved bars 442
- § 143. Analysis of a circular ring 445

Chapter 25. Thick-walled and Thin-walled Vessels 446

- § 144. Analysis of thick-walled cylinders 446
- § 145. Stresses in thick spherical vessels 453
- § 146. Analysis of thin-walled vessels 454

Chapter 26. Design for Permissible Loads. Design for Limiting States 457

- § 147. Design for permissible loads. Application to statically determinate systems 457
- § 148. Design of statically indeterminate systems under tension or compression by the method of permissible loads 458
- § 149. Determination of limiting lifting capacity of a twisted rod 462
- § 150. Selecting beam section for permissible loads 465
- § 151. Design of statically indeterminate beams for permissible loads. The fundamentals. Analysis of a two-span beam 468
- § 152. Analysis of a three-span beam 472
- § 153. Fundamentals of design by the method of limiting states 474

PART VIII. Stability of Elements of Structures**Chapter 27. Stability of Bars Under Compression 477**

- § 154. Introduction. Fundamentals of stability of shape of compressed bars 477
- § 155. Euler's formula for critical force 480
- § 156. Effect of constraining the bar ends 484
- § 157. Limits of applicability of Euler's formula. Plotting of the diagram of total critical stresses 488
- § 158. The stability check of compressed bars 494
- § 159. Selection of the type of section and material 498
- § 160. Practical importance of stability check 502

Chapter 28. More Complicated Questions of Stability in Elements of Structures 504

- § 161. Stability of plane surface in bending of beams 504
- § 162. Design of compressed-bent bars 512
- § 163. Effect of eccentric compressive force and initial curvature of bar 517

PART IX. Dynamic Action of Forces**Chapter 29. Effect of Forces of Inertia. Stresses due to Vibrations 521**

- § 164. Introduction 521
- § 165. Determining stresses in uniformly accelerated motion of bodies 523
- § 166. Stresses in a rotating ring (flywheel rim) 524
- § 167. Stresses in connecting rods 525
- § 168. Rotating disc of uniform thickness 529
- § 169. Disc of uniform strength 533
- § 170. Effect of resonance on the magnitude of stresses 535
- § 171. Determination of stresses in elements subjected to vibration 536
- § 172. The effect of mass of the elastic system on vibrations 541

Chapter 30. Stresses Under Impact Loading 548

- § 173. Fundamental concepts 548
- § 174. General method of determining stresses under impact loading 549

- § 175. Concrete cases of determining stresses and conducting strength checks under impact 554
- § 176. Impact stresses in a non-uniform bar 559
- § 177. Practical conclusions from the derived results 560
- § 178. The effect of mass of the elastic system on impact 562
- § 179. Impact testing for failure 565
- § 180. Effect of various factors on the results of impact testing 568

Chapter 31. Strength Check of Materials Under Variable Loading 571

- § 181. Basic ideas concerning the effect of variable stresses on the strength of materials 571
- § 182. Cyclic stresses 573
- § 183. Strength condition under variable stresses 575
- § 184. Determination of endurance limit in a symmetrical cycle 576
- § 185. Endurance limit in an unsymmetrical cycle 579
- § 186. Local stresses 582
- § 187. Effect of size of part and other factors on endurance limit 589
- § 188. Practical examples of failure under variable loading. Causes of emergence and development of fatigue cracks 593
- § 189. Selection of permissible stresses 597
- § 190. Strength check under variable stresses and compound stressed state 600
- § 191. Practical measures for preventing fatigue failure 602

Chapter 32. Fundamentals of Creep Analysis 605

- § 192. Effect of high temperatures on mechanical properties of metals 605
- § 193. Creep and after-effect 607
- § 194. Creep and after-effect curves 609
- § 195. Fundamentals of creep design 615
- § 196. Examples on creep design 620

Appendix 630

Name index 639

Subject index 641

