

M A T H E M A T I C S

for Elementary Teachers

Second Edition



Eugene F. Krause

M A T H E M A T I C S

for Elementary Teachers
A Balanced Approach

Second Edition

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PREFACE

It is in response to the gratifying reception of *Mathematics for Elementary Teachers: A Balanced Approach* that this second edition of the text has been prepared. Revision was guided by (1) suggestions from users of the first edition, including comments received through a detailed survey conducted by the publisher; (2) extensive notes on student reactions kept by the author during three years of continuous teaching from the first edition; (3) several in-depth reviews; and (4) recommendations from the CUPM (Committee on the Undergraduate Program in Mathematics), the NCSM (National Council of Supervisors of Mathematics), and the NCTM (National Council of Teachers of Mathematics). Because the NCTM's 1989 publication *Curriculum and Evaluation Standards for School Mathematics* had a particularly strong influence on this second edition, a few things should be noted about these *Standards*.

The NCTM *Standards* for School Mathematics

The commission that produced the *Standards* was charged with (1) defining what it will mean to be mathematically literate in the 1990s and (2) writing standards that will serve as criteria for excellence along the path toward that mathematical literacy. The *Standards* that emerged will be to the 1990s what the NCTM's *Agenda for Action* was to the 1980s—a consensus on what school mathematics ought to become. Of course, every independently thinking math educator will find specific items with which to disagree, but the general thrust of the *Standards* represents a shared vision—as evidenced by endorsement of the *Standards* by a long list of educational, scientific, and mathematical organizations including the American Mathematical Association of Two-Year Colleges, the American Mathematical Society, the American Statistical Association, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics.

The curriculum section of the *Standards* is split into three subsections by grade level: K–4, 5–8, and 9–12. The recommendations in the K–4 and 5–8 subsections are of greatest relevance for prospective elementary teachers; we have listed the 13 standards for each of these two grade levels in Appendix B.

As you will see in Appendix B, standards 1 through 4 are qualitatively different from standards 5 through 13. While standards 5–13 are reasonably concrete *content* recommendations (bearing titles like “whole number computation,” “measurement,” “fractions and decimals,” etc.), standards 1–4 (labeled “problem solving,” “communication,” “reasoning,” and “connections”—at all three grade levels) focus more on the *style* in which the mathematics is to be presented. Standards 1–4 envision a classroom in which students use a variety of strategies and draw on a wide range of techniques to solve nonroutine problems both pure and applied; in which students describe and discuss their methods; and in which ideas are critically analyzed and then accepted, discarded, revised, or elaborated.

Our Response to the *Standards*

If the “critical thinking” experiences called for in the *Standards* are good for children, then the opportunity for analogous experiences ought to be good for prospective teachers as well. The implications for a textbook for teachers would seem to include these: (1) the exposition in the book should be so clear and reliable that the instructor can assign reading and cut back on lecturing to free up class time for discussion; (2) there should be good thought-provoking problems in every section to serve as a basis and catalyst for investigation of important mathematical concepts. Users of the first edition of this book listed these two characteristics as its strongest features. We have striven to enhance both clarity of exposition and quality of problems in this second edition. We have also followed the recommendations of the *Standards* in increasing the emphasis on several topics, as described below.

New to the Second Edition

The following topics are given increased attention. Problem solving is strengthened with 60 percent more problems in Chapter 1, additional challenging problems in each of the other chapters, and more specific references to problem-solving strategies throughout the book. Estimation, approximation, and mental arithmetic receive extra attention in both the exposition and in dozens of exercise sets. Our goal here is to develop what is referred to in the *Standards* as an “estimation mindset”—to bring students to the point where they think of estimation as a natural part of any mathematical investigation. The number of applied problems is increased to emphasize the “connections” between mathematics and the real world. The topics of statistics and geometry receive more extensive treatment. More use is made of calculators.

The most noticeable content changes in the second edition are the following. A thorough new section on simple and compound interest has been added to Chapter 8 on decimals and real numbers. Chapter 9 is devoted entirely to probability. A considerable amount of new material in statistics has been in-

cluded so that statistics now constitutes a chapter by itself. In this new Chapter 10 you will find more graphical displays, new problems with up-to-date real data, discussion of z-scores, and a new section on sampling. Coverage of geometry has been increased substantially with many more worked examples, a more concrete approach to transformations, additional constructions, and an extensive investigation of tessellations. There are now four, instead of three, chapters devoted completely to geometry (Chapters 11–14), and large portions of two other chapters (15 and 16) are also geometric in nature.

Changes from the first edition that are not apparent from a perusal of the Contents are these. There are many more figures (over 200 more), worked examples (55 more), and exercises (about 400 more). New exercises have been added to *every* exercise set, and the number of chapter review exercises (which now include challenging as well as routine questions) has been increased by nearly 60 percent. The new exercises were designed to increase the emphasis on problem solving, estimation and mental arithmetic, real-world applications, and the use of calculators. The flowchart symbols have been changed to conform to ANSI standards. More attention has been given to educational theory and terminology. In particular, reference is made to the *Standards* in every chapter to help students see the relevance of their college course to their future careers.

Content Summary

The global organization of content in this second edition is similar to what it was in the first edition. A listing of topics can be found in the Contents, and details about the philosophy, goals, and organization of each chapter can be found in the individual chapter introductions. Here is a brief summary.

Chapter 1 is an expanded introduction to problem solving that names and illustrates the most common strategies, thus providing a conceptual framework for problem solving in subsequent chapters. The strategies of Chapter 1 are revisited throughout the remainder of the text both in the exposition and in the exercise sets, every one of which contains at least one challenging, nonroutine problem.

Chapter 2 covers the fundamental concepts of set, function, and relation that underlie, organize, and unify subsequent work in number systems, probability and statistics, geometry and measurement, algebra, and computer programming.

Chapters 3–8 cover the number systems of school mathematics from whole numbers through reals. Along with the standard topics are some topics that are not always treated: estimation, permutations and combinations, algorithms and flow charts, ratio and proportion, percent and scientific notation, and compound interest. The topic of estimation, in particular, receives heavy emphasis. Specific named estimation strategies (“front end,” “clustering,” “rounding,” “compatible numbers”) are now described in the text. And two entire sections,

Section 10.5 “Sampling” (new in this edition) and Section 13.4 “Approximation and Error,” can be thought of as topics in estimation. But the main emphasis on estimation (and its close relative, mental computation) remains in the exercise sets where students are asked to estimate in literally dozens of places, in contexts ranging from arithmetic through probability and statistics, to geometry and algebra.

Chapter 9 “Probability,” and **Chapter 10** “Statistics,” cover the standard basic concepts of these two closely related fields as well as such recently recommended topics as simulation, stem-and-leaf and box-and-whisker plots, and inference from a sample. The exposition is arranged so that the basic ideas of probability and statistics can be investigated without first having to master the technicalities of permutations and combinations.

Chapters 11–14 are devoted to geometry although geometric representations and references permeate earlier chapters. The content in Chapters 11–14 is organized around a few major concepts and is developed and amplified in an unusually large number of exercises and problems.

Chapter 15 provides a careful, thorough, and self-contained development of algebra and coordinate geometry. Simple algebraic ideas and skills, however, are refreshed and exercised, where appropriate, beginning in Chapter 1 and continuing throughout the text. Chapter 15 should be particularly valuable to prospective upper-elementary and middle-school teachers (who lay the foundation for future work in algebra) because of its emphasis on the underlying unifying concepts and on the use of algebra in problem solving. Students who need a review of the most basic ideas of algebra might profit from covering Section 15.1 at an earlier point in the course, perhaps between Chapters 3 and 4.

Chapter 16 gives an introduction to computer programming in both BASIC and Logo. This largely self-contained chapter could be covered at almost any point in the course, depending on the instructor’s views on the position of programming in the curriculum for elementary teachers.

Appendix A on Logic also deserves comment. While all mathematics instructors want their students to learn to reason logically (see Curriculum Standard 3, “Reasoning,” in Appendix B), there are many opinions on how to achieve that goal. Some instructors think it best to do a fairly thorough and pure unit on logic somewhere in the course, usually early. Others prefer to develop logical concepts gradually, in context, throughout the course, and perhaps summarize the topic at the end. I have tried to accommodate both groups. Appendix A on “Logic” can be inserted into the course at a number of different points. In the body of the text there is also ad hoc discussion of logical concepts as they arise. Instructors who cover Appendix A early may think of these ad hoc remarks as useful reinforcement. Those who prefer to introduce logic gradually can view Appendix A as an organized recapitulation and extension of the logical ideas appearing throughout the text.

Flexibility of Coverage

There is more than enough material for a full year's course. A two-semester arrangement might cover Chapters 1 to 7 or 8 in the first term, and Chapters 8 or 9 to 15 or 16 in the second term. A three-quarter arrangement might cover Chapters 1 to 6, 7 to 12, and 13 to 16. Certain sections have been marked with an asterisk (*) as optional, but, of course, what is optional is a matter of one's own tastes, priorities, and time constraints. The asterisk signifies only that omission of that particular section will not adversely impact the study of subsequent sections. The full 16 chapters, including optional sections, together with Appendix A provide sufficient material for a solid three-semester course.

Features in This Edition

The following special features have been included to make the text more valuable to prospective elementary school teachers.

1. **Reproductions of Pages from Elementary Textbooks.** These are useful for showing students connections between the theory in their college course and the practice in the elementary classroom.
2. **Computer Vignettes.** Each of these twelve strategically located vignettes consists of discussion, a program (written in either BASIC or Logo), and a run. They can be used in several ways. Students who have no access to a machine can look at the printouts and appreciate the kinds of things that can be done using a computer. Students who have access to a machine, but who have no particular expertise in programming, can simply type in the programs and run them (most are interactive). Students who already know how to program in BASIC or Logo can modify or extend the programs as suggested in the accompanying discussion.
3. **Chapter Review Exercises.** As noted earlier, the review exercise sets in this edition (which can be thought of as practice tests) have been lengthened by nearly 60 percent by the addition of new nonroutine as well as routine questions. (Answers to these Review Exercises are available only in the *Instructor's Guide*.)
4. **References.** The list of related references at the end of each chapter has been updated and expanded from the first edition.
5. **Historical Sketches.** The history of mathematics is, among other things, a melancholy record of discrimination against women. Imagine how much more might have been accomplished in the field of mathematics if women had been encouraged (or even allowed!) to contribute. These historical sketches concern five famous male mathematicians and five remarkable women who became mathematicians despite the obstacles. Future teachers

might find these stories useful for encouraging young women to consider careers in mathematics.

6. **Key Concepts.** In response to suggestions from users of the first edition, a checklist of key concepts has been included at each chapter end to help structure a student's review.
7. **Critical Thinking Problems.** In each section one problem has been designated with a chambered nautilus icon (🐚) to indicate that it is particularly well-suited to fostering the step-by-step, essentially limitless growth of students' problem-solving and thinking skills. These problems are intended to trigger the critical thinking and class discussion of concepts that reformers hope will characterize mathematics classrooms in the 1990s.
8. **Calculator Exercises.** The number of exercises for which a calculator is either helpful or essential has been increased in this edition. Such exercises are marked with a calculator symbol (🧮).
9. **Challenging Problems.** The traditional five-point star (★) is used to indicate problems that are particularly challenging. Most sections include at least one of these.

Supplements to the Text

Six supplements are available.

Student Activities and Study Guide. For the student there is now a *Student Activities and Study Guide*, prepared by Professor DeAnn Huinker of the University of Wisconsin at Milwaukee, in collaboration with the author of the text. This new supplement provides an array of math lab and calculator activities, along with additional examples, exercises, and selected answers. It also features an annotated bibliography of computer software. Professor Huinker brought to the writing her experience as an elementary school teacher and, more recently, as the instructor of the mathematics methods course for elementary school teachers at the University of Michigan.

Instructor's Guide. For the professor there is an *Instructor's Guide*, written by the author, with complete solutions to all text exercises. This guide now contains two sample tests for each chapter, with answer keys.

D. C. Heath Exam Computerized Testing. A computerized testing program for the IBM-PC containing a test bank of over 500 questions, each keyed by section number and level of difficulty, is available to professors who want to construct customized tests. Items may be selected randomly by chapter, based on difficulty level, and the program is able to generate multiple scrambled versions of the same exam. The program also allows instructors to import their own test items into the test bank or to edit existing items. Answers and solutions are included.

Test Item File. This is a printed file of all the test items and solutions appearing in the computerized testing program.

D. C. Heath Class Gradebook Software. This new grading software provides a convenient grade management program for the IBM-PC. The program allows professors to track and assess student performance, or the relative performance of sections within a multisection course, and to print out grading reports designed to the individual instructor's specifications.

Elementary School Textbooks. As an additional resource, one set of current editions of grade levels K, 2, 4, 6 and 8 of D. C. Heath's elementary school textbook series are available, upon request, to institutions making class-size adoptions. Contact your local D. C. Heath sales representative to place your request.

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Eugene F. Krause

To the Student

Congratulations on your choice of teaching as your future profession! You will occupy a position of fundamental importance in the lives of students who pass through your class. In your classroom you will be the mathematician-in-residence. Your students' future success in mathematics will depend to a large degree on your competence, confidence, and enthusiasm. Competence, of course, is the key element. Without it there can be neither confidence nor a genuine enthusiasm for the subject.

Mathematical competence, as you probably know, is not what it used to be. The elementary school mathematics curriculum is undergoing fundamental change in response to the ubiquitous presence of calculators and computers and to the increasing mathematical demands that a technological society is placing on its citizens. The central focus of elementary mathematics will no longer be on mechanical, symbol-manipulation skills such as paper-and-pencil procedures for adding, subtracting, multiplying, and dividing. Rather the emphasis will be on applying mathematics to solve nonroutine problems, understanding the mathematical concepts that underlie the symbols, and communicating and evaluating mathematical ideas in a group setting. Perhaps the most influential blueprint for the mathematics curriculum of the 1990s, the NCTM *Standards for Curriculum and Evaluation*, calls for a "conceptually oriented" K–4 curriculum and a "broad, concept-driven" 5–8 curriculum. The teacher who stands at the center of this conceptual curriculum will need a perspective and depth of understanding that does not develop automatically during the successful completion of her or his high school mathematics courses. That teacher will need to have surveyed, purposefully and thoughtfully, the entire breadth of elementary mathematics and to have identified and mastered its fundamental concepts.

This book was written with that conceptual goal in mind. That does not mean it is a difficult or abstract text or that it is unrelated to classroom practice. Quite the opposite. It is possible to explain important mathematical ideas in simple language and to illustrate key concepts with concrete examples, many of which could even be used in an elementary classroom. There is little necessity for formal deduction. I hope that you will find the exposition clear, the problems interesting, and your mathematical power increasing.

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