

LAMINAR COMPOSITES

Second Edition

GEORGE H. STAAB



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George H. Staab



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To Ellen, Dan, Ben, and Jen

Preface

As an introduction to composite materials, the texts that have been published present topics from either a materials science or applied mechanics viewpoint. This text presents the subject from an applied mechanics point of view and limits discussions to continuous fiber composites. Topics are developed at a level suitable for terminal undergraduate students and beginning graduate students. As a prerequisite, students should have completed a course in strength of materials. Additionally, they should be familiar with stress-strain relations for isotropic materials and load-stress relationships. The philosophy behind this text is that it should be fundamentally simple enough for a senior undergraduate to understand and apply the concepts forwarded, while at the same time not too trivial for a beginning graduate student.

The scope of this text is limited to topics associated with the analysis and design of continuous fiber-laminated composite materials. Lamina and laminate analysis is presented with a blend of theoretical developments and examples. The analysis of laminated composites relies heavily on concepts developed in undergraduate statics and mechanics of materials courses. Examples presented in this text require an understanding of free-body diagrams and analysis techniques introduced in undergraduate mechanics courses. Experimental techniques applicable to defining the constitutive relationships for orthotropic lamina are presented, as are failure theories for orthotropic materials.

After establishing the stress-strain relationships, discussing special testing considerations, and covering failure criteria for orthotropic lamina, classical lamination theory is developed. An attempt has been made to present material in an easy-to-follow, logical manner. Loading conditions involving mechanical, thermal, and hygral loads are considered after the effect of each is discussed and developed independently.

Chapters on beams, plates, and shells have been added to the original text. The chapter on beams should prove useful to undergraduates. Beams are a fundamental structural element and this chapter is an extension of what undergraduates learned in their introductory strength of materials courses. Although the plates and shells chapters may be too difficult for undergraduates, they have been added for completeness, and to serve as a starting point for students interested in these topics. These two chapters are necessarily brief since the solutions to many types plate and shell problems require numerical techniques beyond the scope of this text.

Many of the topics covered in this text are a compilation of the topics covered in preceding books, such as *Primer on Composite Materials: Analysis* by Ashton, Halpin, and Petit; *Mechanics of Composite Materials* by Jones; *Introduction to Composite Materials* by Tsai and Hahn; *Experimental Mechanics of Fiber Reinforced*

Composite Materials by Whitney, Daniel, and Pipes; and *The Behavior of Structures Composed of Composite Materials* by Vinson and Sierakowski; *Mechanics of Laminated Composite Plates Theory and Analysis* and *Mechanics of Laminated Composite Plates and Shells* by J.N. Reddy. These texts served as the foundation upon which this text was developed. The present text incorporates many of the standard equations and formulations found in the preceding texts and builds upon them.

The original edition of this text contained an appendix on matrix arithmetic and a section containing additional references. Due to the advances in personal computing, it was felt that a section on matrix arithmetic is no longer needed. Along similar lines, the advancement of web search engines makes a section containing additional references somewhat obsolete. Therefore, this section was similarly deleted.

I am deeply thankful to my longtime friend and colleague Dr. H.R. Busby, emeritus professor of Mechanical and Aerospace Engineering at The Ohio State University. His friendship, helpful comments, suggestions, and notes that we used to develop the composite materials courses at OSU formed the basis of this manuscript. Finally, I wish to thank my wife, Ellen, for her long-term patience and eventual understanding of how engineers are.

Answers to the Problems throughout this book are available on the book's companion website. Go online to access it at: <http://booksite.elsevier.com/9780128024003>

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Introduction to composite materials

1

1.1 Historic and introductory comments

In the most general of terms, a composite is a material which consists of two or more constituent materials or phases. Traditional engineering materials (steel, aluminum, etc.) contain impurities which can represent different phases of the same material and fit the broad definition of a composite, but are not considered composites because the elastic modulus or strength of each phase are nearly identical. The definition of a composite material is flexible and can be augmented to fit specific requirements. In this text, a composite material is considered to be the one which contains two or more distinct constituents with significantly different macroscopic behavior and a distinct interface between each constituent (on the microscopic level). This includes the continuous fiber-laminated composites of primary concern herein, as well as a variety of composites not specifically addressed.

Composite materials have been in existence for many centuries. No record exists as to when people first started using composites. Some of the earliest records of their use date back to the Egyptians, who are credited with the introduction of plywood, paper mache, and the use of straw in mud for strengthening bricks. Similarly, the ancient Inca and Mayan civilizations used plant fibers to strengthen bricks and pottery. Swords and armor were plated to add strength in medieval times. An example is the Samurai sword, which was produced by repeated folding and reshaping to form a multilayered composite (it is estimated that several million layers could have been used). Eskimos use moss to strengthen ice in forming igloos. Similarly, it is not uncommon to find horse hair in plaster for enhanced strength. The automotive industry introduced large scale use of composites with the 1953 Chevrolet Corvette. All of these are examples of man-made composite materials. Bamboo, bone, and celery are examples of cellular composites which exist in nature. Muscle tissue is a multidirectional fibrous laminate. There are numerous other examples of both natural and man-made composite materials.

The structural materials most commonly used in design can be categorized into four primary groups: metals, polymers, composites, and ceramics. These materials have been used to various degrees since the beginning of time. Their relative importance to various societies throughout history has fluctuated. Ashby [1] presents a chronological variation of the relative importance of each group from 10,000 BC, and extrapolates their importance through the year 2020. The information contained in Ref. [1] has been partially reproduced in Figure 1.1. The importance of composites experienced steady growth since about 1960, and is projected to increase in importance through the next several decades. The relative importance of each group of materials is not associated with any specific unit of measure (net tonnage, etc.).

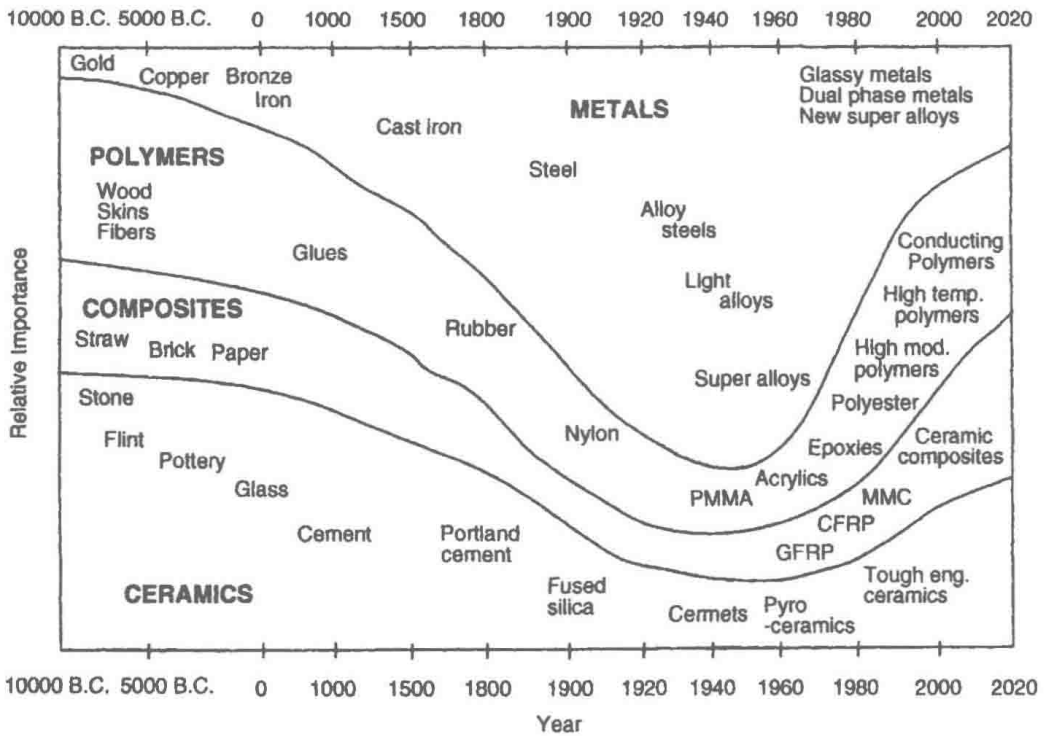


Figure 1.1 Relative importance of material development through history (after Ashby [1]).

As with many advances throughout history, advances in material technology (from both a manufacturing and analysis viewpoint) typically have its origins in military applications. Subsequently this technology filters into the general population and alters many aspects of society. This is most recently seen in the marked increase in relative importance of such structural materials such as composites starting around 1960, when the race for space dominated many aspects of research and development. Similarly, the Strategic Defense Initiative (SDI) program in the 1980s prompted increased research activities in the development of new material systems. Advances in material systems research, manufacturing techniques, and the reduced cost of raw materials have made the use of composite materials a common practice in most aspects of everyday life. The use of composites has grown so much that Roberts [2] estimates the global demand for carbon fibers alone in 2015 will exceed 67,000 metric tons (147,400,000 lbs).

The composites generally used in structural applications are best classified as high performance. They are typically made from synthetic materials, have high strength to weight ratios, and require controlled manufacturing environments for optimum performance. The aircraft industry uses composites to meet performance requirements beyond the capabilities of metals. The Boeing 757, for example, uses approximately 760 ft³ of composites in its body and wing components, with an additional 361 ft³ used in rudder, elevator, edge panels, and tip fairings. An accurate breakdown of specific components and materials can be found in Ref. [3]. The B-2 bomber contains carbon

and glass fibers, epoxy resin matrices, high-temperature polyimides as well as other materials in more than 10,000 composite components. It is considered to be one of the first major steps in making aircraft structures primary from composites. Composites are also used in race cars, tennis rackets, golf clubs, and other sports and leisure products. Although composite materials technology has grown rapidly, it is not fully developed. New combinations of fiber/resin systems, and even new materials are constantly being developed. The best one can hope to do is identify the types of composites that exist through broad characterizations and classifications.

1.2 Characteristics of a composite material

The constituents of a composite are generally arranged so that one or more discontinuous phase is embedded in a continuous phase. The discontinuous phase is termed the *reinforcement* and the continuous phase is the *matrix*. An exception to this is rubber particles suspended in a rigid rubber matrix, which produces a class of materials known as rubber-modified polymers. In general, the reinforcements are much stronger and stiffer than the matrix. Both constituents are required, and each must accomplish specific tasks if the composite is to perform as intended.

A material is generally stronger and stiffer in fiber form than in bulk form. The number of microscopic flaws which act as fracture initiation sites in bulk materials are reduced when the material is drawn into a thinner section. In fiber form, the material will typically contain very few microscopic flaws from which cracks may initiate to produce catastrophic failure. Therefore, the strength of the fiber is greater than that of the bulk material. Individual fibers are hard to control and form into useable components. Without a binder material to separate them, they can become knotted, twisted, and hard to separate. The binder (matrix) material must be continuous and surround each fiber so that they are kept distinctly separate from adjacent fibers and the entire material system is easier to handle and work with.

The physical and mechanical properties of composites are dependent on the properties, geometry, and concentration of the constituents. Increasing the volume content of reinforcements can increase the strength and stiffness of a composite to a point. If the volume content of reinforcements is too high there will not be enough matrix to keep them separate and they can become tangled. Similarly, the geometry of individual reinforcements and their arrangement within the matrix can affect the performance of a composite. There are many factors to be considered when designing with composite materials. The type of reinforcement and matrix, the geometric arrangement and volume fraction of each constituent, the anticipated mechanical loads, the operating environment for the composite, etc., must all be taken into account.

Analysis of composites subjected to various mechanical, thermal, and hygral conditions is the main thrust of this text. Discussions are limited to continuous fiber-laminated composites. In introductory strength of materials, the constitutive relationship between stress and strain was established for homogeneous isotropic materials as Hooke's law. A composite material is analyzed in a similar manner, by establishing a constitutive relationship between stress and strain.

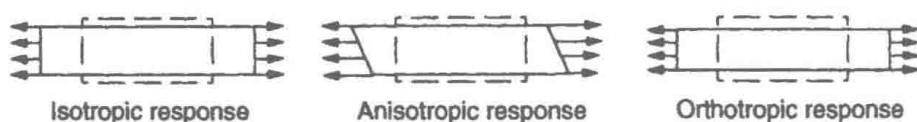


Figure 1.2 Typical material responses for isotropic, anisotropic, and orthotropic materials subjected to axial tension.

Isotropic, homogeneous materials (steel, aluminum, etc.) are assumed to be uniform throughout, and have the same elastic properties in all directions. Upon application of a uniaxial tensile load an isotropic material deforms in a manner similar to that indicated in Figure 1.2 (the dashed lines represent the undeformed specimen). Assuming a unit width and thickness for the specimen, the transverse in-plane and out-of-plane displacements are the same. Unlike conventional engineering materials, a composite material is generally nonhomogeneous and does not behave as an isotropic material. Most composites behave as either an *anisotropic* or *orthotropic* material.

The material properties of an anisotropic material are different in all directions. There is typically a coupling of extension and shear deformation under conditions of uniaxial tension. The response of an anisotropic material subjected to uniaxial tension is also illustrated in Figure 1.2. There are varying degrees of anisotropic material behavior, and the actual deformation resulting from applied loads depends on the material.

The material properties of an orthotropic material are different in three mutually perpendicular planes, but there is generally no shear-extension coupling as with an anisotropic material. The transverse in-plane and out-of-plane displacements are not typically the same since Poisson's ratio is different in these two directions. Figure 1.2 also illustrates orthotropic material response. Although it appears similar to that of an isotropic material, the magnitude of the in-plane and out-of-plane displacements are different.

1.3 Composite materials classifications

Composite materials are usually classified according to the type of reinforcement used. Two broad classes of composites are fibrous and particulate. Each has unique properties and application potential, and can be subdivided into specific categories as discussed below.

Fibrous: A fibrous composite consists of either continuous (long) or chopped (whiskers) fibers suspended in a matrix material. Both continuous fibers and whiskers can be identified from a geometric viewpoint:

Continuous fibers. A continuous fiber is geometrically characterized as having a very high length to diameter ratio. They are generally stronger and stiffer than bulk material. Fiber diameters generally range between 0.00012 and 0.0074-in. (3–200 μm), depending upon the fiber [4].

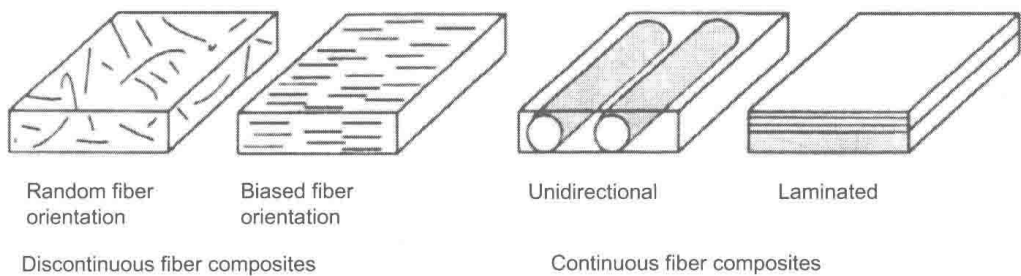


Figure 1.3 Schematic representations of fibrous composites.

Whiskers. A whisker is generally considered to be a short, stubby fiber. It can be broadly defined as having a length to diameter ratio of $5 < l/d < 1000$ and beyond [5]. Whisker diameters generally range between 0.787 and 3937 μm . (0.02 and 100 μm).

Composites in which the reinforcements are discontinuous fibers or whiskers can be produced so that the reinforcements have either a random or biased orientation. Material systems composed of discontinuous reinforcements are considered single-layer composites. The discontinuities can produce a material response which is anisotropic, but in many instances the random reinforcements produce nearly isotropic composites. Continuous fiber composites can be either single layer or multilayered. The single-layer continuous fiber composites can be either unidirectional or woven, and multilayered composites are generally referred to as laminates. The material response of a continuous fiber composite is generally orthotropic. Schematics of both types of fibrous composites are shown in Figure 1.3.

Particulate: A particulate composite is characterized as being composed of particles suspended in a matrix. Particles can have virtually any shape, size, or configuration. Examples of well-known particulate composites are concrete and particle board. There are two subclasses of particulates; flake and filled/skeletal:

Flake. A flake composite is generally composed of flakes with large ratios of planform area to thickness, suspended in a matrix material (e.g., particle board).

Filled/skeletal. A filled/skeletal composite is composed of a continuous skeletal matrix filled by a second material. For example, a honeycomb core filled with an insulating material.

The response of a particulate composite can be either anisotropic or orthotropic. They are used for many applications in which strength is not a significant component of the design. A schematic of several types of particulate composites is shown in Figure 1.4.

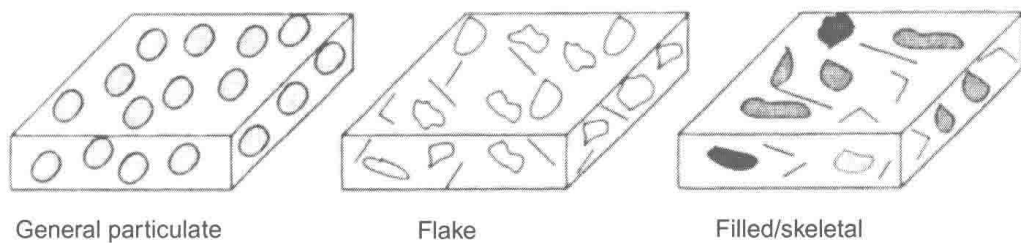


Figure 1.4 Schematic representations of particulate composites.

1.4 Fundamental composite material terminology

Some of the more prominent terms used with composite materials are defined below. A more detailed list can be found in the Glossary as well as Ref. [6].

Lamina. A lamina is a flat (or sometimes curved) arrangement of unidirectional (or woven) fibers suspended in a matrix material. A lamina is generally assumed to be orthotropic, and its thickness depends on the material from which it is made. For example, a graphite/epoxy (graphite fibers suspended in an epoxy matrix) lamina may be on the order of 0.005 in. (0.127 mm) thick. For the purpose of analysis, a lamina is typically modeled as having one layer of fibers through the thickness. This is only a model and not a true representation of fiber arrangement. Both unidirectional and woven lamina are schematically shown in Figure 1.5.

Reinforcements. Reinforcements are used to make the composite structure or component stronger. The most commonly used reinforcements are boron, glass, graphite (often referred to as simply carbon), and Kevlar, but there are other types of reinforcements such as alumina, aluminum, silicon carbide, silicon nitride, and titanium.

Fibers. Fibers are a special case of reinforcements. They are generally continuous and have diameters ranging from 120 to 7400 μm . (3–200 μm). Fibers are typically linear elastic or elastic-perfectly plastic, and are generally stronger and stiffer than the same material in bulk form. The most commonly used fibers are boron, glass, carbon, and Kevlar. Fiber and whisker technology is continuously changing [4,5,7].

Matrix. The matrix is the binder material which supports, separates, and protects the fibers. It provides a path by which load is both transferred to the fibers and redistributed among the fibers in the event of fiber breakage. The matrix typically has a lower density, stiffness, and strength than the fibers. Matrices can be brittle, ductile, elastic, or plastic. They can have either linear or nonlinear stress–strain behavior. In addition, the matrix material must be capable of being forced around the reinforcement during some stage in the manufacture of the composite. Fibers must often be chemically treated in order to assure proper adhesion to the matrix. The most commonly used matrices are polymeric (PMC), ceramic (CMC), metal (MMC), carbon, and

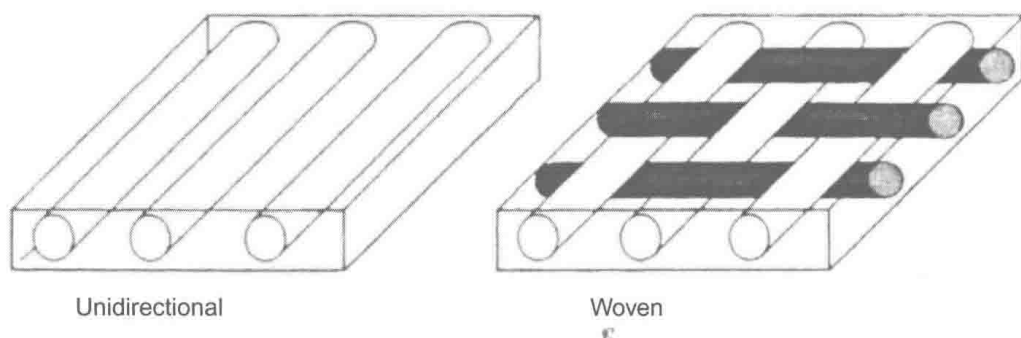


Figure 1.5 Schematic representation of unidirectional and woven composite lamina.