

Kinematic Design of Machines and Mechanisms

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Homer D. Eckhardt

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Preface

The objective of this book is to provide practicing engineers and students with a practical understanding of the principles of kinematics, with an understanding of the connections between these principles and the behavior of actual machines, and with tools for the kinematic design of those machines.

The design of a machine or mechanism or any moving mechanical system always starts with a consideration of kinematics because kinematics is the study of the geometry of motion. That is, kinematics deals with (1) the functional relationships between the parts of a machine or other mechanical system, (2) how those parts are interconnected, and (3) how those parts move relative to each other. Only after choices have been made regarding those three factors can matters such as strengths, materials, fabrication techniques, and costs be seriously addressed. Failure to devote the proper attention to kinematics "up front" can, and often does, result in the design of a system with substandard or nonoptimum performance and/or with unsatisfactory reliability.

Fortunately, today, the ready availability of very powerful personal computers and the associated software allows kinematic synthesis and analysis, which were formerly laborious, to be performed quickly and cheaply. There is no longer an excuse for avoiding doing careful kinematic design up front. Because of the availability of these computer aids and the consequent incentives to apply kinematic principles in design, it is becoming *increasingly important* for the practicing engineer to have a good understanding of those kinematic principles.

Even before engineers can start to use a computer for synthesis or analysis of a machine, they must develop some initial concept of how the machine will operate. To assist engineers in generating such concepts, the first part of each of Chaps. 3 through 7 and all of Chap. 8 describe the operating capabilities of mechanisms which can be used as components of machines. Then step-by-step procedures (principally graphical) for synthesizing such mechanisms to provide the desired

functions are given. If these initially synthesized configurations require further refinement, experience gained in this initial manual synthesis provides understanding and input for any further synthesis and analysis on a computer.

The graphical synthesis and analysis methods described require only the use of compass, ruler, protractor, and simple arithmetic. The CAD (computer-aided drafting or design) systems available today can also be very easily used to perform these graphical procedures and to obtain very accurate answers. Currently available spreadsheet software and equation-solving software can be used with the equations given in this book to obtain numerical values for the kinematic variables involved. Many of the equations have been left in a form in which the unknown variable has *not* been isolated on the left-hand side. When in this form:

- 1. the equations are closer to the derivation from the source phenomena involved and thus the significance of individual terms is more easily seen;
- 2. the equations are already in a form which many equation-solving software packages such as TKSolver®, MathCAD®, and others can evaluate iteratively; and
- 3. the equations are easily converted to a form suitable for programming into a spreadsheet by transposing the single term containing the unknown to the left-hand side and performing a simple operation such as dividing, taking a square root, taking an inverse tangent, etc. The equations can generally be evaluated in the order in which they are presented.

Very often, the answers which are provided by computerized aids are not unique and depend on details of the formulation of the problem as fed to the computer. If the computer-generated answer is not fully satisfactory to the engineer, a decision must be made as to what could and should be changed in that input formulation, how it should be changed, and what resulting change would be expected in the answer. Then new inputs must be fed to the computer and the process must be repeated, in the hope that the next answer will be more satisfactory. If the kinematic principles are not understood, this iteration process becomes one of wasteful trial and error, and valuable solutions can be missed. It is a primary aim of this book to provide help in more effciently guiding such trial-and-error processes by providing engineers with the understanding which will allow them to visualize the connections between the relatively simple mathematics and the physical phenomena involved. Toward that aim, the later portions of each of Chaps. 3 through 7 and all of Chap. 9 describe analytical as well as graphical relationships between mechanism motion variables.

Several mathematical derivations are presented in this book. It is not necessary for readers to be familiar with, to read, or to understand these derivations in order to use the procedures and relationships presented, although derivations are usually helpful in providing understanding of the power and limitations of those procedures and relationships. The derivations are presented largely for readers who may wish to enlarge or build upon them in order to generate more elegant and/or more powerful procedures or relationships.

The only mathematics prerequisite assumed for use of the analytical portions of the book are high-school algebra, including complex numbers, high-school trigonometry, and a knowledge of differential calculus including the ability to differentiate e^x and the trigonometric functions. It is also assumed that the reader is aware that F=ma, and is able to add, subtract, and resolve two-dimensional vectors.

The first three chapters lay the foundations for the synthesis and analysis techniques and procedures which are presented in the remainder of the book.

Chapter 1 presents the definitions and basic concepts. Although the book deals essentially only with planar kinematics, Secs. 1.5 through 1.9 discuss three-dimensional or spatial phenomena. The remainder of the book does not depend upon the material in Secs. 1.6 through 1.9, so they could be skipped or lightly skimmed if time demands. However, because real machines are built and operated in three-dimensional space, it is important that practicing engineers have an appreciation for the phenomena covered in those sections.

Chapter 2 presents methods for analyzing the motions of rigid bodies in planar motion. The chapter covers displacements, velocities, and accelerations of isolated rigid bodies, and discusses the significant relationships between the mathematics and the physical phenomena. It provides analysis techniques which are used in subsequent chapters, where rigid bodies are connected together to form machines.

The crank-slider mechanism is a very useful mechanism, and Chap. 3 presents synthesis and analysis methods for use in its design. Because the geometry and motion of this mechanism are relatively simple and easily visualized, analysis of this mechanism is used as a basis from which the synthesis and analysis of the mechanisms in the subsequent chapters are treated as perturbations and extensions.

Chapters 4 through 7 describe the salient features and capabilities of increasingly complex mechanisms and present procedures for their synthesis. These chapters also briefly illustrate the adaptation of the previously described analysis procedures for use on these more complex mechanisms.

Chapters 8 and 9 present principles and procedures which are useful in preparation of concepts for machines which will be subjects for computer-assisted synthesis and analysis.

Chapters 10 and 11 give the principles of cam systems, gear systems, and timing-belt systems and give procedures for synthesizing such systems.

Although Chaps. 12 and 13 involve forces and inertial reactions and therefore are subjects in kinetics, the intimate involvement of geometry and thus of kinematics in the phenomena covered makes them important subjects for a kinematics text.

Chapter 12 describes the relationships between the static forces which occur in a mechanism, and presents procedures for static balancing of such mechanisms. Examples of such balancing are given. These examples involve the use of balance springs as well as the use of balance weights.

Chapter 13 extends the discussion of Chap. 12 to the relationships between the dynamic forces which occur in a mechanism, and presents procedures for dynamic balancing of mechanisms. Examples of such balancing are given.

Homer D. Eckhardt

How to Use This Book

It is intended that the step-by-step procedures presented in this book be usable without the need to refer to other portions of the text. However, to the extent that time permits, readers should become aware of the existence and nature of the background material in the remainder of this book and in other books on this subject.

Therefore, read this list and then:

- 1. Read the preface. (It takes only a few minutes!)
- 2. Read the table of contents and note the titles of sections.
- 3. Read the Introduction section of each chapter.
- 4. From the table of contents, select a section or sections of particular interest and read the introductory paragraphs of those sections. Also skim those sections for Procedures of interest (see List of Procedures also).
- 5. When you find a particular Procedure which is of immediate interest, read (or at least skim) the section containing that procedure, including Introduction and any Comments and suggestions.
- 6. Skim the index for terms which are of interest and skim the text where those terms appear. The index should be used together with the table of contents because information appears in different forms in these two places.
- 7. Naturally all authors want readers to read, understand, and treasure every word in their books. This particular author realizes that practicing engineers are often pressed for time. Nonetheless, although I, too, have often been pressed for time, I have found the background material in this book to be of great use over the course of many years.

Acknowledgments

I wish to express my appreciation to my many colleagues at Honeywell, Link Aviation, the Massachusetts Institute of Technology, RCA, Draper Loom, Polaroid, the Worcester Polytechnic Institute, and Tufts University for all that they have taught me over the years, and to Professor Robert Norton of the Worcester Polytechnic Institute for inspiring me to write this book. I would especially like to thank Professor Ashok Midha of the University of Missouri at Rolla and Professor Steven Kramer of the University of Toledo who kindly agreed to review the manuscript and bring any egregious errors to my attention.

I am deeply grateful to my children Gretchen, Julie, Jason, Kris, and Lili for all they have taught me about "life, death, and related subjects," and I am particularly grateful to my wife Beverly, who, in addition to teaching me so much, has shown monumental patience with my preoccupation during the preparation of this book.

Despite all the help from the above-mentioned people, I, through my own obstinate efforts, may have included some errors in this book. I apologize to readers for any such errors. I welcome readers' comments and suggestions, which may be sent to me at

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Chapter

1

Basic Concepts and Definitions

1.1 Introduction

This chapter defines the concepts on which this discussion of the kinematic design of machines will be based. Consequently, the chapter starts by defining the terms *kinematics*, *kinematic design*, and *machine*. These definitions introduce the concept of rigid bodies (links) and connections (joints). The remainder of the chapter discusses how the positions and displacements of these bodies may be described and how the motions of these bodies are affected by joining them together.

1.2 Kinematics and Kinematic Design

Mechanics is the branch of science that deals with motions, time, and forces. As indicated in Fig. 1.1, mechanics may be considered to consist of two main branches: statics and dynamics. Statics, as would be expected from its name, deals with stationary systems, so it is concerned only with forces and the geometry of systems. Although it can involve static deflections, it is not concerned with motion or time. Dynamics, on the other hand, deals with systems that are in motion. Dynamics, in turn, can be subdivided into the disciplines of kinematics and kinetics.

Figure 1.1 indicates that *kinematics* consists of the study of the interaction between the geometry of a system and the motions of that system. That is, it is concerned with the magnitudes and directions of the displacements, velocities, and accelerations (and possibly higher motion derivatives also) of the parts of the system. It is also concerned with how these motion parameters change that geometry and with how these parameters are dependent on the geometry. The forces and torques that are required when the motions of a system are prescribed

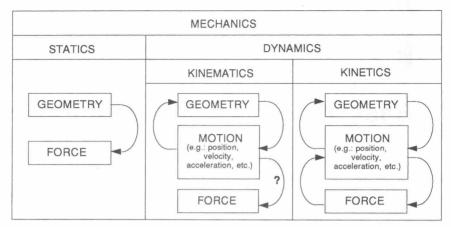


Figure 1.1 The science of mechanics.

can be computed from knowledge of the displacements, velocities, and accelerations in those prescribed motions and by treating inertial reaction forces (d'Alembert forces) and torques as static forces and torques. Such kinematic force and torque computations, then, involve both kinematics and statics and are often referred to as *kinetostatic analyses*. Such a kinetostatic analysis is discussed in Sec. 13.4.

Kinematics, however, does not consider how those forces might interact with the motions of the system. The distinction between kinematics and kinetics is that *kinetics* is concerned with how forces interact with the motions of the system.

One of the first phases in the design of a machine or mechanism is the choosing of the manner in which the parts of the machine must move in order to perform the functions for which the machine is being designed. Decisions must be made as to where the parts must be placed, how far and in what directions the parts must move, which parts must be connected to which other parts and how they must be connected, and what the critical dimensions of the parts must be. This phase will result in drawings or sketches of the general layout of the machine and will indicate how it will operate. This phase obviously involves the interactions between geometry and motions, so it and its resulting drawings will be referred to as the *kinematic design* of the machine. Although this effort may be performed simultaneously with other phases of the design, it is distinct from the design activities in which materials, strengths, responses to forces, wear, power, noise, vibration, costs, manufacturability, and so on are considered.

Although kinematic design is often taken for granted, great care should be exercised in its execution because it has a very powerful effect on all other aspects of the design process! The kinematics of a machine are its very soul.