EMIL WOLF

EDITOR



VOLUME XXXIX

CONTRIBUTORS

L. ALLEN
A.A. ASATRYAN
M. BABIKER
G.W. FORBES
Yu.A. KRAVTSOV
G. LEUCHS
T. OPATRNÝ

M.J. PADGETT
A. SIZMANN
D.J. SOMERFORD
S.K. SHARMA
W. VOGEL
D.-G. WELSCH

PROGRESS IN OPTICS

VOLUME XXXIX

EDITED BY

E. WOLF

University of Rochester, N.Y., U.S.A.

Contributors

L. ALLEN, A.A. ASATRYAN, M. BABIKER, G.W. FORBES, Yu.A. KRAVTSOV, G. LEUCHS, T. OPATRNÝ, M.J. PADGETT, A. SIZMANN, D.J. SOMERFORD, S.K. SHARMA, W. VOGEL, D.-G. WELSCH



1999

 ${\tt ELSEVIER}$ ${\tt AMSTERDAM \cdot LAUSANNE \cdot NEW \; YORK \cdot OXFORD \cdot SHANNON \cdot SINGAPORE \cdot TOKYO}$

PROGRESS IN OPTICS

VOLUME XXXIX

EDITORIAL ADVISORY BOARD

G. S. AGARWAL, Ahmedabad, India

T. ASAKURA, Sapporo, Japan

M.V. BERRY, Bristol, England

С. COHEN-TANNOUDЛ, Paris, France

V. L. GINZBURG, Moscow, Russia

F. GORI, Rome, Italy

A. KUJAWSKI, Warsaw, Poland

J. PEŘINA, Olomouc, Czech Republic

R. M. SILLITTO, Edinburgh, Scotland

H. WALTHER, Garching, Germany

ELSEVIER SCIENCE B.V. SARA BURGERHARTSTRAAT 25 P.O. BOX 211 1000 AE AMSTERDAM THE NETHERLANDS

Library of Congress Catalog Card Number: 61-19297 ISBN Volume XXXIX: 0 444 50104 5

© 1999 Elsevier Science B.V. All rights reserved.

This work is protected under copyright by Elsevier Science B.V., and the following terms and conditions apply to its use:

Photocopying

Single photocopies of single chapters may be made for personal use as allowed by national copyright laws. Permission of the publisher and payment of a fee is required for all other photocopying, including multiple or systematic copying, copying for advertising or promotional purposes, resale, and all forms of document delivery. Special rates are available for educational institutions that wish to make photocopies for non-profit educational classroom use.

Permissions may be sought directly from Elsevier Science Rights & Permissions Department, PO Box 800, Oxford OX5 1DX, UK; phone: (+44) 1865 843830, fax: (+44) 1865 853333, e-mail: permissions@elsevier.co.uk. You may also contact Rights & Permissions directly through Elsevier's home page (http://www.elsevier.nl), selecting first 'Customer Support', then 'General Information', then 'Permissions Query Form'.

In the USA, users may clear permissions and make payments through the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, USA; phone: (978) 7508400, fax: (978) 7504744, and in the UK through the Copyright Licensing Agency Rapid Clearance Service (CLARCS), 90 Tottenham Court Road, London W1P 0LP, UK; phone: (+44) 171 631 5555; fax: (+44) 171 631 5500. Other countries may have a local reprographic rights agency for payments.

Derivative Works

Tables of contents may be reproduced for internal circulation, but permission of Elsevier Science is required for external resale or distribution of such material.

Permission of the publisher is required for all other derivative works, including compilations and translations.

Electronic Storage or Usage

Permission of the publisher is required to store or use electronically any material contained in this work, including any chapter or part of a chapter.

Except as outlined above, no part of this work may be reproduced, stored in a retrieval system or transmitted in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without prior written permission of the publisher.

Address permissions requests to: Elsevier Science Rights & Permissions Department, at the mail, fax and e-mail addresses noted above.

Notice

No responsibility is assumed by the Publisher for any injury and/or damage to persons or property as a matter of products liability, negligence or otherwise, or from any use or operation of any methods, products, instructions or ideas contained in the material herein. Because of rapid advances in the medical sciences, in particular, independent verification of diagnoses and drug dosages should be made.

 \otimes The paper used in this publication meets the requirements of ANSI/NISO Z39.48-1992 (Permanence of Paper).

PRINTED IN THE NETHERLANDS

PREFACE

In this volume five review articles are presented dealing with topics of current research interests in optics.

The first article, by Yu.A. Kravtsov, G.W. Forbes and A.A. Asatryan, is concerned with the analytic extension of the concept of geometrical optics rays into the complex domain. The extension is intimately related to inhomogeneous (evanescent) waves, which are currently of particular interest in connection with the rapidly developing area of near-field optics. The results are also relevant to investigations of wave attenuation in absorbing media, and to the understanding of light penetration into geometrical shadow regions, excitation of surface waves and propagation of Gaussian beams. The article presents the principles, with special emphasis on the physical significance of complex rays and their applications.

The second article, by D.-G. Welsch, W. Vogel and T. Opatrný, describes recent progress in the general area of quantum-state reconstruction, particularly for extracting information about the quantum state of a given object from a set of measurements. The methods are applicable to the optical field as well as to various matter waves. Some methods of processing the measured data and many of the important experiments in this area are discussed.

The next article, by S.K. Sharma and D.J. Somerford, is concerned with the scattering of light in the eikonal approximation. This approximation originated in the theory of high-energy scattering processes and in the broad area of potential scattering. From the well-known analogy between a scattering potential and the distribution of the refractive index, the eikonal approximation was later adapted to the analysis of light scattering by small particles. In this article an account is given of the eikonal approximation in the context of optical scattering, and its domain of validity is discussed. The relationship of this approximation to other approximate techniques as well as some of its possible applications are considered.

The fourth article, by L. Allen, M.J. Padgett and M. Babiker, concerns the orbital angular momentum of light. The orbital angular momentum is shown to be an observable quantity which can be profitably used with certain types of light beams. The phenomenological interaction of the beams with matter in bulk is reviewed and the contributions of the orbital angular momentum to the

vi PREFACE

dissipative and dipole forces on atoms are calculated in detail. Orbital and spin angular momentum of light are compared and contrasted.

The concluding article, by A. Sizmann and G. Leuchs, presents a review of the experimental progress made in recent years in the generation of squeezed light and in quantum nondemolition measurements in optical fibers. The rich nonlinear dynamics of quantum solitons in fibers has led to the discovery of new quantum optical effects, such as intrapulse quantum correlations. The nonlinearity of optical fibers is now used to build passive fiber devices which provide all-optical functions, such as quantum noise reduction, and it is expected that active devices will allow absorption-free measurements of optical signals. The review is also concerned with these and other promising developments in this general area.

Emil Wolf

Department of Physics and Astronomy University of Rochester Rochester, New York 14627, USA

December 1998

CONTENTS

I. THEORY AND APPLICATIONS OF COMPLEX RAYS by Yu.A. Kravtsov (Moscow, Russian Federation), G.W. Forbes and A.A. Asatryan (Sydney, Australia)

§ 1.	Introduction	3
	1.1. Pioneering works	3
	1.2. Character of wavefields described by complex geometrical optics	4
	1.3. Goals of the review	5
§ 2.	BASIC EQUATIONS OF GEOMETRICAL OPTICS	5
	2.1. Eikonal, transfer, and ray equations of traditional geometrical optics	5
	2.2. Rays as the skeleton for the wavefield	8
	2.3. Complex form of the geometrical optics method	10
	2.4. Alternative approach to phenomena described by complex rays	12
§ 3.	PROPERTIES OF COMPLEX RAYS	15
	3.1. Ray paths in the complex space	15
	3.2. Fermat's principle for complex rays	16
	3.3. Selection rules for complex rays	17
	3.4. Complex rays and the saddle-point method	19
		21
		23
	3.7. Electromagnetic waves and complex rays	24
		26
§ 4.		27
	4.1. Complex rays inside a circular caustic	27
	4.2. Wave reflection in a layered medium	29
	4.3. Point source in a layered medium	31
		32
		33
	4.6. Point source in a parabolic layer	33
		36
		37
	4.9. Surface waves	37
4		38
		39
		42
		43
	5.1. Gaussian beams and complex sources	43

viii CONTENTS

	J.Z.	Another description of Gaussian beams in terms of complex rays	. 44
		Transformation of Gaussian beams in optical systems	
	5.4.	Diffraction of Gaussian beams	. 49
§ 6.	DIST	TINCTIVE ASPECTS OF COMPLEX GEOMETRICAL OPTICS	. 50
	6.1.	Nonlocal properties of complex rays	. 50
		Boundaries of applicability of complex geometrical optics	
§ 7.	Con	CLUSION	. 52
		EDGMENTS	
	ERENC		
	I	I. HOMODYNE DETECTION AND QUANTUM-STATE RECONSTRUCTION	
		by DG. Welsch (Jena, Germany), W. Vogel (Rostock,	
		GERMANY) AND T. OPATRNÝ (OLOMOUC, CZECH REPUBLIC)	
§ 1.	Intr	ODUCTION	. 65
		SE-SENSITIVE MEASUREMENTS OF LIGHT	
3		Optical homodyning	
		2.1.1. Basic scheme	
		2.1.2. Quadrature-component statistics	
		2.1.3. Multimode detection	
		2.1.4. Q function	. 81
		2.1.5. Probability operator measures	
		2.1.6. Positive P function	
		2.1.7. Displaced-photon-number statistics	
	2.2	2.1.8. Homodyne correlation measurements	
		Heterodyne detection	
		Parametric amplification	
		Measurement of cavity fields	
§ 3.		NTUM-STATE RECONSTRUCTION	
		Optical homodyne tomography	
		Density matrix in quadrature-component bases	
	3.3.	Density matrix in the Fock basis	. 108
		3.3.1. Sampling of quadrature-components	
		3.3.2. Sampling of the displaced Fock-states on a circle	. 115
		3.3.3. Reconstruction from propensities	
		Multimode density matrices	
	3.5.	Local reconstruction of $P(\alpha; s)$. 122
	3.6.	Reconstruction from test atoms in cavity QED	. 123
		3.6.1. Quantum state endoscopy and related methods	
		3.6.2. Atomic beam deflection	. 128
	3.7.	Alternative proposals	. 131
		Reconstruction of specific quantities	
		3.8.1. Normally ordered photonic moments	
		3.8.2. Quantities admitting normal-order expansion	. 137

CONTENTS ix

	3.8.3. Canonical phase statistics	139
	3.8.4. Hamiltonian and Liouvillian	143
3.9.	Processing of smeared and incomplete data	144
	3.9.1. Experimental inaccuracies	145
	3.9.2. Least-squares method	151
	3.9.3. Maximum-entropy principle	153
	3.9.4. Bayesian inference	155
§ 4. OUA		157
	Molecular vibrations	158
	4.1.1. Harmonic regime	159
	4.1.2. Anharmonic vibrations	160
4.2	Trapped-atom motion	163
	4.2.1. Quadrature measurement	163
	4.2.2. Measurement of the Jaynes–Cummings dynamics	167
	4.2.3. Entangled vibronic states	171
43	Bose–Einstein condensates	173
		175
7.7.	4.4.1. Transverse motion	175
		178
1.5	4.4.2. Longitudinal motion	179
4.3.		180
	4.5.1. Electronic Rydberg wave packets	
	4.5.2. Cyclotron state of a trapped electron	182
	4.5.3. Electron beam	183
	Spin and angular momentum systems	183
	Crystal lattices	185
	EDGMENTS	187
	A. RADIATION FIELD QUANTIZATION	187
	B. QUANTUM-STATE REPRESENTATIONS	189
B.1.	Fock states	189
B.2.	Quadrature-component states	190
	Coherent states	191
B.4.	s-parametrized phase-space functions	192
B.5.	Quantum state and quadrature components	194
APPENDIX	C. PHOTODETECTION	195
APPENDIX	D. ELEMENTS OF LEAST-SQUARES INVERSION	197
REFEREN	ES	200
	III. SCATTERING OF LIGHT IN THE EIKONAL APPROXIMATION by S.K. Sharma (Calcutta, India) and D.J. Somerford (Cardiff, UK)	
		5 6 8
	RODUCTION	
	EIKONAL APPROXIMATION IN NON-RELATIVISTIC POTENTIAL SCATTERING	
	Preliminaries of the problem	
2.2.	The eikonal approximation	219
	2.2.1 Approximation from the Schroedinger equation	210

		2.2.2. Approximation from the integral equation	220
		2.2.3. Propagator approximation	22
		2.2.4. Physical picture of propagation in the EA	222
	2.3.	Scattering amplitude	222
		2.3.1. Eikonal amplitude	222
		2.3.2. Glauber variant of the EA	223
	2.4.	Relationship with partial wave expansion	224
	2.5.	Comparison with the Born series	225
	2.6.	Interpretation of the EA as a long range approximation	226
	2.7.	Numerical comparisons and potential dependence of the EA	226
	2.8.	Modified eikonal approximations: corrections to the EA	227
		2.8.1. The eikonal expansion	227
		2.8.2. The eikonal–Born series	228
		2.8.3. The generalized eikonal approximation	229
	2.9.	Relationship with Rytov approximation	229
§ 3.	Eiko	ONAL APPROXIMATION IN OPTICAL SCATTERING	230
	3.1.	Analogy with potential scattering	231
	3.2.	Validity of scalar scattering approximation	232
	3.3.	Scattering by a homogeneous sphere	233
		3.3.1. The eikonal approximation	233
		3.3.2. Derivation of the EA scattering function from the Mie solutions	236
		3.3.3. Relationship with the anomalous diffraction approximation	
		3.3.4. Corrections to the EA	238
		3.3.5. Numerical comparisons	
			247
			249
			251
	3.4.		253
			254
		3.4.2. Scattering by a homogeneous cylinder	255
		3.4.3. The EA from exact solutions	257
			258
			259
		3.4.6. The EA as $ m-1 \rightarrow 0$ approximation	260
			261
		3.4.8. Scattering at oblique incidence	263
			263
	3.5.		264
			266
			267
§ 4.			268
o .			268
			268
			272
	4.2.	The state of the s	273
			276

CONTENTS xi

4.4. Light scattering by cladded fibers	278
4.5. Diffraction by a volume hologram	279
4.6. Miscellaneous applications	
§ 5. Conclusions and discussions	282
	285
	202
IV. THE ORBITAL ANGULAR MOMENTUM OF LIGHT	
by L. Allen (Colchester/St. Andrews, UK), M.J. Padgett	
(St. Andrews, UK) and M. Babiker (Colchester, UK)	
(entransition, entry that an arrange (established, entry	
§ 1. Introduction	294
§ 2. THE PARAXIAL APPROXIMATION	296
§ 3. Nonparaxial Light Beams	302
§ 4. EIGENOPERATOR DESCRIPTION OF LASER BEAMS	306
§ 5. Generation of Laguerre–Gaussian Modes	
§ 6. OTHER GAUSSIAN LIGHT BEAMS POSSESSING ORBITAL ANGULAR MOMENTUM	319
§ 7. SECOND-HARMONIC GENERATION AND ORBITAL ANGULAR MOMENTUM	322
§ 8. MECHANICAL EQUIVALENCE OF SPIN AND ORBITAL ANGULAR MOMENTUM: OPTICAL SPAN	
	NERS 324
§ 10. Atoms and the Orbital Angular Momentum of Light	328
§ 11. Atoms and Multiple Laguerre–Gaussian Beam Configurations	342
§ 12. MOTION OF MG ⁺ IN MULTIPLE BEAM CONFIGURATIONS	345
§ 13. Atoms and Circularly Polarized Light	356
§ 14. SPIN—ORBIT COUPLING OF LIGHT	363
§ 15. Conclusions	. 366
ACKNOWLEDGEMENTS	. 369
References	. 369
V. THE OPTICAL KERR EFFECT AND QUANTUM OPTICS IN FIBERS	
by A. Sizmann and G. Leuchs (Erlangen, Germany)	
§ 1. Introduction	. 375
§ 2. HISTORICAL PERSPECTIVE	. 377
§ 3. THE OPTICAL KERR EFFECT	. 380
§ 4. QUANTUM OPTICS IN FIBERS – PRACTICAL CONSIDERATIONS	. 388
4.1. Kerr-nonlinearity and power confinement	. 388
4.2. Optical solitons in fibers	. 389
4.3. Guided acoustic-wave Brillouin scattering (GAWBS)	. 393
§ 5. QUADRATURE SQUEEZING	. 397
5.1. Properties of Kerr quadrature squeezed states	. 397
5.1.1. Single-mode interaction Hamiltonian	. 397
5.1.2. Single-mode linearized approach	400

xii CONTENTS

	1	404
5.2.	Experiments with continuous-wave laser light	40
		40
	5.3.1. Ultrashort pulses for GAWBS noise suppression	40
	5.3.2. Generation and detection of pulsed quadrature squeezing using a balanced	
	Sagnac loop	409
	5.3.3. Generation and detection of pulsed quadrature squeezing using a linear	
		41
	_	412
	The state of the s	415
§ 6. OU		418
		418
-	STATE OF STA	418
		421
		423
6.2		428
		431
0.5.	The state of the s	432
		+32 433
		+33 434
		+34 434
e 7 Due		
		435
7.1.		435
	takan panggunanan bank bank bank bank bank bank bank	136
		140
7.2.	The first contraction of the first contraction of the contraction of t	142
7.3.		143
	7.3.1. Noise reduction and enhancement as a function of filter type and cut-off	
		144
		146
		148
	3 7 7 7	149
7.4.		149
	8	151
	First Paris Control of the Control o	152
	7.4.3. Pulsed photon-number squeezing from an asymmetric Sagnac loop 4	153
§ 8. Fut	URE PROSPECTS	158
Acknowi	EDGEMENTS	160
REFERENC	CES	160
AUTHOR 1	INDEX	71
SUBJECT 1	INDEX	87
CONTENT	S OF PREVIOUS VOLUMES	91
CUMULAT	TVE INDEX	01

E. WOLF, PROGRESS IN OPTICS XXXIX © 1999 ELSEVIER SCIENCE B.V. ALL RIGHTS RESERVED

I

THEORY AND APPLICATIONS OF COMPLEX RAYS

BY

Yu.A. Kravtsov¹, G.W. Forbes² and A.A. Asatryan²

Space Research Institute, Russian Academy of Sciences,
 Profsoyuznaya Street 84/32, Moscow 117810, Russian Federation;
 School of Mathematics, Physics, Computing, and Electronics,
 Macquarie University, Sydney, NSW 2109, Australia

CONTENTS

		PAGE
§ 1.	INTRODUCTION	3
§ 2.	BASIC EQUATIONS OF GEOMETRICAL OPTICS	5
§ 3.	PROPERTIES OF COMPLEX RAYS	15
§ 4.	COMPLEX RAYS IN PHYSICAL PROBLEMS	27
§ 5.	GAUSSIAN BEAMS AND COMPLEX RAYS	43
§ 6.	DISTINCTIVE ASPECTS OF COMPLEX GEOMETRICAL OPTICS	50
§ 7.	CONCLUSION	52
ACK	NOWLEDGMENTS	53
REFE	ERENCES	53

§ 1. Introduction

1.1 PIONEERING WORKS

Complex rays are solutions of the ray equations of traditional geometrical optics, but correspond to extremals in the six-dimensional complex space (x',x'',y',y'',z',z''), where $x' = \text{Re}\{x\}$, $x'' = \text{Im}\{x\}$, etc. These trajectories can be used to derive both the phase and amplitude of the associated wavefield. Complex rays were first considered during the 1930s and 1940s in the theory of radio wave propagation through the lossy ionosphere (Epstein [1930a,b], Booker [1939], Bremmer [1949]), but a more general formalization was not developed until the late 1950s and early 1960s. The decisive step in understanding the analytical nature of complex rays was made by Keller [1958], who introduced the notion of complex rays to treat the area of a caustic shadow. He studied the equations for rays passing through points in the interior of a circular caustic in two dimensions and showed that such rays contact the caustic surface at complex points that lie on its analytic continuation.

A year later, Seckler and Keller [1959] studied complex rays in plane-layered media, and Keller and Karal [1960] applied complex rays to the problem of surface wave excitation. Grimshaw [1968] studied these surface waves in more detail for particular surfaces (the sphere, the cone, and the plane with inhomogeneous impedance). Babich [1961] considered the analytic continuation of the wave function into the caustic shadow, and performed calculations that may be interpreted in terms of complex rays. A similar analytic continuation was also applied by Keller and Rubinow [1960] who studied eigenfunctions in both open and closed optical resonators. Complex trajectories are the quantum-mechanical analog of complex rays, and were studied by Maslov [1963] in connection with the quasiclassical asymptotics of solutions to the Schrödinger equation. Complex trajectories appeared there as complex solutions of the classical equations of motion in regions that are inaccessible to classical particles (i.e., areas of tunneling). Maslov [1964] also pointed out that complex rays may form foci and caustics.

The application of complex rays within the theory of radio wave propagation through the lossy ionosphere was resumed by Budden [1961], Sayasov [1962],

Budden and Jull [1964], and Jones [1968, 1970]. Generally speaking, in lossy media all rays become complex because the index of refraction, which enters both the eikonal and ray equations, takes complex values. The growing interest in these ideas led to the first review paper on complex rays, written by Kravtsov [1967a], in which the analytic nature of complex trajectories was refined and the notion of a complex focus introduced for a Gaussian beam. The idea of using a complex point source to model a Gaussian beam was considered almost simultaneously by Deschamps [1967, 1968, 1971], Arnaud [1968, 1969b], and Keller and Streifer [1971].

Of note also, in one-dimensional problems the method of phase integrals (see, e.g., Heading [1962]) can be regarded as a precursor to complex geometrical optics; that is, complex ray methods may be viewed as a generalization of the phase integral method for three-dimensional inhomogeneous media. Similarly, the old idea of a complex angle of refraction for the evanescent component in the case of total internal reflection is a clear ancestor of complex geometrical optics. Thus this field has a long history.

1.2. CHARACTER OF WAVEFIELDS DESCRIBED BY COMPLEX GEOMETRICAL OPTICS

In complex geometrical optics the direction of wave propagation is given by the gradient of the real part of the complex phase, and the direction of exponential decay of the field's magnitude is principally determined by the gradient of the imaginary part. In nonabsorbing media these directions are orthogonal, but in lossy media they are separated by an acute angle. In both cases inhomogeneous waves can enter, and their magnitude changes exponentially on a phase front. In fact, just as homogeneous (propagating) waves are the subject of traditional geometrical optics, inhomogeneous (or evanescent) waves can be regarded as the principal subject of complex geometrical optics (Kravtsov [1967a,b], Choudhary and Felsen [1973], Felsen [1976a]).

Like traditional geometrical optics, complex geometrical optics can involve a multiplicity of rays, so that the total wavefield is then a sum of the waves associated with each ray. As emphasized by Kravtsov [1967a,b], when multiple complex rays are present, selection rules are typically required to exclude nonphysical solutions. Another feature of complex rays is that, unlike real rays, they can describe nonlocal (diffraction-like) processes. A clear demonstration of their nonlocal properties is provided by the example of Gaussian beams, see § 5. It is shown in that section that complex geometrical optics provides a complete description of a Gaussian beam (Kravtsov [1967a,b], Keller and Streifer [1971]).