Finite Mathematics

For Business and Social Science

William J. Adams

Mathematics

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Pace University **William J. Adams**

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To ONUTE, ANDRIUS, RAMUNE, and RASA

Preface

The objective of this book is to present a lucid exposition of mathematical topics that have important applications in business and the social sciences. It is written at a level suitable for beginning college students with minimal training in mathematics: the only background assumed is high school intermediate algebra or its equivalent. The book is intended for one-semester or two-quarter courses in finite mathematics, or as part of multi-semester or quarter sequences in mathematics for business and social science students.

Any new textbook brings its own unique contribution to the course for which it has been written. Among the several postulates that guided the development of this book, the following deserve special mention:

1. The presentation should be well focused and allow a variety of options. The book has been organized into five independent parts: Linear Programming, Probability, Theory of Games, Matrices, and the Mathematics of Finance. Each part is divided into chapters, and each chapter into sections. The five main parts of the book are independent of each other and may be followed in any desired sequence; diagrams that chart various ways in which each part might be taught to meet specific needs have been included at the beginning of each part as a further guide to the instructor.

2. The book should be readable by students. A special effort has been made to present the material in an intuitive and informal manner without sacrificing correctness. Concepts are introduced through examples, and rigorous proofs that are above the expected level of mathematical maturity of the intended audience have been avoided.

3. Emphasis should be placed on realistic applications that are self-contained. The intent of this text is to convey a sense of realism to the reader, to avoid the frequent "Alice in Wonderland" flavor that comes from an overcommitment to artificial applications.

4. The role of assumptions in mathematical model building should be stressed. Too often, students do not understand that mathematical conclusions are only valid with respect to the assumptions on which they are founded. This crucial role of assumptions in mathematical model building is placed in a proper perspective, and the distinction between the validity of a mathematical conclusion and its truth is pointed out throughout the text.

5. The mathematical machinery developed should be appropriate to the applications presented. This text brings a sense of proportion to the mathematics developed and the applications presented. Every attempt has been made to leave the reader with the feeling that any extra effort required to master theory brings with it appropriate rewards in understanding and realistic applications.

6. Answers to problems are not enough. Answers to approximately half of the exercises in the text appear at the end of this book. But answers alone are not enough to indicate to the student where he has made mistakes or to show that a correct solution was reached through incorrect reasoning. Thus a companion Study Guide to the text has been prepared to include answers with detailed solutions to all problems in the book; it should be of benefit to both students and instructors.

The topics included in this book have been selected for their appropriateness and interest to the student of business and the social sciences. The coverage emphasizes a working knowledge and understanding of the most important mathematical topics those students are likely to use or encounter in their future course or professional work, rather than giving a "once over lightly" appreciation of a great number of topics. Exercises and problems have been placed throughout the text where review and practice of material that has been presented is appropriate. Above all, every effort has been made to allow for both flexibility and simplicity in the material.

Some general comments on the material in each part may serve as a useful preface to the text as a whole. Part One presents an exposition of the elements of linear programming, culminating with the development of the simplex method. Only knowledge of basic algebra is assumed, and certain algebraic prerequisites are reviewed within the text before they are used. No matrix or vector machinery is employed: the only requirement for the simplex method is a knowledge of arithmetic.

Part Two develops basic ideas of probability. The heart of this material is Chapter 6, "Probability in Finite Sample Spaces," in which basic probability concepts are introduced. This is followed by a chapter on combinatorial techniques. It has been my experience that an introduction to probability should be separated from the study of combinatorial techniques for a variety of reasons. Students find these techniques difficult to understand, and the inclusion of them along with the basic ideas of probability can make the material overwhelming. In addition, students acquire a distorted perspective of probability when its basic ideas are presented in the setting of combinatorics. They often come away from such a course thinking that all situations in probability have equally-likely outcomes and equating probability to combinatorial methods. Part Two allows the instructor a number of options once Chapter 6 has been covered, and combinatorial methods in probability is only one such choice.

Part 3 provides a short introduction to the theory of games with the spotlight on two-person, zero-sum games. Some very basic knowledge of probability is required for game theory and Chapter 6 has more than enough to satisfy this need. Section 64 shows that game-theory problems can be solved by linear-programming methods. If this section is to be covered, it is suggested that some linear programming be studied first (Chapter 4).

Part 4 is an introduction to basic matrix algebra. A matrix inversion technique is discussed and then used to solve systems of linear equations. The application of matrix methods to input-output analysis and finite Markov chains (Chapter 14 in Part 2) is presented.

Part 5 is an exposition of some basic concepts in the mathematics of finance: simple and compound interest, nominal and effective interest rates, future and present values of an annuity. The mathematics of finance is an optional part of many courses and is included here to complete the text.

A unique addition to the material on linear programming, probability, and the theory of games is the inclusion in each part of a survey of the literature. The aim of these sections is to make the reader aware of the reality of applications in these areas. The references can serve as the basis for term reports and further class study in a particular area of interest.

I would like to express appreciation to my colleagues in the mathematics department of Pace University, who class-tested preliminary versions of certain parts of this work and offered many valuable comments and suggestions, to the reviewers for their constructive criticism, to Carol Beal and Arthur B. Evans of XCP for having made this a better book, and to the rest of the staff at Xerox College Publishing for their contributions in its preparation.

W. J. A.

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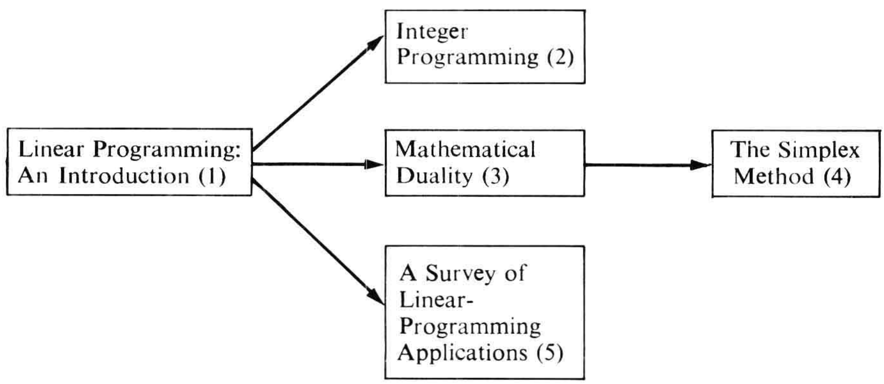

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Linear programming: an introduction

1. The scope of linear programming: a first view

In the late 1930s the Central Laboratory of the Leningrad Plywood Trust appealed to the Institute of Mathematics and Mechanics of Leningrad University for help in solving a production scheduling problem of the following sort. Several different kinds of machines are available for peeling logs for plywood. Various kinds of logs are handled and the productivity of a peeling machine (that is, the number of logs peeled per unit of time) depends on the kind of log being worked on. The productivity of each machine for each kind of log is known. The problem is to determine the distribution of work time of each machine for each kind of log (that is, determine the fraction of the work day that each machine should spend on each kind of log) so that output (number of peeled logs produced) is maximized subject to the condition that logs of the first kind constitute, let us say, 10% of the output; logs of the second kind constitute 12% of the output, and so on.

The problem was taken up by L. V. Kantorovich who discovered that it, together with a wide range of economic problems of diverse nature connected with the organization of production (problems involving the optimum distribution of work time of machines, minimization of scrap in manufacturing processes, best utilization of raw materials, optimum distribution of arable land, optimal fulfillment of a construction plan with given construction materials, the best plan of freight shipments), lead to a group of mathematical problems with the same structure. Such problems are now called linear-programming problems, or linear programs, and are related by virtue of the following structural feature. There is given a linear function of a number of variables which are required to satisfy a given system of linear equations or linear inequalities. The problem is to find values of the variables which satisfy the given conditions and maximize or minimize the linear function. Not only do a wide range of problems in economic planning lead to linear programs but also a variety of problems in accounting, banking, finance, and marketing.

In 1939 Kantorovich published a report[1] on his discoveries which included a "simple" general method (the method of resolving multipliers) which was sufficient for solving all of the problems he had posed. Although Kantorovich had made clear the wide applicability of his approach to economic planning problems, and with a small group of followers had continued the development of his ideas, his methods were almost completely neglected in the Soviet Union until the late 1950s. In the meantime, linear-programming methods were developed independently in the United States in the 1940s, and the 1950s saw rapid progress in the theoretical development and practical applications of linear programming.

To illustrate the nature of situations for which linear-programming models can be developed, let us consider four examples.

PROBLEM 1. The Saturn Company makes air-conditioners and refrigerators. Two plants, I and II, are used. The assembly work is done in plant I, and it is estimated that 5 man-days of work are required to produce a refrigerator and 2 man-days of work are required to produce an air-conditioner. The finishing work is done in plant II, and it is estimated that 3 man-days of work are needed to finish a refrigerator and 2 man-days of work are needed to finish an air-conditioner. Plant I has 220 man-days per week available and plant II has 180 man-days per week available. A market survey indicates that there is an unlimited market for these products.

If the Saturn Company makes a profit of \$50 on each refrigerator and \$30 on each air-conditioner, how many of each should be produced weekly so as to maximize profit?

PROBLEM 2. An animal feed is to be a mixture of oats and hay and must contain at least 6 pounds of protein, 6 pounds of fat, and 5 pounds of mineral matter. Each unit (a unit is 30 pounds) of oats contains 1 pound of protein, 3 pounds of fat, and 1 pound of mineral matter. Each unit of hay contains 3 pounds of protein, 1 pound of fat, and 1 pound of mineral matter.

If oats cost 80 cents per unit and hay costs 76 cents per unit, how many units of each should be used to make up a sack of animal feed which satisfies the nutritional requirements at the least cost? What is the least cost?

PROBLEM 3. Hundred-pound sacks of potatoes are to be shipped from Lewiston and Waterville to distributors in Jackson, Linden, and Clifton. There are 2400 sacks at Lewiston and 3000 sacks at Waterville. 1600 sacks are to be sent to Jackson, 2000 sacks are to be sent to Linden, and 1800 sacks are to be sent to Clifton. It costs 2¢, 4¢, and

1. L. V. Kantorovich, *Mathematical Methods of Organizing and Planning Production*. (See Chapter 5, Section 17, [12].)