# Quantitative Analysis: An Introduction

# Quantitative Analysis

## An Introduction

Roy M. Chiulli

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# Quantitative Analysis

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**About the series editor**

Hamid R. Parsaei, PhD, PE, is a professor of industrial engineering and director of the Manufacturing Research Group at the University of Louisville in Kentucky. He received his MS and PhD in industrial engineering from Western Michigan University and the University of Texas at Arlington, respectively. His teaching and research interests include aspects of engineering economy, robotics, concurrent engineering, computer integrated manufacturing systems and productivity analysis.

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To

my parents *Rosindo and Mary*, who are
with me in my heart forever.

my wife *Joan*, whose understanding
and patience made this book possible.

and

my daughters *Mariana and Irina*,
who make every day a joy for Joan and me.

*If a man will begin with certainties he shall end in doubts; but if he will be content to begin with doubts he shall end in certainties.*

—Francis Bacon

# SERIES PREFACE

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*Hamid R. Parsaei*

# PREFACE

The intent of *Quantitative Analysis: An Introduction* is to provide a survey of optimization techniques most often used in decision making. Since a survey cannot treat any individual topic in detail, readers who wish further knowledge in any particular area addressed in this volume are encouraged to examine books that provide a fuller treatment.

Optimization techniques presented here are not complex and may be mastered quickly. Their simplicity does not diminish their power. In fact, simplicity should be an objective in decision making. The two most important factors in solving a problem effectively are: (1) Propose the simplest problem formulation that captures the essential features of the system being examined; and (2) Solve the problem with the simplest technique appropriate. Proposing an elegant formulation of a problem that cannot be solved within the resources available (time, manpower and money) is not helpful to decision making. For example, equations governing moving objects are complex and difficult to solve when all interactions among celestial bodies are considered. And yet the essentials of physical laws of motion can be modeled satisfactorily using simple equations. This allows NASA to plan and execute interplanetary missions based on solutions to simple, linearized equations of motion.

Basic information on simulation is provided in chapter 11. It is recognized that simulation is a powerful optimization tool for decision making. Simulations can capture the complexities and probabilistic nature of a system to a high level of detail. However, the use of simulation has significant drawbacks to a decision maker. Simulations typically require a large investment of resources (time, manpower and money) to formulate, debug and make enough production runs to gain statistical significance. Moreover, simulations often require a great deal of data to faithfully model the specific details of the system. This data, which typically must be gathered by a population survey or through expert opinion, is also expensive and time-consuming to obtain. While some systems can only be satisfactorily modeled using simulation, it is essential to limit its use to these cases. The techniques addressed in this volume provide quick and easily obtainable solutions. It is obvious that an approximate solution provided in a timely manner is much more useful to a decision maker than an exact solution provided after the decision has been made. It is suggested that analysts use these simple techniques, whenever possible, to provide timely responses to decision makers and use simulation, when appropriate, to *refine* the solutions.

An essential aspect of the application of decision making is the realization that the solution arising from quantitative techniques such as those provided in

*Quantitative Analysis: An Introduction* (or from simulation) cannot be considered "optimal" until the special knowledge of the decision maker is applied. Typically, decisions are affected by political, social, legal and regulatory constraints difficult or impossible to quantify in a satisfactory manner. However, quantitative techniques do provide valuable information to the decision maker to be considered along with all other appropriate factors when deciding upon the true optimum solution.

# CONTENTS

*Series Preface* xiii

*Preface* xv

1 PROBABILITY & STATISTICS 1

1.1 Basic Probability 1
1.2 Events 3
1.3 Combining Events 4
1.4 Complements 6
1.5 Expected Value and Standard Deviation 7
1.6 Normal (Gaussian) Distribution 10
1.7 Confidence Intervals 13
1.8 Poisson Distribution 14
1.9 Exponential Distribution 16
1.10 Statistics Definitions 18
1.11 Goodness of Fit Test: Chi-Square Test 19
1.12 Goodness of Fit Test: F-Statistic 24
1.13 Student t Distribution 27
1.14 Conditional Probability 30
1.15 Bayes' Theorem 34
Exercise 1 Probability & Statistics 37
Probability & Statistics Additional Solved Problems 40

2 DECISION ANALYSIS 45

2.1 Expected Value Decision 45
2.2 Maximin Decision 48
2.3 Decision Trees 50
2.4 Sequential Decision Trees 55
2.5 Utility & Subjective Probability 65
2.6 Expected Value of Information 67
Exercise 2 Decision Analysis 79
Decision Analysis Additional Solved Problems 81

3 Break-Even Analysis 91

3.1 Introduction 91
3.2 Break-Even Volume 93
3.3 Make vs. Buy Decision 95
3.4 Sensitivity Analysis 98
3.5 Extensions to Make vs. Buy Decision 101
3.6 Time Value of Money 106
3.7 Net Present Value Method 112
Exercise 3 Break-Even Analysis 117
Break-Even Analysis Additional Solved Problems 119

4 INVENTORY DECISION ANALYSIS 125

4.1 Deterministic Demand 125
4.2 Economic Order Quantity (EOQ) 129
4.3 Sensitivity Analysis 136
4.4 Production Order Quantity (POQ) 139
4.5 Stochastic Demand 143
Exercise 4 Inventory Decision Analysis 152
Inventory Decision Analysis Additional Solved Problems 154

5 PROJECT MANAGEMENT (PERT/CPM) 159

5.1 Introduction 159
5.2 Building the PERT Network 161
5.3 Solving the PERT Network 165
5.4 Gantt Charts 174
5.5 PERT with Stochastic Time Estimates 176
Exercise 5 Project Management (PERT/CPM) 183
Project Management (PERT/CPM) Additional Solved Problems 185

6 SIMPLE REGRESSION 195

6.1 Introduction 195
6.2 Method of Least Squares 197
6.3 Testing the Linear Trend (F-Statistic) 203
6.4 Measuring the Goodness of the Linear Fit ($R^2$-Statistic) 208
6.5 Error Terms for the Regression 210
6.6 General Regression 213
6.7 Future Forecasts and Confidence Intervals 218

Exercise 6 Simple Regression 223
Simple Regression Additional Solved Problems 226

7 FORECASTING 231

7.1 Introduction 231
7.2 Method of Moving Averages 234
7.3 Method of Exponential Smoothing 243
7.4 Method of Seasonal Indices 249
7.5 Measures of Forecasting Error 253
7.6 Demand Times Series vs. Forecasting Methods (Notional) 257
Exercise 7 Forecasting 261
Forecasting Additional Solved Problems 265

8 MULTIPLE REGRESSION 267

8.1 Introduction 267
8.2 Method of Least Squares Applied to Multiple Regression 269
8.3 Assumptions of Multiple Regression Analysis 281
8.4 Specification of the Multiple Regression Model 289
8.5 Conclusions 298
Exercise 8 Multiple Regression 300
Multiple Regression Additional Solved Problems 306

9 LINEAR PROGRAMMING (LP) 311

9.1 Introduction 311
9.2 Linear Program Formulation 313
9.3 Solving the LP (Graphical Method) 318
9.4 Solving the LP (Quick Overview of Simplex Method) 323
9.5 Mechanics of the Simplex Method 326
9.6 Assignment Problem 334
9.7 Transportation Problem 339
9.8 Integer Programming 354
9.9 Duality 360
Exercise 9 Linear Programming (LP) 364
Linear Programming (LP) Additional Solved Problems 369

**10** QUEUEING 377

10.1 Introduction 377
10.2 Arrival Process 381
10.3 Service Process 388
10.4 Solving the Queueing Problem 394
10.5 Final Notes 401
Exercise 10 Queueing 404
Queueing Additional Solved Problems 406

**11** SIMULATION 409

11.1 Definitions 409
11.2 Simulation Overview 413
11.3 Simulation of Complex Models 414
11.4 Summary 426
Exercise 11 Simulation 429
Simulation Additional Solved Problems 431

APPENDICES

A Design of Experiments 435
B Management of Information Systems 459
C Solutions to Exercises 481
D References 551
E Tables 553

Index 571

*"Probability is the very guide of life."*

Cicero

# 1 Probability & Statistics

## 1.1 Basic Probability

Definition: Probability represents the proportion of the time an outcome will occur *in the long run.*

**Example 1:** Consider the toss of a fair die. What is the probability of rolling a "6"?

**Answer:** There are six equally likely outcomes {1,2,3,4,5,6} and the probability of the outcome "6" is 1/6.

For any given sequence of tosses of the die, the number of "6's" will not exactly be one-sixth of the total tosses due to the statistical variation (randomness) of the process. However, for a large number of tosses of a fair die (in the long run), the proportion of "6's" will approach 1/6.

Notes: Two basic axioms of probability are as follows:

- The probability of any single outcome is between 0 and 1, inclusive.
- The sum of the probabilities of all possible outcomes equals 1.

For Example 1, define $O_i$ = The outcome of "$i$" on a roll of a fair die. Then:

$$P(O_1) = P(O_2) = \cdots = P(O_6) = \frac{1}{6}$$

$$P(O_1) + P(O_2) + \cdots + P(O_6) = 1$$

Typically, the process being examined will be more complex than the process of rolling a die and be capable of many outcomes. However, the set of all possible outcomes for any process (experiment, system) is called the sample space.

**Example 2:** The arcade at the Big Top Circus has a game where three darts are thrown at a large number of Blue (B) and Green (G) balloons arranged randomly on a target. Each throw will either a Blue or a Green balloon. Provide the sample space of all possible outcomes for the three throws.

**Answer:** There are eight possible outcomes:

$$\frac{2\ choices}{(Throw\ \#1)} \cdot \frac{2\ choices}{(Throw\ \#2)} \cdot \frac{2\ choices}{(Throw\ \#3)} = 8\ possible\ \ outcomes\ (permutations)$$

$\{B, B, B\} = O_1$
$\{B, B, G\} = O_2$
$\{B, G, B\} = O_3$
$\{B, G, G\} = O_4$
$\{G, B, B\} = O_5$
$\{G, B, G\} = O_6$
$\{G, G, B\} = O_7$
$\{G, G, G\} = O_8$

If we assume that Blue and Green are equally likely to be hit (i.e. $P(B) = P(G) = \frac{1}{2}$ ), then:
$P(O_1) = P(O_2) = \cdots = P(O_8) = \frac{1}{8}$

The outcomes may also be enumerated using an outcome tree as shown in Figures 1. The probability tree is shown in Figure 2.

**Outcome Tree**
(2·2·2 = 8 Events)

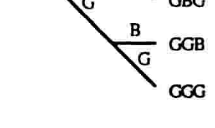


**Figure 1**

**Probability Tree**
($\Sigma$ P(O$_i$) = 1)

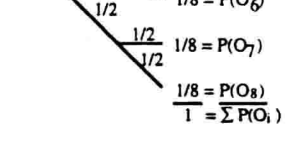


**Figure 2**

If we assume that the number of Blue and Green balloons is such that Blue is hit $\frac{2}{3}$ of the time, (i.e. $P(B) = \frac{2}{3}$ and $P(G) = \frac{1}{3}$ ), then the outcome/probability tree structure is as shown in Figure 3.

**Outcome/Probability Tree**

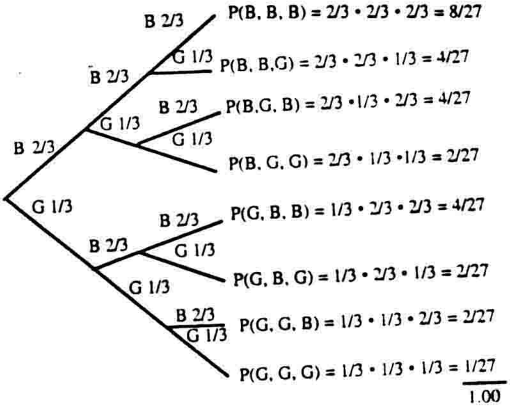


**Figure 3**

## 1.2 Events

An <u>Event</u> is defined as a subset of possible outcomes in the sample space.

**Example 3:** In Example 2 where a group of three balloons is hit, what outcomes represent the Event E = "At least two green in the group of three balloons"? What is P(E)?

**Answer:** The outcomes for Event E = $\{O_4, O_6, O_7, O_8\}$. The probability of an Event E is the sum of the probabilities of the outcomes that represent the event.

$$P(E) = \Sigma P(O_i) \qquad \text{for all } O_i \text{ belonging to Event E}$$

If P(G) = P(B) = 1/2,

$$\begin{aligned} P(E) &= P(O_4) + P(O_6) + P(O_7) + P(O_8) \\ &= P(B, G, G) + P(G, B, G) + P(G, G, B) + P(G, G, G) \\ &= (\tfrac{1}{2} \cdot \tfrac{1}{2} \cdot \tfrac{1}{2}) + (\tfrac{1}{2} \cdot \tfrac{1}{2} \cdot \tfrac{1}{2}) + (\tfrac{1}{2} \cdot \tfrac{1}{2} \cdot \tfrac{1}{2}) + (\tfrac{1}{2} \cdot \tfrac{1}{2} \cdot \tfrac{1}{2}) \\ &= \tfrac{1}{8} + \tfrac{1}{8} + \tfrac{1}{8} + \tfrac{1}{8} \\ &= \tfrac{1}{2} \end{aligned}$$

If P(B) = 2/3 and P(G) = 1/3,

$$\begin{aligned} P(E) &= P(O_4) + P(O_6) + P(O_7) + P(O_8) \\ &= P(B, G, G) + P(G, B, G) + P(G, G, B) + P(G, G, G) \\ &= (\tfrac{2}{3} \cdot \tfrac{1}{3} \cdot \tfrac{1}{3}) + (\tfrac{1}{3} \cdot \tfrac{2}{3} \cdot \tfrac{1}{3}) + (\tfrac{1}{3} \cdot \tfrac{1}{3} \cdot \tfrac{2}{3}) + (\tfrac{1}{3} \cdot \tfrac{1}{3} \cdot \tfrac{1}{3}) \\ &= \tfrac{2}{27} + \tfrac{2}{27} + \tfrac{2}{27} + \tfrac{1}{27} \end{aligned}$$