ACTIVE NETWORK ANALYSIS

Problems & Solutions

WAI-KAI CHEN

World Scientific

ACTIVE NETWORK ANALYSIS

Problems & Solutions

WAI-KAI CHEN
The University of Illinois at Chicago

World Scientific
Singapore • New Jersey • London • Hong Kong

Published by

World Scientific Publishing Co. Pte. Ltd.

P O Box 128, Farrer Road, Singapore 9128

USA office: Suite 1B, 1060 Main Street, River Edge, NJ 07661

UK office: 73 Lynton Mead, Totteridge, London N20 8DH

ACTIVE NETWORK ANALYSIS – PROBLEMS & SOLUTIONS

ISBN 981-02-1336-0 (pbk)

Printed in Singapore by Stamford Press Pte Ltd

ACTIVE NETWORK ANALYSIS

Problems & Solutions

PREFACE

The solutions to the problems in the text ***Active Network Analysis*** are presented in this book. Attempt has been made to conform the notation of the text, and the approach of the text is used in solving the problems. The book is intended as an aid for the instructors using the textbook, not for the use of students. It contains solutions to every problem in the text except some problems involving proofs of the identities and the verification of solutions. The book gives only the major steps and manipulations, skipping most of the details.

The solutions to the problems were worked out by graduate students, teaching assistants, and visiting scholars. Most of the solutions to the first five chapters were contributed by my doctoral students Dr. Eishi Yasui and Ms. Hui Tang. The solutions to the last four chapters were provided by Professor Mao-Da Tong of the University of Science and Technology of China, who was a visiting scholar at the University of Illinois at Chicago. The material was initially typed by Messrs. Xin Shu and Chi-Kuang Chao. Professor Yi-Sheng Zhu of Dalian Marine College proofread the first five chapters, and Professor Tong for the remainder.

I wish to express my appreciation to all the persons mentioned above whose valuable contributions made the project possible.

Wai-Kai Chen
Department of Electrical Engineering
and Computer Science
University of Illinois at Chicago
Chicago, Illinois 60680

November 15, 1992

CONTENTS

Chapter 1

Characterizations of Networks

Problem 1.4 *Consider a linear time-varying resistor whose resistance is characterized by the equation*

$$R(t) = r_0 + r_1 \cos(\omega_0 t + \theta)$$

where $r_0 \geq r_1 > 0$ and $\omega_0 > 0$. *Determine whether this device is passive or active.*

SOLUTION:

$$\varepsilon(t) = \int_{-\infty}^{t} v(x)i(x)\,dx = \int_{-\infty}^{t} R(x)i^2(x)\,dx$$

$$= \int_{-\infty}^{t} r_0\left[1 + \frac{r_1}{r_0}\cos(\omega_0 x + \theta)\right]i^2(x)\,dx$$

Since $r_0 \geq r_1 > 0$

$$1 + \frac{r_1}{r_0}\cos(\omega_0 x + \theta) \geq 0$$

showing that

$$\varepsilon(t) \geq 0$$

for all t and all excitations. Thus the device is passive.

Problem 1.7 *For the given one-port network of Fig. 1.29, determine the regions of passivity and activity in the closed RHS. Choose a point in the passive region and demonstrate that this one-port has a passive equivalent one-port at the chosen point. Choose a point in the active region and show that this one-port can be employed for energy gain at the chosen complex frequency.*

SOLUTION: The input admittance of the one-port is given by

$$y(s) = -\frac{1}{R} + Cs$$

On the σ-axis, the passivity condition becomes

$$y(\sigma) = -\frac{1}{R} + C\sigma \geq 0, \text{ or } \sigma \geq \sigma_m = \frac{1}{RC}$$

On the $j\omega$-axis,

$$\text{Re}\, y(s) = -\frac{1}{R} < 0$$

and the one-port is active for all real frequencies. For the remainder of the closed RHS, we consider $s = \sigma + j\omega$, $\omega \neq 0$. When

$$\sigma \le \sigma_m = \frac{1}{RC}$$

we have

$$\mathrm{Re}\, y(s) = -\frac{1}{R} + C\sigma \le 0$$

So $\mathrm{Re}\, y(s) - \frac{\sigma}{|s|}|y(s)| < 0$, and the one-port is active.

When $\sigma > \sigma_m = \frac{1}{RC}$, $\mathrm{Re}\, y(s) = -\frac{1}{R} + C\sigma > 0$ and

$$\mathrm{Re}\, y(s) - \frac{\sigma}{|s|}|y(s)| = \mathrm{Re}\, y(s)\left[1 - \frac{\left|y(s)/\mathrm{Re}\, y(s)\right|}{\left|s/\sigma\right|}\right]$$

$$= \left(-\frac{1}{R} + C\sigma\right)\left[1 - \frac{\left|1 + j\dfrac{\omega}{(\sigma - \sigma_m)}\right|}{\left|1 + j\dfrac{\omega}{\sigma}\right|}\right] < 0$$

because

$$0 < \sigma - \sigma_m < \sigma, \qquad \left|1 + j\frac{\omega}{(\sigma - \sigma_m)}\right| > \left|1 + j\frac{\omega}{\sigma}\right|$$

This shows that the one-port is again active. Thus, the passive region is on the σ-axis for all

$$\sigma \ge \sigma_m = \frac{1}{RC}$$

Choose a point in the passive region, say $s_0 = 2\sigma_m$. This one-port is equivalent to a single resistor with resistance

$$r = \frac{1}{y(s_0)} = \frac{1}{y(2\sigma_m)} = \frac{1}{-\frac{1}{R} + C\frac{2}{RC}} = R$$

Pick a point $s_0 = \frac{1}{2}\sigma_m = \frac{1}{2RC}$ in the active region. Then

$$y(s_0) = -\frac{1}{R} + \frac{1}{2}\sigma_m C = -\frac{1}{R} + \frac{1}{2R} = -\frac{1}{2R}$$

Let R_L be a resistor. The energy delivered to R_L by the current source

$$i(t) = I_0\, e^{\frac{1}{2}\sigma_m t}$$

being connected in parallel with the resistor, is given by

$$\varepsilon(t) = \int_{-\infty}^{t} R_L i^2(x)\, dx$$

Now connect the active one-port $y(s_0)$ in parallel with $i(t)$ and R_L. Then the terminal voltage $v_l(t)$ and current $i_l(t)$ of R_L are found to be

$$v_l(t) = \frac{i(t)}{y(s_0) + \frac{1}{R_L}} = \frac{i(t)}{-\frac{1}{2R} + \frac{1}{R_L}}$$

$$= \frac{2RR_L}{2R - R_L}\, i(t)$$

$$i_l(t) = \frac{1}{R_L} v_l(t) = \frac{2R}{2R - R_L}\, i(t)$$

The energy delivered to R_L under this condition is given by

$$\varepsilon_d(t) = \int_{-\infty}^{t} i_l(x) v_l(x) dx = \frac{4R^2 R_L}{(2R - R_L)^2} \int_{-\infty}^{t} i^2(x) dx$$

or

$$\frac{\varepsilon_d(t)}{\varepsilon(t)} = \frac{4R^2 R_L}{(2R - R_L)^2 R_L} = \frac{4R^2}{(2R - R_L)^2}$$

When $4R > R_L > 0$,

$$\frac{\varepsilon_d(t)}{\varepsilon(t)} > 1$$

Pick another point $s_0 = 2\sigma_m + j\omega_0 = \frac{2}{RC} + j\omega_0$ in the active region.

$$y(s_0) = -\frac{1}{R} + C\left(\frac{2}{RC} + j\omega_0\right) = \frac{1}{R} + j\omega_0 C$$

Let the admittance $y_p(s)$ of a passive one-port at

$$s = s_0 = 2\sigma_m + j\omega_0$$

be given by

$$y_p(s_0) = \frac{1}{R_L} - j\omega_0 C$$

To ensure passivity of $y_p(s_0)$, we must have [from (1.139)]

$$\frac{1}{R_L} \geq \frac{2\sigma_m |\omega_0 C|}{|\omega_0|} \quad or \quad R \geq 2R_L$$

Then following the same steps of Example 1.15, the maximum energy delivered to $y_p(s_0)$ by the current source of the type

$$i(t) = \frac{1}{2}(I_0 e^{s_0 t} + \bar{I}_0 e^{\bar{s}_0 t}) = \frac{1}{2}\left[I_0 e^{(2\sigma_m + j\omega_0)t} + \bar{I}_0 e^{(2\sigma_m - j\omega_0)t}\right]$$

under arbitrary passive admittance $y_b(s_0)$ of Fig. 1.27(b) is obtained as

$$\varepsilon_m(t) = \frac{|I_0|^2 e^{2\sigma_0 t}}{4\sigma_0\left[\mathrm{Re}\, y_p(s_0) + \sigma_0 \left|\mathrm{Im}\, y_p(s_0)\middle/\omega_0\right|\right]^2}\left[\mathrm{Re}\, y_p(s_0) + \frac{\sigma_0}{|s_0|}|y_p(s_0)|\cos(2\omega_0 t + \theta)\right]$$

where $\theta = 2\angle I_0 + \angle y_p(s_0) - \angle s_0$.

If we replace $y_b(s_0)$ by $y(s)$, the energy $\varepsilon_d(t)$ provided to $y_p(s_0)$ is given by

$$\varepsilon_d(t) = \frac{|I_0|^2 e^{2\sigma_0 t}}{4\sigma_0|y(s_0) + y_p(s_0)|^2}\left[\mathrm{Re}\, y_p(s_0) + \frac{\sigma_0}{|s_0|}|y_p(s_0)|\cos(2\omega_0 t + \theta)\right]$$

giving

$$\frac{\varepsilon_d(t)}{\varepsilon_m(t)} = \frac{\left[\mathrm{Re}\, y_p(s_0) + \sigma_0\left|\mathrm{Im}\, y_p(s_0)\middle/\omega_0\right|\right]^2}{|y(s_0) + y_p(s_0)|^2}$$

$$= \frac{\left[\frac{1}{R_L} + \frac{2}{RC}\left|-\omega_0 C\middle/\omega_0\right|\right]^2}{\left|\frac{1}{R} + j\omega_0 C + \frac{1}{R_L} - j\omega_0 C\right|^2}$$

$$= \frac{\left(\frac{1}{R_L} + \frac{2}{R}\right)^2}{\left(\frac{1}{R} + \frac{1}{R_L}\right)^2} = \frac{(R + 2R_L)^2}{(R + R_L)^2} > 1$$

Thus the active one-port can be employed for energy gain at the chosen complex frequency.

Problem 1.8 *For the two-port network considered in Example 1.4, compute the hybrid parameters $h_{12}(s)$ and $h_{22}(s)$, the impedance parameters $z_{12}(s)$ and $z_{22}(s)$, the admittance parameters $y_{12}(s)$ and $y_{22}(s)$, and the transmission parameters $B(s)$ and $D(s)$.*

SOLUTION:

$$h_{12}(s) = \frac{C_2 s}{G_1 + (C_1 + C_2)s}$$

$$h_{22}(s) = (G_2 + C_2 s) + \frac{(g_m - C_2 s)C_2 s}{G_1 + (C_1 + C_2)\, s}$$

$$z_{12}\,(s) = \frac{C_2 s}{(G_1 + C_1 s\,)(G_2 + C_2 s\,) + (g_m + G_2)C_2 s}$$

$$z_{22}\,(s) = \frac{1}{h_{22}\,(s)} = \frac{G_1 + (C_1 + C_2)s}{(G_1 + C_1 s)(G_2 + C_2 s) + (g_m + G_2)C_2 s}$$

$$y_{12}\,(s) = -\,C_2 s$$

$$y_{22}\,(s) = G_2 + C_2 s$$

$$B(s) = \frac{1}{C_2 s - g_m}$$

$$D(s) = \frac{G_1 + (C_1 + C_2)s}{C_2 s - g_m}$$

Problem 1.10 *Determine the condition under which the impedance matrix (1.69) is positive real.*

SOLUTION:

$$\mathbf{Z}(s) = \frac{1}{q(s)}\begin{bmatrix} G_2 + sC_2 & sC_2 \\ sC_2 - g_m & G_1 + s(C_1 + C_2) \end{bmatrix}$$

$$q(s) = (G_1 + sC_1)(G_2 + sC_2) + sC_2(G_2 + g_m)$$

Obviously $\mathbf{Z}(s)$ has no poles in the closed RHS, and $\overline{\mathbf{Z}}(s) = \mathbf{Z}(\bar{s})$. So conditions 1, 2 and 3 of Theorem 1.3 are satisfied.

$$\mathbf{Z}(j\omega) = \frac{1}{q(j\omega)}\begin{bmatrix} G_2 + j\omega C_2 & j\omega C_2 \\ j\omega C_2 - g_m & G_1 + j\omega(C_1 + C_2) \end{bmatrix}$$

where

$$\begin{aligned} q(j\omega) &= \alpha + j\beta \\ &= G_1 G_2 - sC_1 C_2 \omega^2 + j\omega(C_1 G_2 + C_2 G_1 + C_2 G_2 + C_2 g_m) \end{aligned}$$

Compute

$$\mathbf{Z}_h(j\omega) = \frac{1}{2}[\mathbf{Z}(j\omega) + \mathbf{Z}^*(j\omega)] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

where

$$a_{11} = \frac{G_1G_2^2 + \omega^2 C_2^2(G_1 + G_2 + g_m)}{\left|q(j\omega)\right|^2} > 0$$

$$a_{12} = \overline{a_{21}} = \frac{2\omega C_2\beta - g_m\alpha - jg_m\beta}{2\left|q(j\omega)\right|^2}$$

$$a_{22} = \frac{G_1^2G_2 + \omega C_2\beta + \omega^2 C_1(C_1G_2 + C_2G_2 + C_2g_m)}{\left|q(j\omega)\right|^2} > 0$$

$$\begin{aligned}\left|q(j\omega)\right|^4 \det \mathbf{Z}_h(j\omega) &= G_1G_2\alpha^2 - \tfrac{1}{4}g_m^2(\alpha^2 + \beta^2) + \omega^2 C_1C_2\beta^2 \\ &\quad + \alpha\omega\beta(C_1G_2 + C_2G_1 + C_2G_2 + C_2g_m) \\ &= (G_1G_2 - \tfrac{1}{4}g_m^2)\left|q(j\omega)\right|^2\end{aligned}$$

or

$$\det \mathbf{Z}_h(j\omega) = \frac{G_1G_2 - \frac{1}{4}g_m^2}{\left|q(j\omega)\right|^2} \geq 0$$

We obtain

$$4G_1G_2 \geq g_m^2$$

Problem 1.17 *A nonlinear capacitor is characterized by the equation*

$$i(t) = 2v(t)\frac{dv(t)}{dt}$$

Determine whether this capacitor is passive or active.

SOLUTION:

$$\varepsilon(t) = \int_{-\infty}^{t} i(x)v(x)dx = \frac{2}{3}v^3(t) \qquad \text{if } v(-\infty) = 0$$

This capacitor is active because $\varepsilon(t) < 0$ for some excitations $v(t)$ of negative values. For example, let

$$v(t) = \begin{cases} 0 & t < 0 \\ -t & t \geq 0 \end{cases}$$

Then