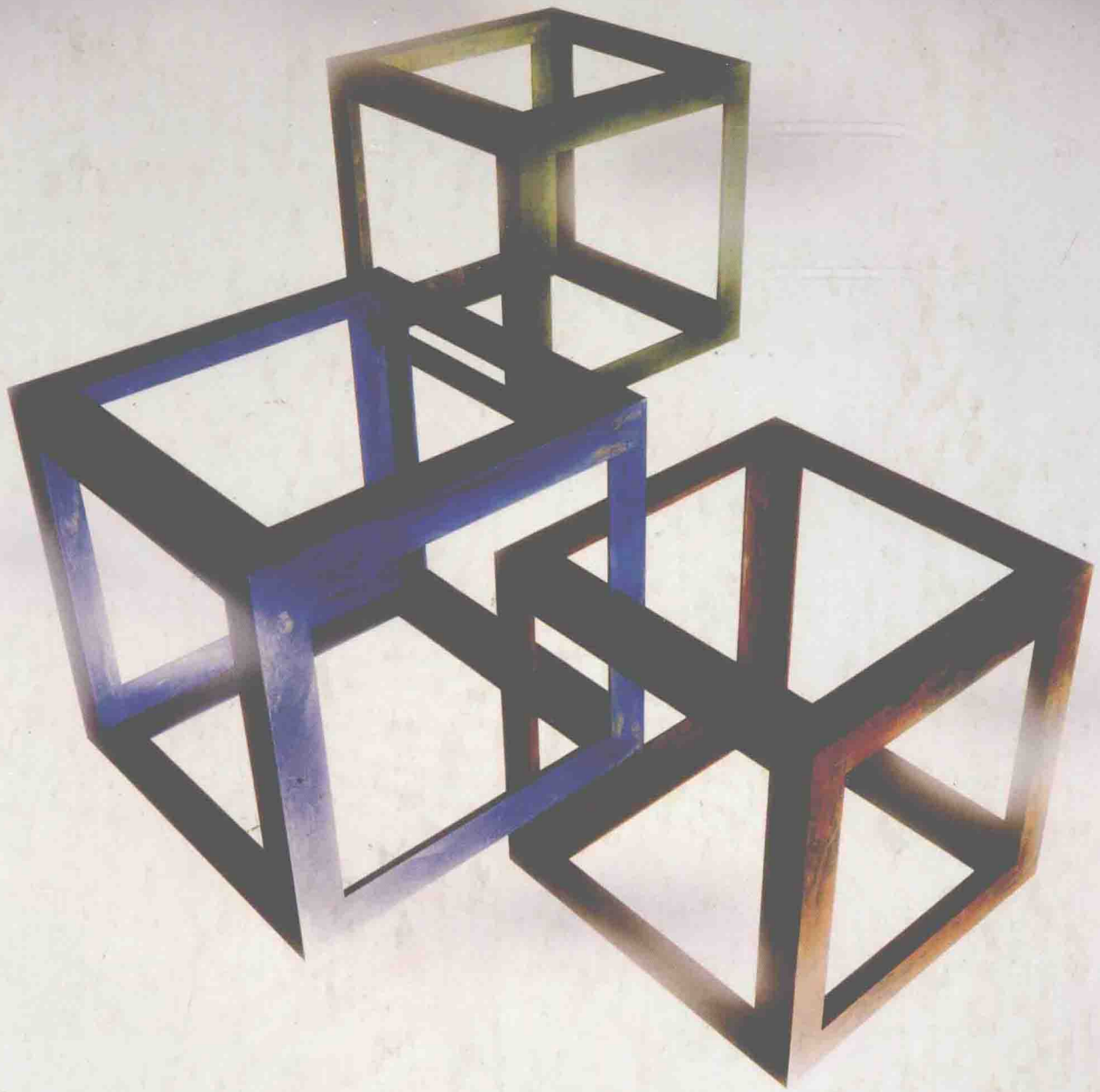
PRECALCULUS

FUNCTIONS & GRAPHS



SWOKOWSKI / COLE

NINTH EDITION

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PRECALCULUS: Functions and Graphs

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PREFACE

The ninth edition of *Precalculus: Functions and Graphs* improves upon the eighth edition in several ways, but the most significant is the inclusion of many graphing calculator examples and inserts, which feature specific color-coded keystrokes and screens for the TI-83 Plus and the TI-86. These give added VALUE to the text for students—especially those who are working with a graphing calculator for the first time. They also give professors more flexibility in terms of the way they approach a solution.

This feature should save class time, as students have the directions right in front of them, not in an additional optional manual. The design of the text makes the technology inserts easily identifiable. I view the inserts as such an important addition to the text that I have included a special technology table of contents to make looking them up easier.

There are many new examples, figures, and exercises. All have been incorporated without sacrificing the mathematical soundness that has been paramount to the success of this text.

The principal changes made to each chapter are highlighted below. For the reader who is unfamiliar with the previous editions of this text, a list of the general features of the text follows this list of changes.

Changes for This Edition

Chapter 1 Graphing calculator operations are introduced early, and there is an emphasis on using a graphing calculator to verify algebraic operations. Specific directions are given for most of the basic graphing features, such as finding zeros and points of intersection, as well as some of the more difficult topics, such as finding a regression model.

Chapter 2 New calculator coverage includes sketching a piecewise-defined function, finding maximum or minimum values, and graphing the inverse of a function.

Chapter 3 More attention is devoted to the relationship between rational expressions and graphs of rational functions.

Chapter 4 Modeling an exponential function and the relationship between a^x and e^x are given added attention.

Chapter 6 New alternative solutions of examples give students valuable insight into solving exercises using a variety of methods.

Chapter 7 The sections on vectors and the dot product have been moved to follow the law of cosines, which can be used for many of the solutions of the exercises involving vectors. There are additional exercises requiring work with unit vectors.

Chapter 8 Systems of inequalities and linear programming immediately follow solving systems. The solution of a system using matrices has been moved closer to the algebra of matrices.

Chapter 9 Substantial technology support has been included for sequences and series. The topic of odds and their relationship to probability has been added.

Chapter 10 The section on polar coordinates has been rewritten so that graphing polar equations is discussed after polar-rectangular point and equation conversions.

Features

Illustrations Brief demonstrations of the use of definitions, laws, and theorems are provided in the form of illustrations.

Charts Charts give students easy access to summaries of properties, laws, graphs, relationships, and definitions. These charts often contain simple illustrations of the concepts that are being introduced.

Examples Titled for easy reference, all examples provide detailed solutions of problems similar to those that appear in exercise sets. Many examples include graphs, charts, or tables to help the student understand procedures and solutions.

Step-by-Step Explanations In order to help students follow them more easily, many of the solutions in examples contain step-by-step explanations.

Discussion Exercises Each chapter ends with several exercises that are suitable for small-group discussions. These exercises range from easy to difficult and from theoretical to application-oriented.

Checks The solutions to some examples are explicitly checked, to remind students to verify that their solutions satisfy the conditions of the problems.

Graphing Calculator Examples Wherever appropriate, examples requiring the use of a graphing utility have been added to the text. These are designated by a calculator icon (shown to the left) and illustrated with a figure reproduced from a graphing calculator screen.

Graphing Calculator Inserts In addition to the graphing calculator examples, these inserts are included to highlight some of the capabilities of graphing calculators and/or illustrate their use in performing the operations under discussion. See, for example, "Using the TI-86 POLY Feature" in Section 3.1 and "Using the TI-83 Plus Sequence Mode" in Section 9.1.

Graphing Calculator Exercises Exercises specifically designed to be solved with a graphing utility are included in appropriate sections. These exercises are also designated by a calculator icon (shown to the left).

Applications To arouse student interest and to help students relate the exercises to current real-life situations, applied exercises have been titled. One look at the Index of Applications in the back of the book reveals the wide array of topics. Many professors have indicated that the applications constitute one of the strongest features of the text.

Exercises Exercise sets begin with routine drill problems and gradually progress to more difficult problems. An ample number of exercises contain graphs and tabular data; others require the student to find a mathematical model for the given data. Applied problems generally appear near the end of an exercise set, to allow students to gain confidence in working with the new ideas that have been presented before they attempt problems that require greater analysis and synthesis of these ideas. Review exercises at the end of each chapter may be used to prepare for examinations.

Guidelines Boxed guidelines enumerate the steps in a procedure or technique, to help students solve problems in a systematic fashion.

Warnings Interspersed throughout the text are warnings to alert students to common mistakes.

Text Art Forming a total art package that is second to none, figures and graphs have been computer-generated for accuracy, using the latest technology. Colors are employed to distinguish between different parts of figures. For example, the graph of one function may be shown in blue and that of a second function in red. Labels are the same color as the parts of the figure they identify.

Text Design The text has been designed to ensure that discussions are easy to follow and important concepts are highlighted. Color is used pedagogically to clarify complex graphs and to help students visualize applied problems. Previous adopters of the text have confirmed that the text strikes a very appealing balance in terms of color use.

Endpapers The endpapers in the front and back of the text provide useful summaries from algebra, geometry, and trigonometry.

Appendixes Appendix I, "Common Graphs and Their Equations," is a pictorial summary of graphs and equations that students commonly encounter in precalculus mathematics. Appendix II, "A Summary of Graph Transformations," is an illustrative synopsis of the basic graph transformations discussed in the text: shifting, stretching, compressing, and reflecting. Appendix III, "Graphs of Trigonometric Functions and Their Inverses," contains graphs, domains, and ranges of the six trigonometric functions and their inverses. Appendix IV, "Values of the Trigonometric Functions of Special Angles on a Unit Circle," is a full-page reference for the most common angles on a unit circle—valuable for students who are trying to learn the basic trigonometric function values.

Answer Section The answer section at the end of the text provides answers for most of the odd-numbered exercises, as well as answers for all chapter review exercises. Considerable thought and effort were devoted to making this section a learning device for the student instead of merely a place to check answers. For instance, proofs are given for mathematical induction problems. Numerical answers for many exercises are stated in both an exact and an approximate form. Graphs, proofs, and hints are included whenever appropriate. Author-prepared solutions and answers ensure a high degree of consistency among the text, the solutions manuals, and the answers.

Teaching Tools for the Instructor

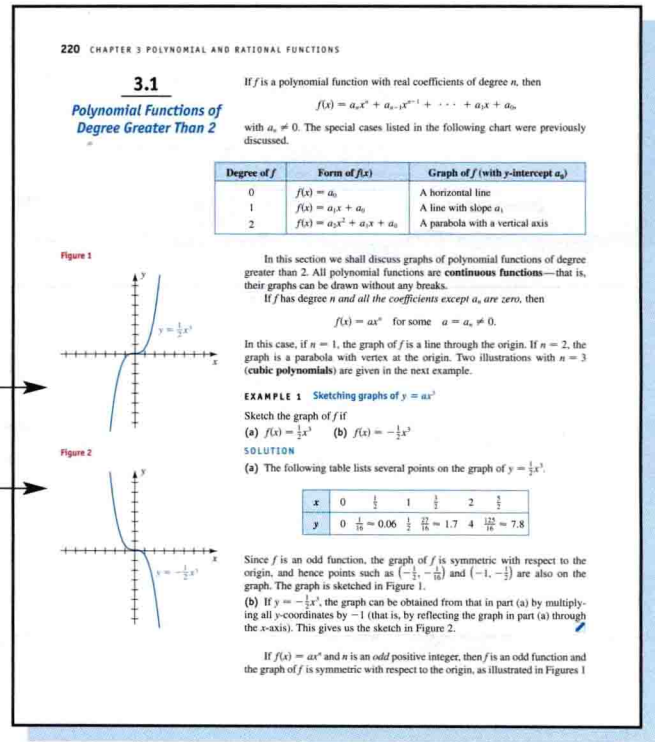
Printed Materials

Annotated Instructor's Edition (0-534-38461-7) This special version of the complete student text has answers printed in blue next to the respective exercises. Graphs, tables, and other answers too long to appear next to their exercises are in a special answer section in the back of the text.

Instructor's Solutions Manual by Jeff Cole (0-534-37848-X) This author-prepared manual includes answers to all exercises and detailed solutions to most exercises. The manual has been thoroughly reviewed for accuracy.

Software

BCA Testing (0-534-37847-1) With a balance of efficiency and high performance, simplicity, and versatility, *Brooks/Cole Assessment* gives you the power to transform the learning and teaching experience. This revolutionary, internet-ready testing suite is text-specific and allows you to customize exams and track student progress in an accessible, browser-based format. BCA offers full algorithmic generation of problems and free response mathematics. No longer are you limited to multiple-choice or true/false test questions. The complete integration of the testing and course management components simplifies routine tasks. Test results flow automatically to your gradebook, and you can communicate easily with individuals, sections, or entire courses.



220 CHAPTER 3 POLYNOMIAL AND RATIONAL FUNCTIONS

3.1

Polynomial Functions of Degree Greater Than 2

If f is a polynomial function with real coefficients of degree n, then

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

with $a_n \neq 0$. The special cases listed in the following chart were previously discussed.

Degree of f	Form of $f(x)$	Graph of f (with y-intercept a_0)
0	$f(x) = a_0$	A horizontal line
1	$f(x) = a_1x + a_0$	A line with slope a_1
2	$f(x) = a_2x^2 + a_1x + a_0$	A parabola with a vertical axis

Figure 1

Figure 2

In this section we shall discuss graphs of polynomial functions of degree greater than 2. All polynomial functions are **continuous functions**—that is, their graphs can be drawn without any breaks.

If f has degree n *and all the coefficients except* a_n *are zero*, then

$$f(x) = ax^n \quad \text{for some} \quad a = a_n \neq 0.$$

In this case, if $n = 1$, the graph of f is a line through the origin. If $n = 2$, the graph is a parabola with vertex at the origin. Two illustrations with $n = 3$ (**cubic polynomials**) are given in the next example.

EXAMPLE 1 Sketching graphs of $y = ax^3$

Sketch the graph of f if

(a) $f(x) = \frac{1}{2}x^3$ (b) $f(x) = -\frac{1}{2}x^3$

SOLUTION

(a) The following table lists several points on the graph of $y = \frac{1}{2}x^3$.

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$
y	0	$\frac{1}{16} \approx 0.06$	$\frac{1}{2}$	$\frac{27}{16} \approx 1.7$	4	$\frac{125}{16} \approx 7.8$

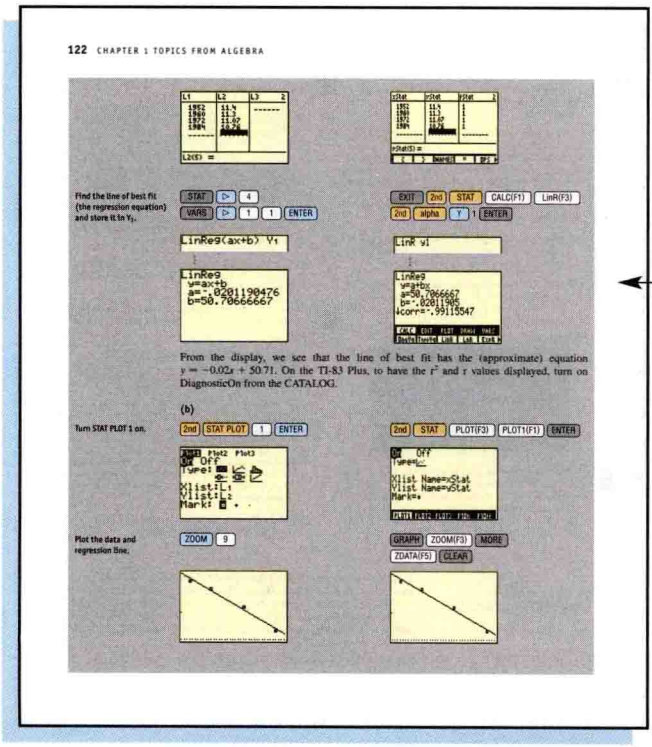
Since f is an odd function, the graph of f is symmetric with respect to the origin, and hence points such as $\left(-\frac{1}{2}, -\frac{1}{16}\right)$ and $\left(-1, -\frac{1}{2}\right)$ are also on the graph. The graph is sketched in Figure 1.

(b) If $y = -\frac{1}{2}x^3$, the graph can be obtained from that in part (a) by multiplying all y-coordinates by -1 (that is, by reflecting the graph in part (a) through the x-axis). This gives us the sketch in Figure 2.

If $f(x) = ax^n$ and n is an *odd* positive integer, then f is an odd function and the graph of f is symmetric with respect to the origin, as illustrated in Figures 1

Examples, well structured and graded by difficulty, are titled for easy reference.

Many examples contain graphs, charts, or tables to help the student understand procedures and solutions.



122 CHAPTER 1 TOPICS FROM ALGEBRA

Find the line of best fit (the regression equation) and store it in Y_1.

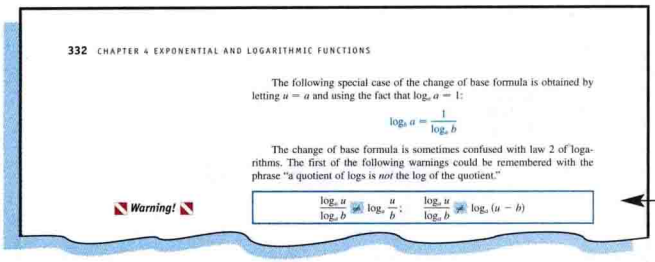
From the display, we see that the line of best fit has the (approximate) equation $y = -0.02x + 50.71$. On the TI-83 Plus, to have the r^2 and r values displayed, turn on DiagnosticOn from the CATALOG.

(b)

Turn STAT PLOT 1 on.

Plot the data and regression line.

Graphing calculator inserts are included to highlight some of the capabilities of graphing calculators and/or illustrate their use in performing the operations under discussion.



332 CHAPTER 4 EXPONENTIAL AND LOGARITHMIC FUNCTIONS

The following special case of the change of base formula is obtained by letting $u = a$ and using the fact that $\log_a a = 1$:

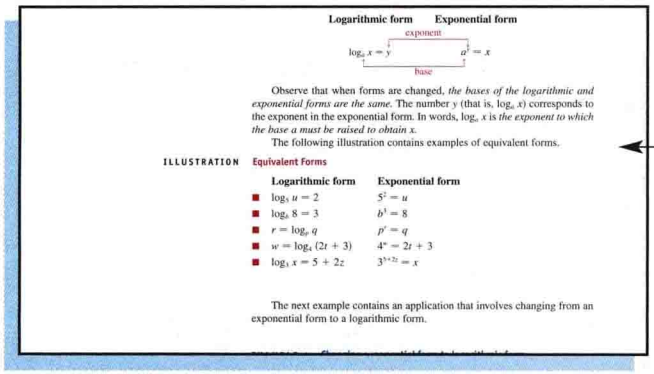
$$\log_a a = \frac{1}{\log_a b}$$

The change of base formula is sometimes confused with law 2 of logarithms. The first of the following warnings could be remembered with the phrase "a quotient of logs is *not* the log of the quotient."

Warning!

$$\frac{\log_a u}{\log_a b} \neq \log_a \frac{u}{b}; \qquad \frac{\log_a u}{\log_a b} \neq \log_a (u - b)$$

Warnings are interspersed throughout the text to alert students to common mistakes.



Logarithmic form **Exponential form**

exponent

$\log_a x = y$ $\qquad a^y = x$

base

Observe that when forms are changed, *the bases of the logarithmic and exponential forms are the same.* The number y (that is, $\log_a x$) corresponds to the exponent in the exponential form. In words, $\log_a x$ *is the exponent to which the base a must be raised to obtain x.*

The following illustration contains examples of equivalent forms.

ILLUSTRATION Equivalent Forms

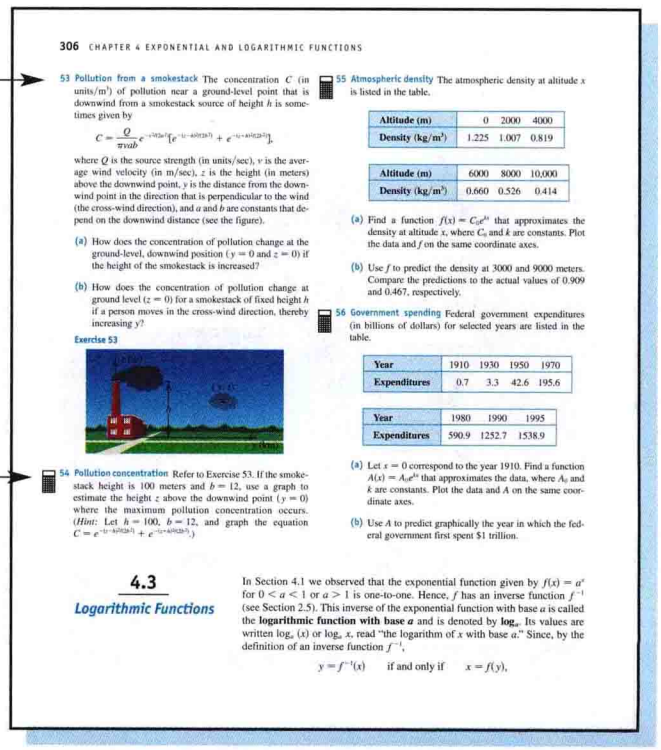
Logarithmic form	Exponential form
$\log_5 u = 2$	$5^2 = u$
$\log_b 8 = 3$	$b^3 = 8$
$r = \log_p q$	$p^r = q$
$w = \log_4 (2t + 3)$	$4^w = 2t + 3$
$\log_3 x = 5 + 2z$	$3^{5+2z} = x$

The next example contains an application that involves changing from an exponential form to a logarithmic form.

Illustrations provide brief demonstrations of the use of definitions, laws, and theorems.

Exercise sets begin with drill problems and then progress to more challenging problems, including applications designed to show students how the mathematical procedures can be applied to current real-life situations.

Graphing calculator exercises, designated by a , are included in many sections.



306 CHAPTER 4 EXPONENTIAL AND LOGARITHMIC FUNCTIONS

53 Pollution from a smokestack The concentration C (in units/m^3) of pollution near a ground-level point that is downwind from a smokestack source of height h is sometimes given by

$$C = \frac{Q}{\pi v a b} e^{-y^2/a^2} \left[e^{-(z-h)^2/b^2} + e^{-(z+h)^2/b^2} \right],$$

where Q is the source strength (in units/sec), v is the average wind velocity (in m/sec), z is the height (in meters) above the downwind point, y is the distance from the downwind point in the direction that is perpendicular to the wind (the cross-wind direction), and a and b are constants that depend on the downwind distance (see the figure).

(a) How does the concentration of pollution change at the ground-level, downwind position ($y = 0$ and $z = 0$) if the height of the smokestack is increased?

(b) How does the concentration of pollution change at ground level ($z = 0$) for a smokestack of fixed height h if a person moves in the cross-wind direction, thereby increasing y?

Exercise 53

54 Pollution concentration Refer to Exercise 53. If the smokestack height is 100 meters and $b = 12$, use a graph to estimate the height z above the downwind point ($y = 0$) where the maximum pollution concentration occurs. (*Hint:* Let $h = 100$, $b = 12$, and graph the equation $C = e^{-(z-h)^2/(2b^2)} + e^{-(z+h)^2/(2b^2)}$.)

55 Atmospheric density The atmospheric density at altitude x is listed in the table.

Altitude (m)	0	2000	4000
Density (kg/m^3)	1.225	1.007	0.819

Altitude (m)	6000	8000	10,000
Density (kg/m^3)	0.660	0.526	0.414

(a) Find a function $f(x) = C_0 e^{kx}$ that approximates the density at altitude x, where C_0 and k are constants. Plot the data and f on the same coordinate axes.

(b) Use f to predict the density at 3000 and 9000 meters. Compare the predictions to the actual values of 0.909 and 0.467, respectively.

56 Government spending Federal government expenditures (in billions of dollars) for selected years are listed in the table.

Year	1910	1930	1950	1970
Expenditures	0.7	3.3	42.6	195.6

Year	1980	1990	1995
Expenditures	590.9	1252.7	1538.9

(a) Let $x = 0$ correspond to the year 1910. Find a function $A(x) = A_0 e^{kx}$ that approximates the data, where A_0 and k are constants. Plot the data and A on the same coordinate axes.

(b) Use A to predict graphically the year in which the federal government first spent $1 trillion.

4.3 Logarithmic Functions

In Section 4.1 we observed that the exponential function given by $f(x) = a^x$ for $0 < a < 1$ or $a > 1$ is one-to-one. Hence, f has an inverse function f^{-1} (see Section 2.5). This inverse of the exponential function with base a is called the **logarithmic function with base a** and is denoted by $\log_a$. Its values are written $\log_a(x)$ or $\log_a x$, read "the logarithm of x with base a." Since, by the definition of an inverse function f^{-1},

$$y = f^{-1}(x) \quad \text{if and only if} \quad x = f(y),$$

Videotapes

Text-Specific Videotapes (0-534-37906-0) A set of book-specific videotapes is available without charge. The videotapes feature instruction by Jeff Cole, who takes students step by step through the key concepts from the text, and computer-generated art.

Learning Tools for the Student

Printed Materials

Student Solutions Manual by Jeff Cole (0-534-37897-8) This author-prepared manual provides solutions for all of the odd-numbered exercises, as well as strategies for solving additional exercises. Many helpful hints and warnings are also included.

Quick Reference Card Packaged with this edition of the text is a formula card for solving exercises. This perforated card, found in the back of the book, will aid students in mastering key formulas and minimize the need for page turning. Because the card reduces time spent on tedious tasks, the student can focus on the central concepts and principles of the course.

Explorations in Precalculus Using the TI-82/TI-83/TI-83 Plus/TI-85/TI-86, Second Edition by Deborah J. Cochener and Bonnie M. Hodge of Austin Peay State University (0-534-38197-9) This workbook helps students quickly learn to use the graphing calculator to develop problem-solving and critical-thinking skills that will improve performance in their college algebra course. The authors include hands-on applications, Further Explorations, key charts, a Troubleshooting list, and more!

Software

Technical Support Toll-free technical support is available for any Brooks/Cole software product by phoning (800) 423-0563.

BCA Tutorial (student: 0-534-37905-2; instructor: 0-534-38846-9) This text-specific, interactive tutorial software, delivered via the web (at http://bca.brookscole.com), is offered in both student and instructor versions. Like *BCA Testing,* it has a browser-based format, making it an intuitive mathematical guide, ideal even for students with little technological proficiency. So sophisticated it's simple, *BCA Tutorial* allows students to work with real math notation in real time and provides instant analysis and feedback. The tracking program built into the instructor version of the software enables instructors to carefully monitor student progress.

Interactive Video Skillbuilder (0-534-38373-4) This interactive tutorial CD-ROM covers mathematical concepts that have been found to be the most difficult for students. It features short videos of problem-solving lessons as well as chapter tests.

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In addition to all the persons named here, I would like to express my sincere gratitude to the many students and teachers who have helped shape my views on mathematics education. Please feel free to write to me about any aspect of this text—I value your opinion.

Jeffery A. Cole

CONTENTS

LIST OF GRAPHING CALCULATOR TOPICS

There are many other places where a graphing calculator is used—these are the ones that include specific keystrokes.