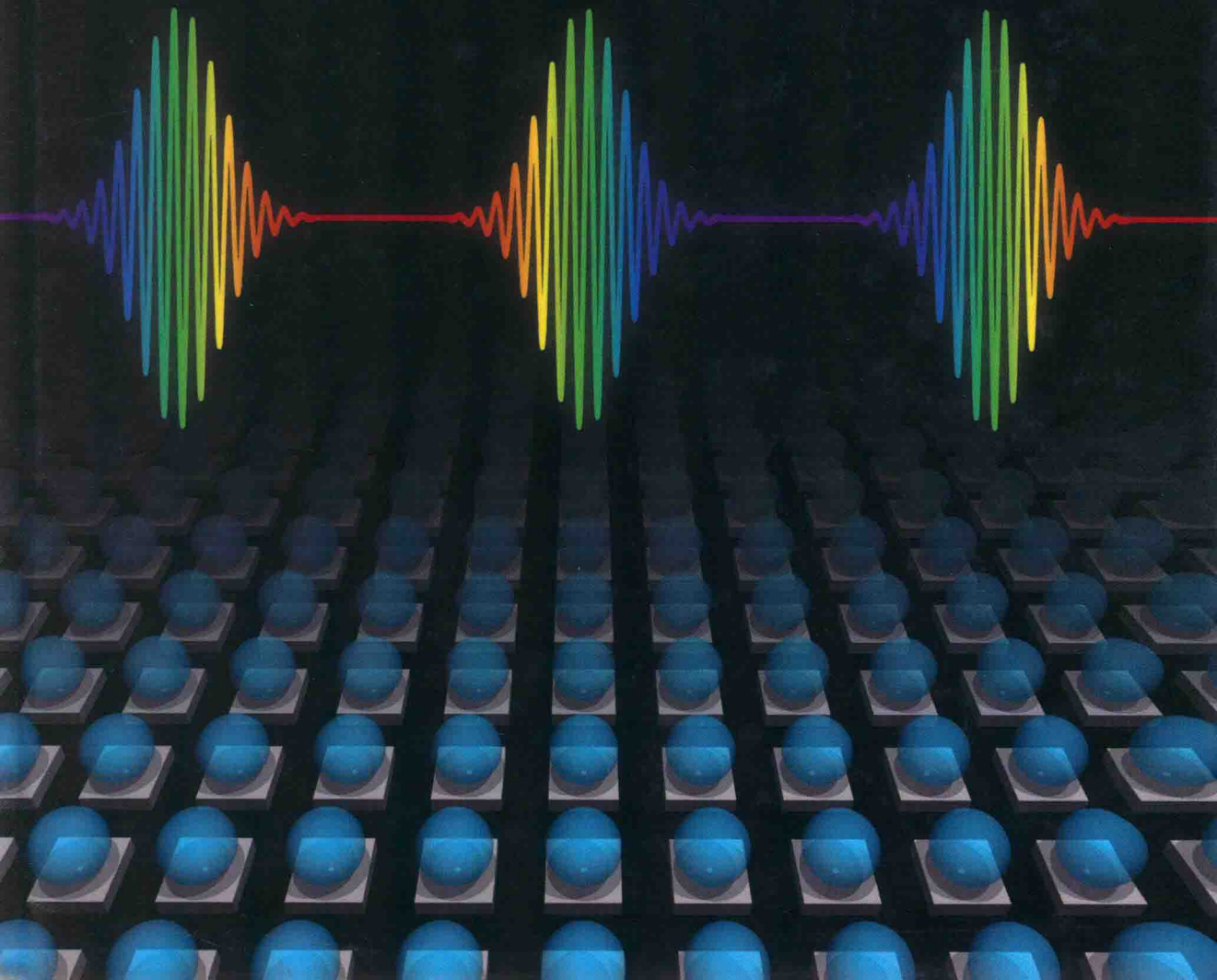


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# Atomic Physics

PRECISE MEASUREMENTS & ULTRACOLD MATTER

Massimo Inguscio | Leonardo Fallani



# Atomic Physics: Precise Measurements and Ultracold Matter

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*a Manuela*

# Atomic Physics: Precise Measurements and Ultracold Matter

# Foreword

In 1958, Arthur L. Schawlow and Charles H. Townes published a seminal paper that triggered the race to build the first laser. The advent of the laser ushered in a period of extraordinary advances in science and technology. Atomic physics moved again to the centre stage. No less than 21 Nobel Prizes have since been awarded for laser-related research, many of them recognizing achievements in precision spectroscopy (Bloembergen, Schawlow, Hall, Hänsch) and in laser cooling (Chu, Cohen-Tannoudji, Phillips, Cornell, Wieman, and Ketterle). The most recent Nobel Prize in Physics has been awarded to Haroche and Wineland “for ground-breaking experimental methods that enable measuring and manipulation of individual quantum systems”. This book by Massimo Inguscio and Leonardo Fallani illuminates the extraordinary evolution of atomic physics during the past decades, and it leads the reader to the fast-moving frontier of current research. The text conveys the fascination and excitement of the field through the eyes of pioneering researchers, so that it can provide inspiration to students and seasoned colleagues alike.

Reading this book made me realize how fortunate I have been to witness the development of this field as an active participant from the beginning. Here and there, I could even influence the evolution with contributions of my own. Essential parts of my work have been motivated by the goal of precision spectroscopy of the simple hydrogen atom, which permits unique confrontations between experiment and fundamental theory. This continuing quest has inspired inventions such as the first highly monochromatic, widely tunable dye laser, techniques for Doppler-free laser spectroscopy, and the frequency comb technique for measuring the frequency of laser light with extreme precision. Recent laser measurements of the Lamb shift in muonic hydrogen have unveiled a “proton size puzzle” that may hint at a possible dent in the armour of quantum electrodynamics. Future precision spectroscopy of antihydrogen might detect small differences between matter and antimatter.

Laser cooling was originally invented as a tool for precision spectroscopy. It has enabled the construction of microwave caesium fountain clocks, which form the basis for the definition of the second and for international time keeping. Today, laser-cooled trapped atoms or ions serve as pendulums in optical atomic clocks, which have reached a reproducibility of better than  $10^{-17}$ . Such clocks enable laboratory experiments that can search for slow changes of fundamental constants and they permit stringent new tests of special and general relativity.

Laser cooling followed by evaporative cooling has been the key to Bose–Einstein condensation of ultracold atoms. Today, experimental studies of degenerate quantum gases have developed into a rich field of research at the interface between atomic physics and condensed matter physics. Ultracold atoms in optical lattice potentials offer a particularly intriguing playground. Starting with the observation of a quantum phase

transition from a wave-like superfluid state to a strongly correlated crystal-like Mott insulator state, it has become possible to simulate many-body quantum phenomena from solid-state physics to particle physics and astrophysics.

The evolution of atomic physics along the path outlined in this book has led to a veritable Renaissance in science, engaging researchers from many different fields, including atomic physics, condensed matter physics, particle physics, astrophysics, chemistry, mathematics, and computer science. What can we expect from atomic physics in the future? We can safely predict more Nobel Prizes for surprising discoveries. However, we cannot predict the evolution of atomic physics. As in the past, the most important research results will be those that make all planning obsolete.

Munich, January 2013

Prof. T. W. Hänsch

# Preface

## A long story of atoms and light

The observation that atoms and molecules only absorb or emit electromagnetic waves at discrete frequencies is probably the most striking experimental evidence that led to the birth of quantum mechanics. Measuring the “colour” of atomic lines, which is the basic aim of spectroscopy, gives us an exceedingly powerful instrument to understand Nature. The history of atomic physics has been shaped by a constant quest for more and more precision. Indeed, precision atomic spectroscopy of simple atoms (such as hydrogen or helium) can be used to measure fundamental physical constants, to perform extremely precise measurements of feeble effects, to validate existing physical models, to search for possible discrepancies that could hint at novel exotic theories. It could also reveal whether the quantities that we use to treat as fundamental constants are really constant or whether they are instead very slowly changing in time, which could implicate major revisions in our understanding of the Universe.

The history of atomic physics is marked by fundamental discoveries as well as by revolutionary technological advances. Among these, the invention of lasers 50 years ago has disclosed unprecedented horizons of precision in atomic spectroscopy, providing sources of very monochromatic light for probing atomic or molecular structures. A second revolution started in the 1970s, when it was realized that lasers are not only the primary tools that allow us to probe atomic spectra but can also be used to control the motion of the atoms, slowing them down to amazingly low temperatures, less than a millionth of a degree above absolute zero, indeed much lower than what is possible with traditional cryogenic techniques. The realization of ultracold atomic gases opened new horizons in spectroscopy thanks to the possibility of eliminating all the limiting effects connected to the motion of the atoms. Laser cooling was indeed the key to detect tinier and tinier effects and to realize extremely precise atomic clocks which are so precise as to lose or gain less than one second over the entire age of the Universe! These accurate measurements have been possible also thanks to recent important developments in laser physics: the production of extremely monochromatic lasers with ultra-narrow linewidth and the revolutionary invention of the *frequency comb*, which has fundamentally changed the field of spectroscopy by allowing direct accurate measurements of optical frequencies.

And when matter becomes ultracold, new fascinating effects emerge. Ultracold atoms move so slow that, when viewed at a sufficiently small length scale, it becomes impossible to distinguish them one from another, owing to the Heisenberg uncertainty principle. As a consequence, quantum statistics becomes important. At a sufficiently small temperature, a gas of bosonic atoms undergoes Bose–Einstein condensation in which the atoms suffer an “identity crisis” and they collectively occupy the same quantum state. This state of matter, predicted by A. Einstein more than 80 years



ago and observed only in the 1990s with ultracold atomic dilute gases, share many similarities with laser light, since it manifests as the macroscopic occupation of the same quantum state, and it is accompanied by the same properties of macroscopic coherence as superfluids and superconductors. With similar experimental techniques, dilute gases of fermionic atoms can be cooled down to Fermi degeneracy, which is characterized by the unity occupation of all the single-particle energy levels up to the Fermi energy of the system.

Lasers can also confine atoms in very small traps, which are used to provide an ideal setting for high-precision spectroscopy. Atoms trapped in *optical lattices*, i.e. periodic arrays of laser traps, are isolated from the environment and can be trapped for long times in the absence of perturbing fields: furthermore, their motion can be controlled and even frozen to the ground state of the trap in such a way as not to cause any perturbation to their spectroscopy. Besides constituting valuable tools for the realization of accurate optical clocks, optical lattices have opened a new field of research which goes beyond spectroscopy and is founded on the close analogy between neutral atoms trapped in periodic structures made of light and electrons moving through the periodic crystalline structure of a solid. Ideal solid-state physics models can be experimentally realized and fundamental effects such as Bloch oscillations, Anderson localization, and conductor–insulator quantum phase transitions can be very finely investigated, thus making atoms powerful and precise *quantum simulators* of quantum dynamics and many-body systems.

## Why this book?

There are many excellent textbooks of atomic physics; why another one? Fifty years of international conferences in atomic physics have shown that during this time atomic physics has been characterized by a continuous redefinition of its meaning: from pre-laser spectroscopy (described in classic Oxford books such as the ones by G. W. Series and A. Corney), to the success of laser and non-linear spectroscopy in the 1970s, to more recent milestones represented by the possibility of using the laser to manipulate atomic velocities (which means colder and colder samples for more and more precise measurements) and by the possibility of performing direct frequency measurements in the optical domain. Nowadays atomic physics has moved from the investigation of atomic structures to a field in which the aforementioned possibilities of control on both the internal and the external degrees of freedom allow us to use atoms to test the validity of quantum theories, to measure fundamental constants, to build precise atomic clocks, to realize quantum simulators of condensed-matter systems. We have decided to write this book to illustrate this change of paradigm in atomic physics, from conventional laser spectroscopy to laser manipulation of the atomic motion and its implications for precise measurements: in a sense, this book takes a snapshot of what atomic physics has now become and what the new challenges are.

This is not a classical atomic physics textbook, in which the focus is on the theory of atomic structures and light–matter interaction. This book focuses on the experimental investigations, both illustrating milestone experiments and key experimental techniques, and discussing the results and perspectives of current research activity. Emphasis is given on the investigation of precision Physics: from the determination of fundamental

constants of Nature to tests of General Relativity and Quantum Electrodynamics, from the realization of ultra-stable atomic clocks to the precise simulation of condensed-matter theories with ultracold gases. The book discusses these topics while tracing the evolution of experimental atomic physics from traditional laser spectroscopy to the revolution introduced by laser cooling.

## Book structure

The first part of the book is structured following the increase of complexity in atoms. As most of the books in atomic physics, Chapter 1 starts with hydrogen, the simplest of the atoms, which is one of the main doors that allowed physicists to discover the quantum world and a primary testing ground for fundamental physical theories. The history of hydrogen spectroscopy will be used to illustrate fundamental steps in laser spectroscopy, from Doppler-free spectroscopy to direct frequency measurements with optical frequency combs. Chapter 2 is devoted to alkali atoms, which have a very similar electronic structure to hydrogen: their simple and accessible transitions were used for the first demonstration of laser cooling, that will be introduced in this chapter together with its applications to microwave atomic clocks (that currently provide the definition of the SI second) and atom interferometry. The ultimate frontiers of cooling will be discussed in Chapter 3, which is devoted to the investigation of ultracold quantum degenerate gases, with a particular focus on the properties and applications of Bose–Einstein condensates. In Chapter 4 we will move to the physics of helium, which shows qualitatively different properties from hydrogen and alkali atoms, owing to the presence of two electrons. Important applications of helium will be discussed, from high-precision spectroscopy for the determination of fundamental constants to the applications of degenerate helium gases for experiments of quantum atom optics. Chapter 5 starts with alkaline-earth atoms, which feature the same two-electron structure as helium: these atoms are of utmost importance in frontier atomic physics, since their extremely narrow transitions enable the realization of ultra-accurate optical atomic clocks. In this chapter we will also discuss laser cooling and spectroscopy of ions, which represent the ultimate frontier of accuracy (below  $10^{-17}$ !) in the realization of optical frequency standards, as well as the most precise measurements in physics.

The last two chapters will be devoted to the physics which arises when ultracold atoms are trapped into optical lattices. In Chapter 6 we will focus on the physics of quantum transport in optical lattices, which both provides a testing ground for ideal solid-state physics and allows very promising applications for the determination of fundamental constants and for the precise measurements of forces where ultracold atoms are used as sensors. Chapter 7 extends this possibility to the emerging field of *quantum simulation*, in which ultracold atoms are used to experimentally realize basic condensed-matter models to precisely investigate their properties and their quantum phase transitions in an ultimately clean setting where decoherence or unwanted interactions with the environment can be avoided.

## Acknowledgements

This book is the result of exciting years of research and teaching carried out at LENS (European Laboratory for Nonlinear Spectroscopy) and at the University of Florence. Since its foundation 22 years ago, the LENS research in atomic physics has spanned from high-precision laser spectroscopy to the new frontiers of ultracold quantum gases, following the evolution of the field which is described in this book.

We would like to thank the many researchers, visitors, and students who have walked along this path with us. In particular, we acknowledge the friendship of important LENS colleagues who have been participating in this adventure since the early days: Marco Bellini, Pablo Cancio Pastor, Francesco S. Cataliotti, Jacopo Catani, Paolo De Natale, Marco Fattori, Francesca Ferlaino, Gabriele Ferrari, Chiara Fort (to whom we are particularly grateful for critical suggestions and careful reading of the manuscript), Giovanni Giusfredi, Francesco Marin, Francesco Minardi, Giovanni Modugno, Francesco S. Pavone, Marco Prevedelli, Leonardo Ricci, Giacomo Roati, and Guglielmo M. Tino. We are particularly indebted to two distinguished scientists, Theodor W. Hänsch (who wrote the foreword to this book) and Eric A. Cornell, for their inspiring advice and continuous support in pushing the LENS research in atomic physics to frontier topics in high-precision spectroscopy and ultracold matter. Profound discussions with theorists like Franco Dalfovo, Michele Modugno, Lev P. Pitaevskii, Augusto Smerzi, and Sandro Stringari are also acknowledged.

Finally, we would like to thank our Oxford University Press publisher Sonke Adlung, without whose constant encouragement this book would have never been published.

Firenze, January 2013

Massimo Inguscio  
Leonardo Fallani

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# 1

# Hydrogen

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*Never measure anything but frequency!*  
A. Schawlow

High-precision spectroscopy of simple atoms is a wonderful tool for precision tests of fundamental physics. Among the simple atoms hydrogen has an undisputed prime role. Its minimal internal structure, only a proton and an electron bound together, provides the possibility of formulating extremely precise theoretical predictions, which can be compared with the results coming from high-precision spectroscopy. Nowadays, atomic transition frequencies can be measured with spectacular accuracy. As an example of this possibility, the most precise experimental determination of the hydrogen  $1s - 2s$  transition frequency reported up to now is

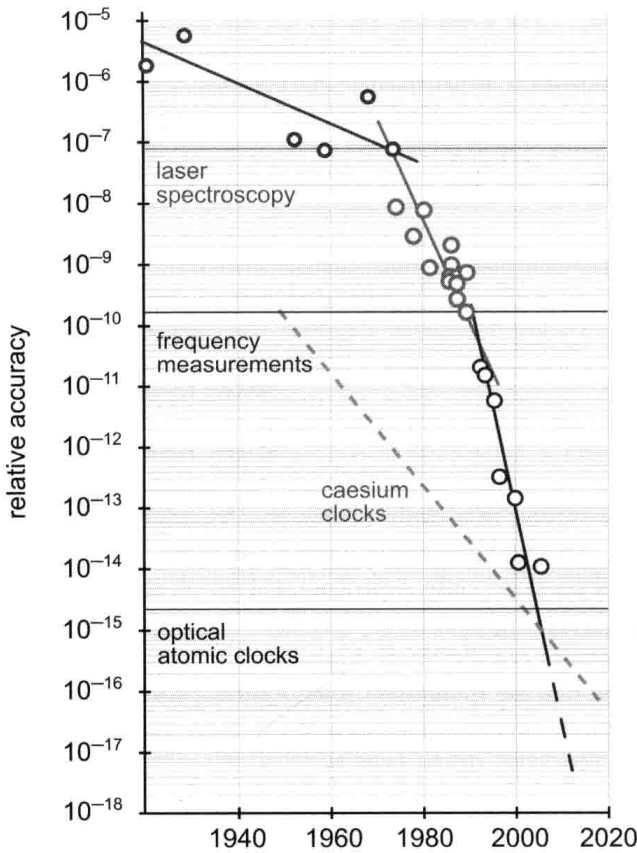
$$\delta\nu(1s - 2s) = 2\,466\,061\,413\,187\,035\,(10)\,\text{Hz}, \quad (1.1)$$

which was obtained in a recent measurement performed by the group of T. W. Hänsch (Nobel Prize for Physics in 2005) at MPQ (Garching) (Parthey *et al.*, 2011). This amazing precision of  $4.2 \times 10^{-15}$  is the result of decades of advances in atomic physics, which have been marked by outstanding scientific discoveries and technological progress, leading to a continuous increase in the measurement accuracy, as illustrated in Fig. 1.1. Among the milestones in this journey to the land of precision one cannot avoid mentioning the invention of lasers, the development of nonlinear spectroscopic techniques, and the recent possibility of performing direct frequency measurements of light with optical frequency combs. In this first chapter we will present a brief history of hydrogen spectroscopy, evidencing the most important steps and focusing on the state of the art and on future perspectives and implications of this research.

## 1.1 The hydrogen spectrum

Since the dawn of quantum mechanics, the hydrogen spectrum has represented a benchmark for testing the predictions of quantum theories (Series, 1957; Series, 1988). The problem of two quantum particles at a distance  $r$  interacting with a Coulomb  $\sim 1/r$  potential is indeed one of the few relevant physical situations for which quantum mechanics can provide results in analytic form. Its solution, which the reader can find discussed in any general quantum mechanics textbook, dates back to a famous paper by E. Schrödinger in 1926 (Schrödinger, 1926). The Schrödinger equation for the two-body





**Fig. 1.1** Historical evolution of the accuracy in the measurement of optical hydrogen transitions (not including the last 2011 MPQ measurement). Reprinted with permission from Hänsch (2006). © American Physical Society.

problem of an electron bound to a proton through the Coulomb interaction gives the well-known Bohr energies

$$E_n = -hcR_\infty \frac{1}{n^2}, \tag{1.2}$$

where  $h$  is the Planck constant,  $c$  is the speed of light,  $n$  is the atomic principal quantum number, and  $R_\infty$  is the *Rydberg constant*, which can be expressed in terms of other fundamental constants as

$$R_\infty = \frac{m_e e^4}{8\epsilon_0^2 h^3 c} \simeq 1.097 \times 10^7 \text{ m}^{-1}, \tag{1.3}$$

where  $m_e$  is the electron mass,  $e$  is the elementary charge, and  $\epsilon_0$  is the permittivity of free space.

The hydrogen spectrum is represented in Fig. 1.2, together with a sketch of some possible transitions between different energy levels. These transitions are traditionally grouped in series, which correspond to sets of lines involving the same low-energy level. Historically, the first identified series was the Balmer series (from J. Balmer, who