VOLUME II

SUPERSTRING THEORY AND BEYOND

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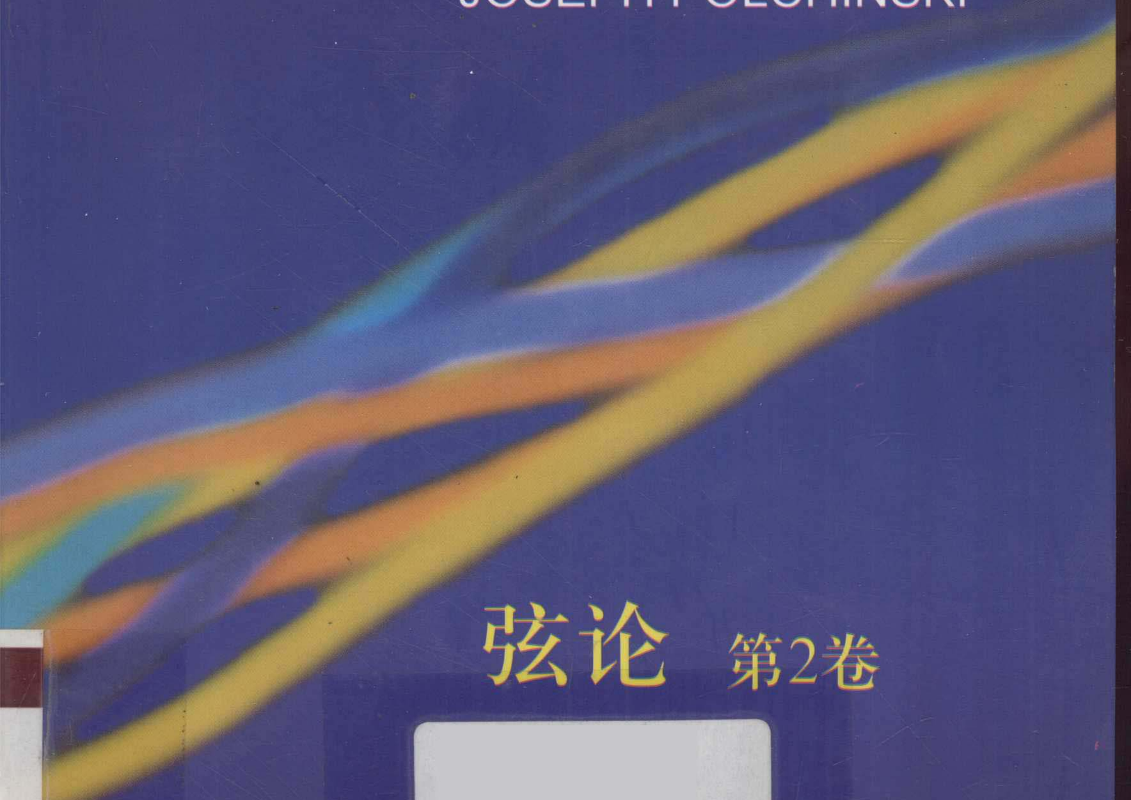
JOSEPH POLCHINSKI



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VOLUME II

Superstring Theory and Beyond

JOSEPH POLCHINSKI
Institute for Theoretical Physics
University of California at Santa Barbara

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Outline of volume one

Foreword

From the beginning it was clear that, despite its successes, the Standard Model of elementary particles would have to be embedded in a broader theory that would incorporate gravitation as well as the strong and electroweak interactions. There is at present only one plausible candidate for such a theory: it is the theory of strings, which started in the 1960s as a not-very-successful model of hadrons, and only later emerged as a possible theory of all forces.

There is no one better equipped to introduce the reader to string theory than Joseph Polchinski. This is in part because he has played a significant role in the development of this theory. To mention just one recent example: he discovered the possibility of a new sort of extended object, the 'Dirichlet brane', which has been an essential ingredient in the exciting progress of the last few years in uncovering the relation between what had been thought to be different string theories.

Of equal importance, Polchinski has a rare talent for seeing what is of physical significance in a complicated mathematical formalism, and explaining it to others. In looking over the proofs of this book, I was reminded of the many times while Polchinski was a member of the Theory Group of the University of Texas at Austin, when I had the benefit of his patient, clear explanations of points that had puzzled me in string theory. I recommend this book to any physicist who wants to master this exciting subject.

Steven Weinberg
Series Editor
Cambridge Monographs on Mathematical Physics
1998

Preface

When I first decided to write a book on string theory, more than ten years ago, my memories of my student years were much more vivid than they are today. Still, I remember that one of the greatest pleasures was finding a text that made a difficult subject accessible, and I hoped to provide the same for string theory.

Thus, my first purpose was to give a coherent introduction to string theory, based on the Polyakov path integral and conformal field theory. No previous knowledge of string theory is assumed. I do assume that the reader is familiar with the central ideas of general relativity, such as metrics and curvature, and with the ideas of quantum field theory through non-Abelian gauge symmetry. Originally a full course of quantum field theory was assumed as a prerequisite, but it became clear that many students were eager to learn string theory as soon as possible, and that others had taken courses on quantum field theory that did not emphasize the tools needed for string theory. I have therefore tried to give a self-contained introduction to those tools.

A second purpose was to show how some of the simplest four-dimensional string theories connect with previous ideas for unifying the Standard Model, and to collect general results on the physics of four-dimensional string theories as derived from world-sheet and spacetime symmetries. New developments have led to a third goal, which is to introduce the recent discoveries concerning string duality, M-theory, D-branes, and black hole entropy.

In writing a text such as this, there is a conflict between the need to be complete and the desire to get to the most interesting recent results as quickly as possible. I have tried to serve both ends. On the side of completeness, for example, the various path integrals in chapter 6 are calculated by three different methods, and the critical dimension of the bosonic string is calculated in seven different ways in the text and exercises.

On the side of efficiency, some shorter paths through these two volumes are suggested below.

A particular issue is string perturbation theory. This machinery is necessarily a central subject of volume one, but it is somewhat secondary to the recent nonperturbative developments: the free string spectrum plus the spacetime symmetries are more crucial there. Fortunately, from string perturbation theory there is a natural route to the recent discoveries, by way of T-duality and D-branes.

One possible course consists of chapters 1–3, section 4.1, chapters 5–8 (omitting sections 5.4 and 6.7), chapter 10, sections 11.1, 11.2, 11.6, 12.1, and 12.2, and chapters 13 and 14. This sequence, which I believe can be covered in two quarters, takes one from an introduction to string theory through string duality, M-theory, and the simplest black hole entropy calculations. An additional shortcut is suggested at the end of section 5.1.

Readers interested in T-duality and related stringy phenomena can proceed directly from section 4.1 to chapter 8. The introduction to Chan–Paton factors at the beginning of section 6.5 is needed to follow the discussion of the open string, and the one-loop vacuum amplitude, obtained in chapter 7, is needed to follow the calculation of the D-brane tension.

Readers interested in supersymmetric strings can read much of chapters 10 and 11 after section 4.1. Again the introduction to Chan–Paton factors is needed to follow the open string discussion, and the one-loop vacuum amplitude is needed to follow the consistency conditions in sections 10.7, 10.8, and 11.2.

Readers interested in conformal field theory might read chapter 2, sections 6.1, 6.2, 6.7, 7.1, 7.2, 8.2, 8.3 (concentrating on the CFT aspects), 8.5, 10.1–10.4, 11.4, and 11.5, and chapter 15. Readers interested in four-dimensional string theories can follow most of chapters 16–19 after chapters 8, 10, and 11.

In a subject as active as string theory — by one estimate the literature approaches 10 000 papers — there will necessarily be important subjects that are treated only briefly, and others that are not treated at all. Some of these are represented by review articles in the lists of references at the end of each volume. The most important omission is probably a more complete treatment of compactification on curved manifolds. Because the geometric methods of this subject are somewhat orthogonal to the quantum field theory methods that are emphasized here, I have included only a summary of the most important results in chapters 17 and 19. Volume two of Green, Schwarz, and Witten (1987) includes a more extensive introduction, but this is a subject that has continued to grow in importance and clearly deserves an introductory book of its own.

This work grew out of a course taught at the University of Texas at Austin in 1987–88. The original plan was to spend a year turning the lecture notes into a book, but a desire to make the presentation clearer and more complete, and the distraction of research, got in the way. An early prospectus projected the completion date as June 1989 $\pm$ one month, off by 100 standard deviations. For eight years the expected date of completion remained approximately one year in the future, while one volume grew into two. Happily, finally, one of those deadlines didn't slip.

I have also used portions of this work in a course at the University of California at Santa Barbara, and at the 1994 Les Houches, 1995 Trieste, and 1996 TASI schools. Portions have been used for courses by Nathan Seiberg and Michael Douglas (Rutgers), Steven Weinberg (Texas), Andrew Strominger and Juan Maldacena (Harvard), Nathan Berkovits (Sâo Paola) and Martin Einhorn (Michigan). I would like to thank those colleagues and their students for very useful feedback. I would also like to thank Steven Weinberg for his advice and encouragement at the beginning of this project, Shyamoli Chaudhuri for a thorough reading of the entire manuscript, and to acknowledge the support of the Departments of Physics at UT Austin and UC Santa Barbara, the Institute for Theoretical Physics at UC Santa Barbara, and the National Science Foundation.

During the extended writing of this book, dozens of colleagues have helped to clarify my understanding of the subjects covered, and dozens of students have suggested corrections and other improvements. I began to try to list the members of each group and found that it was impossible. Rather than present a lengthy but incomplete list here, I will keep an updated list at the erratum website

http://www.itp.ucsb.edu/~joep/bigbook.html.

In addition, I would like to thank collectively all who have contributed to the development of string theory; volume two in particular seems to me to be largely a collection of beautiful results derived by many physicists. String theory (and the entire base of physics upon which it has been built) is one of mankind's great achievements, and it has been my privilege to try to capture its current state.

Finally, to complete a project of this magnitude has meant many sacrifices, and these have been shared by my family. I would like to thank Dorothy, Steven, and Daniel for their understanding, patience, and support.

Joseph Polchinski
Santa Barbara, California
1998

Notation

This book uses the +++ conventions of Misner, Thorne, & Wheeler (1973). In particular, the signature of the metric is $(- + + \ldots +)$. The constants $\hbar$ and c are set to 1, but the Regge slope α' is kept explicit.

A bar $\bar{}$ is used to denote the conjugates of world-sheet coordinates and moduli (such as z, τ and q), but a star $*$ is used for longer expressions. A bar on a spacetime fermion field is the Dirac adjoint (this appears only in volume two), and a bar on a world-sheet operator is the Euclidean adjoint (defined in section 6.7). For the degrees of freedom on the string, the following terms are treated as synonymous:

$$\text{holomorphic} = \text{left-moving},$$

$$\text{antiholomorphic} = \text{right-moving},$$

as explained in section 2.1. Our convention is that the supersymmetric side of the heterotic string is right-moving. Antiholomorphic operators are designated by tildes $\tilde{}$; as explained in section 2.3, these are not the adjoints of holomorphic operators. Note also the following conventions:

$$d^2z \equiv 2dxdy \ , \quad \delta^2(z, \bar{z}) \equiv \frac{1}{2}\delta(x)\delta(y) \ ,$$

where $z = x + iy$ is any complex variable; these differ from most of the literature, where the coefficient is 1 in each definition.

Spacetime actions are written as $\boldsymbol{S}$ and world-sheet actions as S. This presents a problem for D-branes, which are T-dual to the former and S-dual to the latter; $\boldsymbol{S}$ has been used arbitrarily. The spacetime metric is $G_{\mu\nu}$, while the world-sheet metric is γ_{ab} (Minkowskian) or g_{ab} (Euclidean). In volume one, the spacetime Ricci tensor is $\boldsymbol{R}_{\mu\nu}$ and the world-sheet Ricci tensor is R_{ab}. In volume two the former appears often and the latter never, so we have changed to $R_{\mu\nu}$ for the spacetime Ricci tensor.

The following are used:

$\equiv$ defined as

$\cong$ equivalent to

$\approx$ approximately equal to

$\sim$ equal up to nonsingular terms (OPEs), or rough correspondence.

Contents

10
Type I and type II superstrings

Having spent volume one on a thorough development of the bosonic string, we now come to our real interest, the supersymmetric string theories. This requires a generalization of the earlier framework, enlarging the world-sheet constraint algebra. This idea arises naturally if we try to include spacetime fermions in the spectrum, and by guesswork we are led to *superconformal symmetry*. In this chapter we discuss the (1,1) superconformal algebra and the associated type I and II superstrings. Much of the structure is directly parallel to that of the bosonic string so we can proceed rather quickly, focusing on the new features.

10.1 The superconformal algebra

In bosonic string theory, the mass-shell condition

$$p_\mu p^\mu + m^2 = 0 \tag{10.1.1}$$

came from the physical state condition

$$L_0|\psi\rangle = 0\,, \tag{10.1.2}$$

and also from $\tilde{L}_0|\psi\rangle = 0$ in the closed string. The mass-shell condition is the Klein–Gordon equation in momentum space. To get spacetime fermions, it seems that we need the Dirac equation

$$ip_\mu\Gamma^\mu + m = 0 \tag{10.1.3}$$

instead. This is one way to motivate the following generalization, and it will lead us to all the known consistent string theories.

Let us try to follow the pattern of the bosonic string, where L_0 and $\tilde{L}_0$ are the center-of-mass modes of the world-sheet energy-momentum tensor $(T_B, \tilde{T}_B)$. A subscript B for 'bosonic' has been added to distinguish these from the fermionic currents now to be introduced. It seems then that we need new conserved quantities T_F and $\tilde{T}_F$, whose center-of-mass modes give the Dirac equation, and which play the same role as T_B and $\tilde{T}_B$ in the bosonic theory. Noting further that the spacetime momenta p^μ are the center-of-mass modes of the world-sheet current $(\partial X^\mu, \bar{\partial} X^\mu)$, it is natural to guess that the gamma matrices, with algebra

$$\{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu} , \tag{10.1.4}$$

are the center-of-mass modes of an anticommuting world-sheet field ψ^μ.

With this in mind, we consider the world-sheet action

$$S = \frac{1}{4\pi} \int d^2 z \left(\frac{2}{\alpha'} \partial X^\mu \bar{\partial} X_\mu + \psi^\mu \bar{\partial} \psi_\mu + \tilde{\psi}^\mu \partial \tilde{\psi}_\mu \right) . \tag{10.1.5}$$

For reference we recall from chapter 2 the XX operator product expansion (OPE)

$$X^\mu(z,\bar{z}) X^\nu(0,0) \sim -\frac{\alpha'}{2} \eta^{\mu\nu} \ln |z|^2 . \tag{10.1.6}$$

The ψ conformal field theory (CFT) was described in section 2.5. The fields ψ^μ and $\tilde{\psi}^\mu$ are respectively holomorphic and antiholomorphic, and the operator products are

$$\psi^\mu(z)\psi^\nu(0) \sim \frac{\eta^{\mu\nu}}{z} , \quad \tilde{\psi}^\mu(\bar{z})\tilde{\psi}^\nu(0) \sim \frac{\eta^{\mu\nu}}{\bar{z}} . \tag{10.1.7}$$

The *world-sheet supercurrents*

$$T_F(z) = i(2/\alpha')^{1/2} \psi^\mu(z) \partial X_\mu(z) , \quad \tilde{T}_F(\bar{z}) = i(2/\alpha')^{1/2} \tilde{\psi}^\mu(\bar{z}) \bar{\partial} X_\mu(\bar{z}) \tag{10.1.8}$$

are also respectively holomorphic and antiholomorphic, since they are just the products of (anti)holomorphic fields. The annoying factors of $(2/\alpha')^{1/2}$ could be eliminated by working in units where $\alpha' = 2$, and then be restored if needed by dimensional analysis. Also, throughout this volume the : : normal ordering of coincident operators will be implicit.

This gives the desired result: the modes ψ_0^μ and $\tilde{\psi}_0^\mu$ will satisfy the gamma matrix algebra, and the centers-of-mass of T_F and $\tilde{T}_F$ will have the form of Dirac operators. We will see that the resulting string theory has spacetime fermions as well as bosons, and that the tachyon is gone.

From the OPE and the Ward identity it follows (exercise 10.1) that the currents

$$j^\eta(z) = \eta(z) T_F(z) , \quad \tilde{j}^\eta(\bar{z}) = \bar{\eta}(\bar{z}) \tilde{T}_F(\bar{z}) \tag{10.1.9}$$

generate the *superconformal transformation*

$$\epsilon^{-1}(2/\alpha')^{1/2} \delta X^\mu(z,\bar{z}) = -\eta(z)\psi^\mu(z) - \eta(z)^* \tilde{\psi}^\mu(\bar{z}) , \tag{10.1.10a}$$

$$\epsilon^{-1}(\alpha'/2)^{1/2} \delta\psi^\mu(z) = \eta(z) \partial X^\mu(z) , \tag{10.1.10b}$$

$$\epsilon^{-1}(\alpha'/2)^{1/2} \delta\tilde{\psi}^\mu(\bar{z}) = \eta(z)^* \bar{\partial} X^\mu(\bar{z}) . \tag{10.1.10c}$$

This transformation mixes the commuting field X^μ with the anticommuting fields ψ^μ and $\tilde{\psi}^\mu$, so the parameter $\eta(z)$ must be anticommuting. As with conformal symmetry, the parameters are arbitrary holomorphic or antiholomorphic functions. That this is a symmetry of the action (10.1.5) follows at once because the current is (anti)holomorphic, and so conserved.

The commutator of two superconformal transformations is a conformal transformation,

$$\delta_{\eta_1}\delta_{\eta_2} - \delta_{\eta_2}\delta_{\eta_1} = \delta_v \,, \quad v(z) = -2\eta_1(z)\eta_2(z) \,, \tag{10.1.11}$$

as the reader can check by acting on the various fields. Similarly, the commutator of a conformal and superconformal transformation is a superconformal transformation. The conformal and superconformal transformations thus close to form the *superconformal algebra.* In terms of the currents, this means that the OPEs of T_F with itself and with

$$T_B = -\frac{1}{\alpha'}\partial X^\mu \partial X_\mu - \frac{1}{2}\psi^\mu \partial \psi_\mu \tag{10.1.12}$$

close. That is, only T_B and T_F appear in the singular terms:

$$T_B(z)T_B(0) \sim \frac{3D}{4z^4} + \frac{2}{z^2}T_B(0) + \frac{1}{z}\partial T_B(0) \,, \tag{10.1.13a}$$

$$T_B(z)T_F(0) \sim \frac{3}{2z^2}T_F(0) + \frac{1}{z}\partial T_F(0) \,, \tag{10.1.13b}$$

$$T_F(z)T_F(0) \sim \frac{D}{z^3} + \frac{2}{z}T_B(0) \,, \tag{10.1.13c}$$

and similarly for the antiholomorphic currents. The $T_B T_F$ OPE implies that T_F is a tensor of weight $(\frac{3}{2}, 0)$. Each scalar contributes 1 to the central charge and each fermion $\frac{1}{2}$, for a total

$$c = (1 + \tfrac{1}{2})D = \tfrac{3}{2}D \,. \tag{10.1.14}$$

This enlarged algebra with T_F and $\tilde{T}_F$ as well as T_B and $\tilde{T}_B$ will play the same role that the conformal algebra did in the bosonic string. That is, we will impose it on the states as a constraint algebra — it must annihilate physical states, either in the sense of old covariant quantization (OCQ) or of Becchi–Rouet–Stora–Tyutin (BRST) quantization. Because of the Minkowski signature of spacetime the timelike ψ^0 and $\tilde{\psi}^0$, like X^0, have opposite sign commutators and lead to negative norm states. The fermionic constraints T_F and $\tilde{T}_F$ will remove these states from the spectrum.

More generally, the $N = 1$ superconformal algebra in operator product form is

$$T_B(z)T_B(0) \sim \frac{c}{2z^4} + \frac{2}{z^2}T_B(0) + \frac{1}{z}\partial T_B(0)\,, \qquad (10.1.15a)$$

$$T_B(z)T_F(0) \sim \frac{3}{2z^2}T_F(0) + \frac{1}{z}\partial T_F(0)\,, \qquad (10.1.15b)$$

$$T_F(z)T_F(0) \sim \frac{2c}{3z^3} + \frac{2}{z}T_B(0) \qquad (10.1.15c)$$

The Jacobi identity requires the same constant c in the $T_B T_B$ and $T_F T_F$ products (exercise 10.5). Here, $N = 1$ refers to the number of $(\frac{3}{2}, 0)$ currents. In the present case there is also an antiholomorphic copy of the same algebra, so we have an $(N, \tilde{N}) = (1, 1)$ *superconformal field theory (SCFT)*. We will consider more general algebras in section 11.1.

Free SCFTs

The various free CFTs described in chapter 2 have superconformal generalizations. One free SCFT combines an anticommuting bc theory with a commuting $\beta\gamma$ system, with weights

$$h_b = \lambda\,, \quad h_c = 1 - \lambda\,, \qquad (10.1.16a)$$

$$h_\beta = \lambda - \tfrac{1}{2}\,, \quad h_\gamma = \tfrac{3}{2} - \lambda\,. \qquad (10.1.16b)$$

The action is

$$S_{BC} = \frac{1}{2\pi}\int d^2z\,(b\bar{\partial}c + \beta\bar{\partial}\gamma)\,, \qquad (10.1.17)$$

and

$$T_B = (\partial b)c - \lambda\partial(bc) + (\partial\beta)\gamma - \frac{1}{2}(2\lambda - 1)\partial(\beta\gamma)\,, \qquad (10.1.18a)$$

$$T_F = -\frac{1}{2}(\partial\beta)c + \frac{2\lambda - 1}{2}\partial(\beta c) - 2b\gamma\,. \qquad (10.1.18b)$$

The central charge is

$$[-3(2\lambda - 1)^2 + 1] + [3(2\lambda - 2)^2 - 1] = 9 - 12\lambda\,. \qquad (10.1.19)$$

Of course there is a corresponding antiholomorphic theory.

We can anticipate that the superconformal ghosts will be of this form with $\lambda = 2$, the anticommuting $(2, 0)$ ghost b being associated with the commuting $(2, 0)$ constraint T_B as in the bosonic theory, and the commuting $(\frac{3}{2}, 0)$ ghost β being associated with the anticommuting $(\frac{3}{2}, 0)$ constraint T_F. The ghost central charge is then $-26 + 11 = -15$, and the condition that the total central charge vanish gives the critical dimension

$$0 = \frac{3}{2}D - 15 \Rightarrow D = 10\,. \qquad (10.1.20)$$