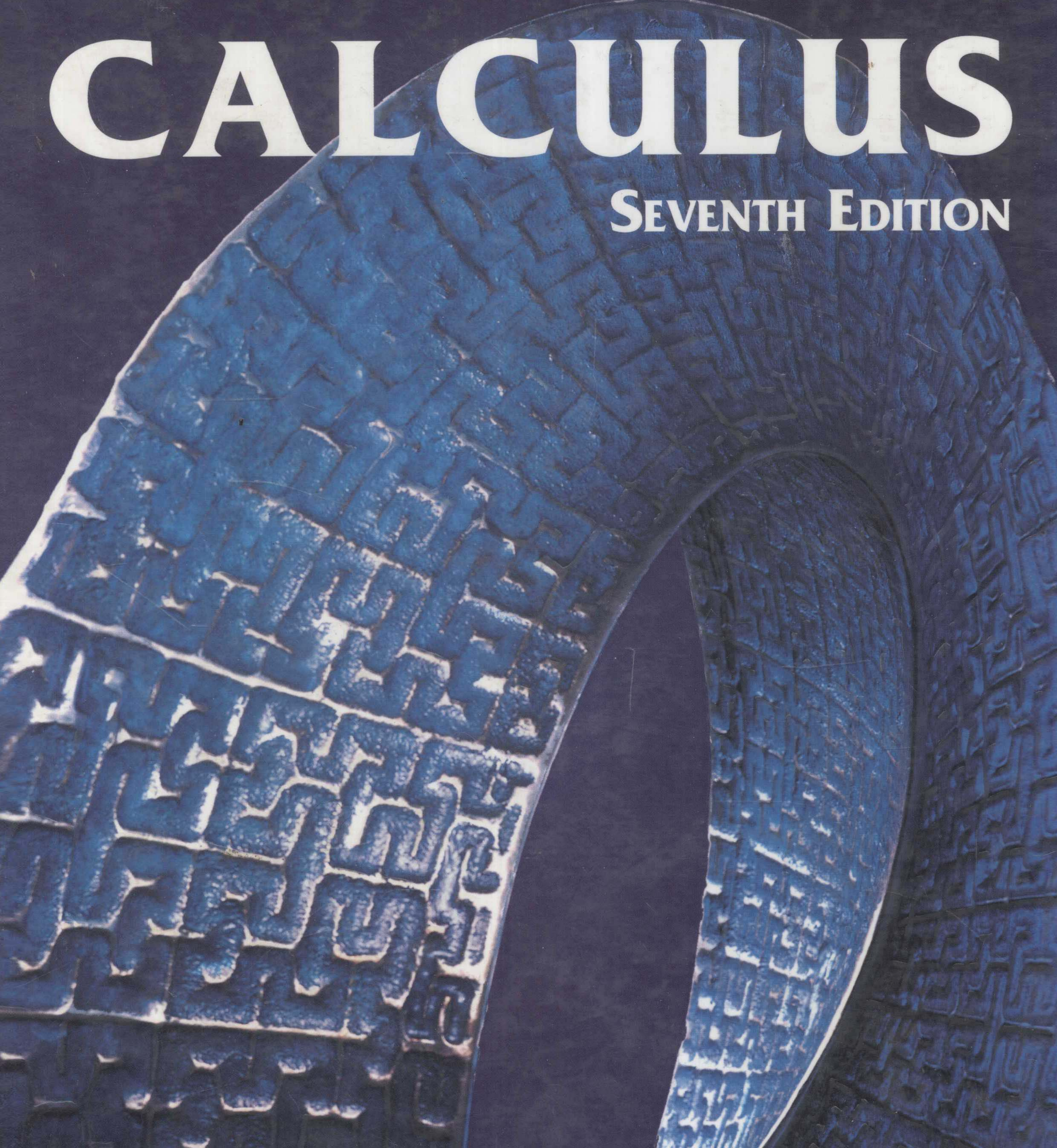
LARSON HOSTETLER EDWARDS

CALCULUS

SEVENTH EDITION



Calculus
with Analytic Geometry

Seventh Edition

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The Behrend College

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We have included examples and exercises that use real-life data as well as technology output from a variety of software. This would not have been possible without the help of many people and organizations. Our wholehearted thanks goes to all for their time and effort.

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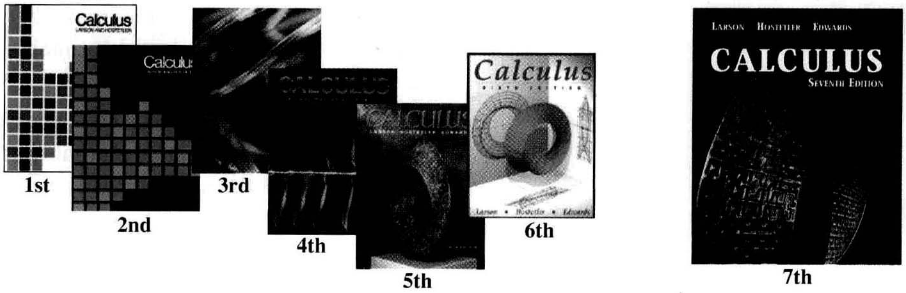
*Available in *e-solutions Calculus Learning Tools Student CD-ROM* and at the text-specific website at *college.hmco.com.*

A Word from the Authors

Welcome to *Calculus with Analytic Geometry*, Seventh Edition. Much has changed since we wrote the first edition—nearly 25 years ago. With each edition, we have listened to you, our users, and have tried to incorporate your suggestions for improvement.



A Text Formed by Its Users

Through your support and suggestions, the text has evolved over seven editions to include these extensive enhancements:

- Expanded exercise sets containing a greater variety of tasks such as skill building, applications, explorations, writing, critical thinking, and theoretical problems
- Additional applications that more accurately represent the diverse uses of calculus in the world
- Many more open-ended activities and investigations
- Clearer, less cluttered text, full annotations and labels—carefully planned page layout
- Additional art, composed with more color, accuracy, and realism
- A more comprehensive and more mathematically rigorous text, particularly the third semester of the Seventh Edition, which is quite different when compared with the First Edition
- Increased technology use, as both a problem-solving tool and an investigative tool
- References to the history of calculus and to the mathematicians who developed it
- Updated references to current mathematical journals
- Considerably more help in the supplements package for both students and instructors
- Alternatives to the traditional print medium, particularly in the CD-ROM version
- Five different volumes from which to choose your preferred teaching approach—a great development in flexibility from the single volume in the First Edition (see page xx)

What's New and Different in the Seventh Edition

In the Seventh Edition, we continue to offer instructors and students a text that is pedagogically sound, mathematically precise, and comprehensible. There are many minor changes in the mathematics, prose, art, and design. The more significant changes are noted here.

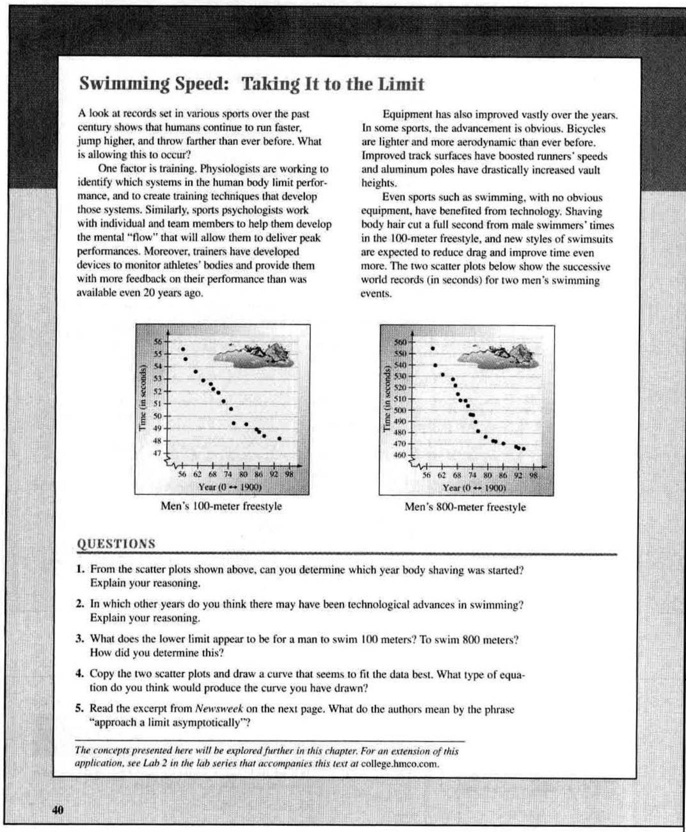
- *New* **P.S. Problem Solving** At the end of each chapter, we have included a two-page collection of new applied and theoretical exercises. These exercises offer problems that have some unusual characteristics that set them apart from exercises in a regular exercise set.
- *New* **Getting at the Concept** Midway through each section exercise set we have added a set of problems that check a student's understanding of the basic concepts presented in the section.
- *New* **Section Objectives** Each section in the Seventh Edition begins with a list of learning objectives. These enable students to identify and focus on the key points of the section.
- *New* **Downloadable Graphs** Many exercise sets contain problems in which students are asked to draw on the graph that is provided. Because this is not feasible in the actual text, we now provide printable enlargements of these graphs on the website *www.mathgraphs.com.*
- *New* **Journal Articles on the Web** The Seventh Edition contains over 60 references to articles from mathematics journals noted in the feature *For Further Information.* In order to make the articles easily accessible to instructors and students, they are now available on the website *www.matharticles.com.*
- *Revised* **Chapter Openers** The chapter openers have been redesigned as two-page spreads in the Seventh Edition. Included in the chapter openers is a real-world application designed to motivate the calculus topics of the chapter.
- *Revised* **Review Exercises** In order to provide a more effective study tool, we have grouped the Review Exercises by text section. This reorganization allows students to target specific concepts that may require additional study and review.
- **Exercise Sets** Approximately 20 percent of the exercises in the Seventh Edition are new. The new exercises include skill, concept, applied, and theoretical problems.
- **Table of Contents** Although the organization of the table of contents is much the same as in the Sixth Edition, some notable changes are as follows. In an effort to cut back on the length of the text, we have moved Section 3.10 *Business and Economic Applications* (Appendix G in the Seventh Edition), Chapter 15 *Differential Equations*, Appendix A *Precalculus Review* (Appendix D in the Seventh Edition), Appendix E *Rotation and the General Second-Degree Equation*, and Appendix F *Complex Numbers* to the text-specific website at *college.hmco.com.* We removed Appendix C *Basic Differentiation Rules for Elementary Functions* from the text; however, that material appears on the inside front cover of the text. Although Chapter 15 has been moved from the text, some of the differential equations topics have been retained and other topics have been expanded in a new Appendix A (*Additional Topics in Differential Equations*). Coverage includes slope fields, Euler's Method, and first-order linear differential equations.

Although we carefully and thoroughly revised the text by enhancing the usefulness of some features and topics and by adding others, we did not change many of the things that our colleagues and the two million students who have used this book have told us work for them. We still offer comprehensive coverage of the material required by students in a three-semester or four-quarter calculus course, including carefully stated theories and proofs.

We hope you will enjoy the Seventh Edition. We are proud to have it as our first calculus book to be published in the twenty-first century.

Ron Larson Robert P. Hostetler Bruce H. Edwards

Features



Swimming Speed: Taking It to the Limit

A look at records set in various sports over the past century shows that humans continue to run faster, jump higher, and throw farther than ever before. What is allowing this to occur?

One factor is training. Physiologists are working to identify which systems in the human body limit performance, and to create training techniques that develop those systems. Similarly, sports psychologists work with individual and team members to help them develop the mental "flow" that will allow them to deliver peak performances. Moreover, trainers have developed devices to monitor athletes' bodies and provide them with more feedback on their performance than was available even 20 years ago.

Equipment has also improved vastly over the years. In some sports, the advancement is obvious. Bicycles are lighter and more aerodynamic than ever before. Improved track surfaces have boosted runners' speeds and aluminum poles have drastically increased vault heights.

Even sports such as swimming, with no obvious equipment, have benefited from technology. Shaving body hair cut a full second from male swimmers' times in the 100-meter freestyle, and new styles of swimsuits are expected to reduce drag and improve time even more. The two scatter plots below show the successive world records (in seconds) for two men's swimming events.

Men's 100-meter freestyle

Men's 800-meter freestyle

QUESTIONS

1. From the scatter plots shown above, can you determine which year body shaving was started? Explain your reasoning.
2. In which other years do you think there may have been technological advances in swimming? Explain your reasoning.
3. What does the lower limit appear to be for a man to swim 100 meters? To swim 800 meters? How did you determine this?
4. Copy the two scatter plots and draw a curve that seems to fit the data best. What type of equation do you think would produce the curve you have drawn?
5. Read the excerpt from *Newsweek* on the next page. What do the authors mean by the phrase "approach a limit asymptotically"?

The concepts presented here will be explored further in this chapter. For an extension of this application, see Lab 2 in the lab series that accompanies this text at college.hmco.com.

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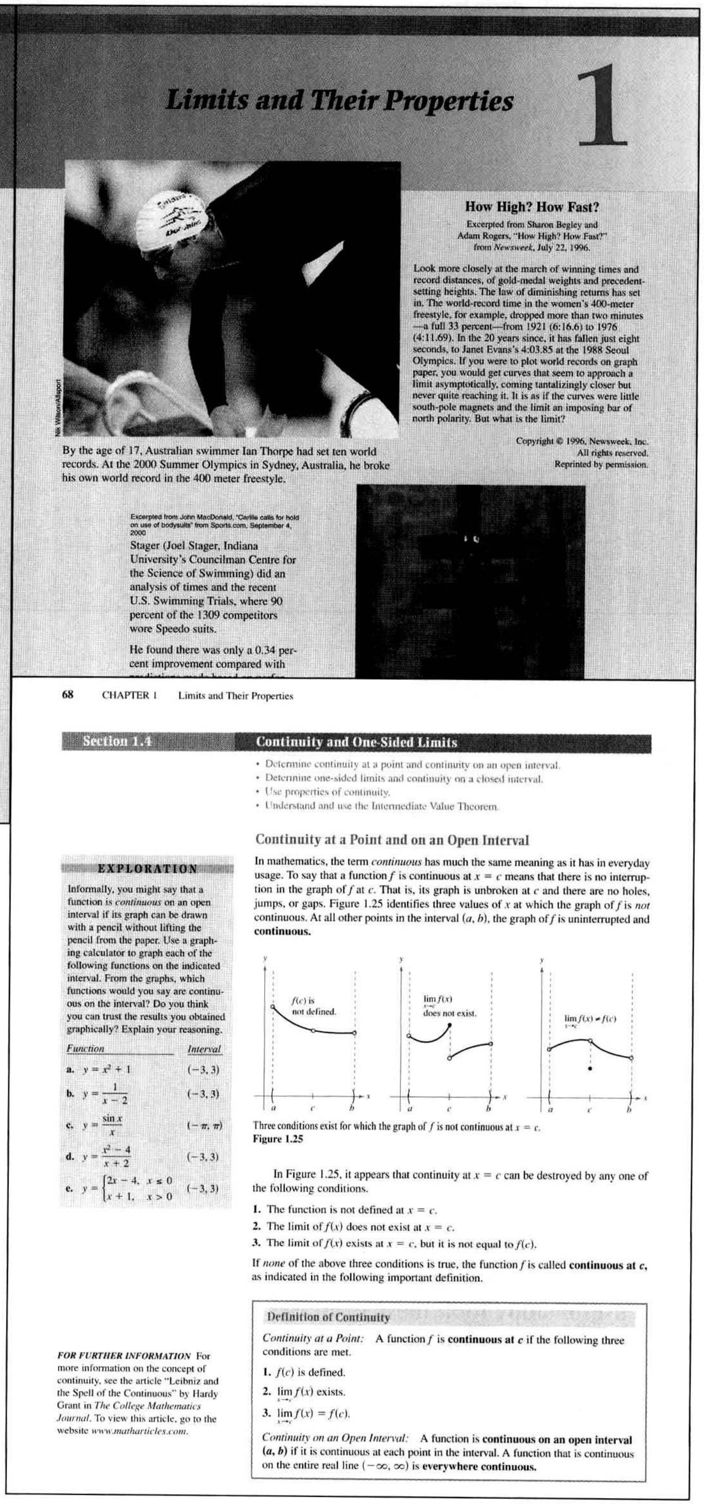

Limits and Their Properties 1

How High? How Fast?

Excerpted from Sharon Begley and Adam Rogers, "How High? How Fast?" from *Newsweek*, July 22, 1996.

Look more closely at the march of winning times and record distances, of gold-medal weights and precedent-setting heights. The law of diminishing returns has set in. The world-record time in the women's 400-meter freestyle, for example, dropped more than two minutes —a full 33 percent—from 1921 (6:16.6) to 1976 (4:11.69). In the 20 years since, it has fallen just eight seconds, to Janet Evans's 4:03.85 at the 1988 Seoul Olympics. If you were to plot world records on graph paper, you would get curves that seem to approach a limit asymptotically, coming tantalizingly closer but never quite reaching it. It is as if the curves were little south-pole magnets and the limit an imposing bar of north polarity. But what is the limit?

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Nik Wilson/Allsport

By the age of 17, Australian swimmer Ian Thorpe had set ten world records. At the 2000 Summer Olympics in Sydney, Australia, he broke his own world record in the 400 meter freestyle.

Excerpted from John MacDonald, "Carlile calls for hold on use of bodysuits" from Sports.com, September 4, 2000

Stager (Joel Stager, Indiana University's Councilman Centre for the Science of Swimming) did an analysis of times and the recent U.S. Swimming Trials, where 90 percent of the 1309 competitors wore Speedo suits.

He found there was only a 0.34 percent improvement compared with

68 CHAPTER 1 Limits and Their Properties

Section 1.4 Continuity and One-Sided Limits

- Determine continuity at a point and continuity on an open interval.
- Determine one-sided limits and continuity on a closed interval.
- Use properties of continuity.
- Understand and use the Intermediate Value Theorem.

Continuity at a Point and on an Open Interval

EXPLORATION

Informally, you might say that a function is *continuous* on an open interval if its graph can be drawn with a pencil without lifting the pencil from the paper. Use a graphing calculator to graph each of the following functions on the indicated interval. From the graphs, which functions would you say are continuous on the interval? Do you think you can trust the results you obtained graphically? Explain your reasoning.

Function	*Interval*
a. $y = x^2 + 1$	$(-3, 3)$
b. $y = \frac{1}{x-2}$	$(-3, 3)$
c. $y = \frac{\sin x}{x}$	$(-\pi, \pi)$
d. $y = \frac{x^2-4}{x+2}$	$(-3, 3)$
e. $y = \begin{cases} 2x-4, & x \le 0 \\ x+1, & x > 0 \end{cases}$	$(-3, 3)$

In mathematics, the term *continuous* has much the same meaning as it has in everyday usage. To say that a function f is continuous at $x = c$ means that there is no interruption in the graph of f at c. That is, its graph is unbroken at c and there are no holes, jumps, or gaps. Figure 1.25 identifies three values of x at which the graph of f is *not* continuous. At all other points in the interval (a, b), the graph of f is uninterrupted and **continuous.**

$f(c)$ is not defined. $\lim_{x\to c} f(x)$ does not exist. $\lim_{x\to c} f(x) \ne f(c)$

Three conditions exist for which the graph of f is not continuous at $x = c$.
Figure 1.25

In Figure 1.25, it appears that continuity at $x = c$ can be destroyed by any one of the following conditions.

1. The function is not defined at $x = c$.
2. The limit of $f(x)$ does not exist at $x = c$.
3. The limit of $f(x)$ exists at $x = c$, but it is not equal to $f(c)$.

If *none* of the above three conditions is true, the function f is called **continuous at c,** as indicated in the following important definition.

FOR FURTHER INFORMATION For more information on the concept of continuity, see the article "Leibniz and the Spell of the Continuous" by Hardy Grant in *The College Mathematics Journal*. To view this article, go to the website *www.matharticles.com*.

Definition of Continuity

Continuity at a Point: A function f is **continuous at c** if the following three conditions are met.

1. $f(c)$ is defined.
2. $\lim_{x\to c} f(x)$ exists.
3. $\lim_{x\to c} f(x) = f(c)$.

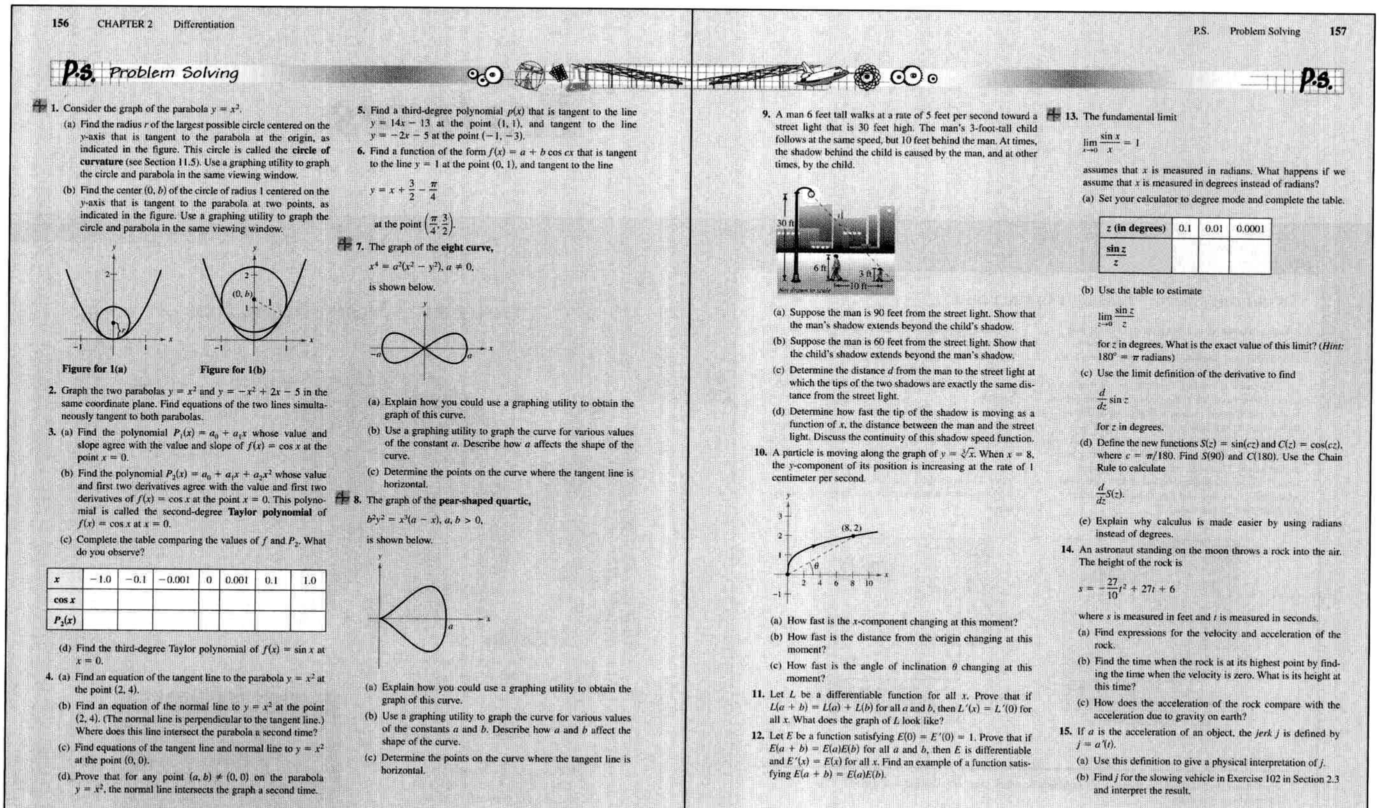
Continuity on an Open Interval: A function is **continuous on an open interval (a, b)** if it is continuous at each point in the interval. A function that is continuous on the entire real line $(-\infty, \infty)$ is **everywhere continuous.**

Chapter Openers

Each chapter opens with a real-world application designed to motivate the calculus concepts covered in the chapter. Following a brief introduction, open-ended questions guide students through an introduction to the main themes of the chapter. In addition, photographs and interesting facts related to the application are included in the chapter opener.

Section Objectives

Every section begins with a list of learning objectives that outline the key concepts of the section. This list helps instructors with class planning and provides students a study guide for the section.



156 CHAPTER 2 Differentiation

P.S. Problem Solving

1. Consider the graph of the parabola $y = x^2$.
(a) Find the radius r of the largest possible circle centered on the y-axis that is tangent to the parabola at the origin, as indicated in the figure. This circle is called the **circle of curvature** (see Section 11.5). Use a graphing utility to graph the circle and parabola in the same viewing window.
(b) Find the center $(0, b)$ of the circle of radius 1 centered on the y-axis that is tangent to the parabola at two points, as indicated in the figure. Use a graphing utility to graph the circle and parabola in the same viewing window.

Figure for 1(a) **Figure for 1(b)**

2. Graph the two parabolas $y = x^2$ and $y = -x^2 + 2x - 5$ in the same coordinate plane. Find equations of the two lines simultaneously tangent to both parabolas.
3. (a) Find the polynomial $P_1(x) = a_0 + a_1x$ whose value and slope agree with the value and slope of $f(x) = \cos x$ at the point $x = 0$.
(b) Find the polynomial $P_2(x) = a_0 + a_1x + a_2x^2$ whose value and first two derivatives agree with the value and first two derivatives of $f(x) = \cos x$ at the point $x = 0$. This polynomial is called the second-degree **Taylor polynomial** of $f(x) = \cos x$ at $x = 0$.
(c) Complete the table comparing the values of f and P_2. What do you observe?

x	-1.0	-0.1	-0.001	0	0.001	0.1	1.0
$\cos x$							
$P_2(x)$							

(d) Find the third-degree Taylor polynomial of $f(x) = \sin x$ at $x = 0$.
4. (a) Find an equation of the tangent line to the parabola $y = x^2$ at the point $(2, 4)$.
(b) Find an equation of the normal line to $y = x^2$ at the point $(2, 4)$. (The normal line is perpendicular to the tangent line.) Where does this line intersect the parabola a second time?
(c) Find equations of the tangent line and normal line to $y = x^2$ at the point $(0, 0)$.
(d) Prove that for any point $(a, b) \neq (0, 0)$ on the parabola $y = x^2$, the normal line intersects the graph a second time.
5. Find a third-degree polynomial $p(x)$ that is tangent to the line $y = 14x - 13$ at the point $(1, 1)$, and tangent to the line $y = -2x - 5$ at the point $(-1, -3)$.
6. Find a function of the form $f(x) = a + b \cos cx$ that is tangent to the line $y = 1$ at the point $(0, 1)$, and tangent to the line
$y = x + \frac{3}{2} - \frac{\pi}{4}$
at the point $\left(\frac{\pi}{4}, \frac{3}{2}\right)$.
7. The graph of the **eight curve,**
$x^4 = a^2(x^2 - y^2), a \neq 0,$
is shown below.
(a) Explain how you could use a graphing utility to obtain the graph of this curve.
(b) Use a graphing utility to graph the curve for various values of the constant a. Describe how a affects the shape of the curve.
(c) Determine the points on the curve where the tangent line is horizontal.
8. The graph of the **pear-shaped quartic,**
$b^2y^2 = x^3(a - x), a, b > 0,$
is shown below.
(a) Explain how you could use a graphing utility to obtain the graph of this curve.
(b) Use a graphing utility to graph the curve for various values of the constants a and b. Describe how a and b affect the shape of the curve.
(c) Determine the points on the curve where the tangent line is horizontal.

P.S. Problem Solving 157

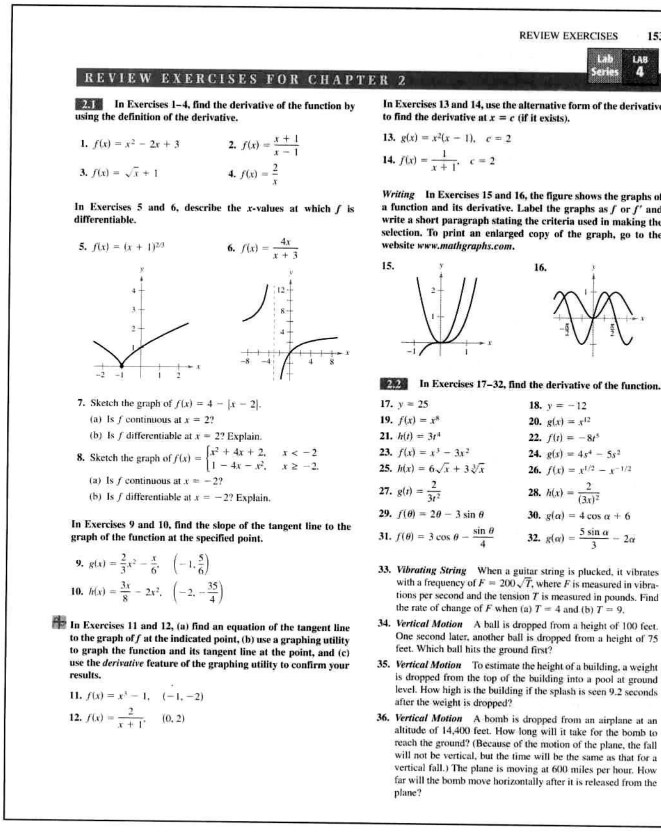
9. A man 6 feet tall walks at a rate of 5 feet per second toward a street light that is 30 feet high. The man's 3-foot-tall child follows at the same speed, but 10 feet behind the man. At times, the shadow behind the child is caused by the man, and at other times, by the child.
(a) Suppose the man is 90 feet from the street light. Show that the man's shadow extends beyond the child's shadow.
(b) Suppose the man is 60 feet from the street light. Show that the child's shadow extends beyond the man's shadow.
(c) Determine the distance d from the man to the street light at which the tips of the two shadows are exactly the same distance from the street light.
(d) Determine how fast the tip of the shadow is moving as a function of x, the distance between the man and the street light. Discuss the continuity of this shadow speed function.
10. A particle is moving along the graph of $y = \sqrt[3]{x}$. When $x = 8$, the y-component of its position is increasing at the rate of 1 centimeter per second.
(a) How fast is the x-component changing at this moment?
(b) How fast is the distance from the origin changing at this moment?
(c) How fast is the angle of inclination θ changing at this moment?
11. Let L be a differentiable function for all x. Prove that if $L(a + b) = L(a) + L(b)$ for all a and b, then $L'(x) = L'(0)$ for all x. What does the graph of L look like?
12. Let E be a function satisfying $E(0) = E'(0) = 1$. Prove that if $E(a + b) = E(a)E(b)$ for all a and b, then E is differentiable and $E'(x) = E(x)$ for all x. Find an example of a function satisfying $E(a + b) = E(a)E(b)$.
13. The fundamental limit
$\lim_{x \to 0} \frac{\sin x}{x} = 1$
assumes that x is measured in radians. What happens if we assume that x is measured in degrees instead of radians?
(a) Set your calculator to degree mode and complete the table.

z (in degrees)	0.1	0.01	0.0001
$\frac{\sin z}{z}$			

(b) Use the table to estimate
$\lim_{z \to 0} \frac{\sin z}{z}$
for z in degrees. What is the exact value of this limit? (*Hint:* $180° = \pi$ radians)
(c) Use the limit definition of the derivative to find
$\frac{d}{dz} \sin z$
for z in degrees.
(d) Define the new functions $S(z) = \sin(cz)$ and $C(z) = \cos(cz)$, where $c = \pi/180$. Find $S(90)$ and $C(180)$. Use the Chain Rule to calculate
$\frac{d}{dz} S(z)$.
(e) Explain why calculus is made easier by using radians instead of degrees.
14. An astronaut standing on the moon throws a rock into the air. The height of the rock is
$s = -\frac{27}{10}t^2 + 27t + 6$
where s is measured in feet and t is measured in seconds.
(a) Find expressions for the velocity and acceleration of the rock.
(b) Find the time when the rock is at its highest point by finding the time when the velocity is zero. What is its height at this time?
(c) How does the acceleration of the rock compare with the acceleration due to gravity on earth?
15. If a is the acceleration of an object, the *jerk* j is defined by $j = a'(t)$.
(a) Use this definition to give a physical interpretation of j.
(b) Find j for the slowing vehicle in Exercise 102 in Section 2.3 and interpret the result.

New! P.S. Problem Solving

Each chapter concludes with a collection of thought-provoking and challenging exercises that further explore and expand upon the concepts of the chapter. These exercises have unusual characteristics that set them apart from traditional calculus exercises.



REVIEW EXERCISES 153

Lab Series LAB 4

REVIEW EXERCISES FOR CHAPTER 2

2.1 In Exercises 1–4, find the derivative of the function by using the definition of the derivative.

1. $f(x) = x^2 - 2x + 3$
2. $f(x) = \frac{x + 1}{x - 1}$
3. $f(x) = \sqrt{x} + 1$
4. $f(x) = \frac{2}{x}$

In Exercises 5 and 6, describe the x-values at which f is differentiable.

5. $f(x) = (x + 1)^{2/3}$
6. $f(x) = \frac{4x}{x + 3}$

7. Sketch the graph of $f(x) = 4 - |x - 2|$.
(a) Is f continuous at $x = 2$?
(b) Is f differentiable at $x = 2$? Explain.
8. Sketch the graph of $f(x) = \begin{cases} x^2 + 4x + 2, & x < -2 \\ 1 - 4x - x^2, & x \geq -2. \end{cases}$
(a) Is f continuous at $x = -2$?
(b) Is f differentiable at $x = -2$? Explain.

In Exercises 9 and 10, find the slope of the tangent line to the graph of the function at the specified point.

9. $g(x) = \frac{2}{3}x^2 - \frac{x}{6}, \quad \left(-1, \frac{5}{6}\right)$
10. $h(x) = \frac{3x}{8} - 2x^2, \quad \left(-2, -\frac{35}{4}\right)$

In Exercises 11 and 12, (a) find an equation of the tangent line to the graph of f at the indicated point, (b) use a graphing utility to graph the function and its tangent line at the point, and (c) use the *derivative* feature of the graphing utility to confirm your results.

11. $f(x) = x^3 - 1, \quad (-1, -2)$
12. $f(x) = \frac{2}{x + 1}, \quad (0, 2)$

In Exercises 13 and 14, use the alternative form of the derivative to find the derivative at $x = c$ (if it exists).

13. $g(x) = x^2(x - 1), \quad c = 2$
14. $f(x) = \frac{1}{x + 1}, \quad c = 2$

Writing **In Exercises 15 and 16, the figure shows the graphs of a function and its derivative. Label the graphs as f or f' and write a short paragraph stating the criteria used in making the selection. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.**

15. 16.

2.2 In Exercises 17–32, find the derivative of the function.

17. $y = 25$
18. $y = -12$
19. $f(x) = x^8$
20. $g(x) = x^{12}$
21. $h(t) = 3t^4$
22. $f(t) = -8t^5$
23. $f(x) = x^3 - 3x^2$
24. $g(s) = 4s^4 - 5s^2$
25. $h(x) = 6\sqrt{x} + 3\sqrt[3]{x}$
26. $f(x) = x^{1/2} - x^{-1/2}$
27. $g(t) = \frac{2}{3t^2}$
28. $h(x) = \frac{2}{(3x)^2}$
29. $f(\theta) = 2\theta - 3 \sin \theta$
30. $g(\alpha) = 4 \cos \alpha + 6$
31. $f(\theta) = 3 \cos \theta - \frac{\sin \theta}{4}$
32. $g(\alpha) = \frac{5 \sin \alpha}{3} - 2\alpha$

33. ***Vibrating String*** When a guitar string is plucked, it vibrates with a frequency of $F = 200\sqrt{T}$, where F is measured in vibrations per second and the tension T is measured in pounds. Find the rate of change of F when (a) $T = 4$ and (b) $T = 9$.
34. ***Vertical Motion*** A ball is dropped from a height of 100 feet. One second later, another ball is dropped from a height of 75 feet. Which ball hits the ground first?
35. ***Vertical Motion*** To estimate the height of a building, a weight is dropped from the top of the building into a pool at ground level. How high is the building if the splash is seen 9.2 seconds after the weight is dropped?
36. ***Vertical Motion*** A bomb is dropped from an airplane at an altitude of 14,400 feet. How long will it take for the bomb to reach the ground? (Because of the motion of the plane, the fall will not be vertical, but the time will be the same as that for a vertical fall.) The plane is moving at 600 miles per hour. How far will the bomb move horizontally after it is released from the plane?

Review Exercises

A set of *Review Exercises* is included at the end of each chapter. In order to provide students with a more useful study tool, these exercises are grouped by section. This organization allows students to identify specific problem types related to chapter concepts for study and review.

Getting at the Concept

45. Let f be continuous on $[a, b]$ and differentiable on (a, b). If there exists c in (a, b) such that $f'(c) = 0$, does it follow that $f(a) = f(b)$? Explain.

46. Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). Also, suppose that $f(a) = f(b)$ and that c is a real number in the interval such that $f'(c) = 0$. Find an interval for the function g over which Rolle's Theorem can be applied, and find the corresponding critical number of g (k is a constant).

(a) $g(x) = f(x) + k$ (b) $g(x) = f(x - k)$

(c) $g(x) = f(kx)$

47. A plane begins its takeoff at 2:00 P.M. on a 2500-mile flight. The plane arrives at its destination at 7:30 P.M. Explain why there were at least two times during the flight when the speed of the plane was 400 miles per hour.

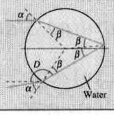
48. When an object is removed from a furnace and placed in an environment with a constant temperature of 90°F, its core temperature is 1500°F. Five hours later the core temperature is 390°F. Explain why there must exist a time in the interval when the temperature is decreasing at a rate of 222°F per hour.

SECTION PROJECT RAINBOWS

Rainbows are formed when light strikes raindrops and is reflected and refracted, as shown in the figure. (This figure shows a cross section of a spherical raindrop.) The Law of Refraction states that $(\sin \alpha)/(\sin \beta) = k$, where $k \approx 1.33$ (for water). The angle of deflection is given by $D = \pi + 2\alpha - 4\beta$.

(a) Sketch the graph of D for $0 \le \alpha \le \pi/2$. Use a graphing utility with

$$D = \pi + 2\alpha - 4 \sin^{-1}\left(\frac{1}{k} \sin \alpha\right).$$



(b) Prove that the minimum angle of deflection occurs when

$$\cos \alpha = \sqrt{\frac{k^2 - 1}{3}}.$$

For water, what is the minimum angle of deflection, D_{min}? (The angle $\pi - D_{min}$ is called the *rainbow angle*.) What value of α produces this minimum angle? (A ray of sunlight that strikes a raindrop at this angle, α, is called a *rainbow ray*.)

FOR FURTHER INFORMATION For more information about the mathematics of rainbows, see the article "Somewhere Within the Rainbow" by Steven Janke in *The UMAP Journal*. To view this article, go to the website *www.matharticles.com*.

Getting at the Concept

These exercises contain questions that check a student's understanding of the basic concepts of the section. They are generally located midway through the section exercise sets and are boxed and titled for easy reference.

Section Projects

Appearing at the end of selected exercise sets, the *Section Projects* contain extended applications, which can be assigned as an individual or group activity.

Open Explorations

The *Interactive* CD-ROM version of this text contains open explorations, which further investigate selected examples throughout the text using computer algebra systems (*Maple*, *Mathematica*, *Derive*, and *Mathcad*). The icon identifies an example for which an open exploration exists.

Additional Features

Additional teaching and learning resources can be found throughout the text. These resources include explorations, technology notes, historical vignettes, study tips, journal references, lab series, and notes. For a complete description of these resources, go to the text-specific website at *college.hmco.com*.

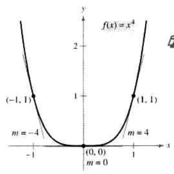
SECTION 2.2 Basic Differentiation Rules and Rates of Change 107

Example 2 Using the Power Rule

Function	*Derivative*
a. $f(x) = x^3$	$f'(x) = 3x^2$
b. $g(x) = \sqrt[3]{x}$	$g'(x) = \frac{d}{dx}[x^{1/3}] = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$
c. $y = \frac{1}{x^2}$	$\frac{dy}{dx} = \frac{d}{dx}[x^{-2}] = (-2)x^{-3} = -\frac{2}{x^3}$

In Example 2c, note that *before* differentiating, $1/x^2$ was rewritten as x^{-2}. Rewriting is the first step in *many* differentiation problems.

Given:		Rewrite:		Differentiate:		Simplify:
$y = \frac{1}{x^2}$	⇨	$y = x^{-2}$	⇨	$\frac{dy}{dx} = (-2)x^{-3}$	⇨	$\frac{dy}{dx} = -\frac{2}{x^3}$



The slope of a graph at a point is the value of the derivative at that point.
Figure 2.16

Example 3 Finding the Slope of a Graph

Find the slope of the graph of $f(x) = x^4$ when

a. $x = -1$ **b.** $x = 0$ **c.** $x = 1$.

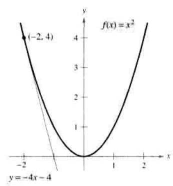
Solution The derivative of f is $f'(x) = 4x^3$.

a. When $x = -1$, the slope is $f'(-1) = 4(-1)^3 = -4$. Slope is negative.

b. When $x = 0$, the slope is $f'(0) = 4(0)^3 = 0$. Slope is zero.

c. When $x = 1$, the slope is $f'(1) = 4(1)^3 = 4$. Slope is positive.

In Figure 2.16, note that the slope of the graph is negative at the point $(-1, 1)$, the slope is zero at the point $(0, 0)$, and the slope is positive at the point $(1, 1)$.



The line $y = -4x - 4$ is tangent to the graph of $f(x) = x^2$ at the point $(-2, 4)$.
Figure 2.17

Example 4 Finding an Equation of a Tangent Line

Find an equation of the tangent line to the graph of $f(x) = x^2$ when $x = -2$.

Solution To find the *point* on the graph of f, evaluate the original function at $x = -2$.

$(-2, f(-2)) = (-2, 4)$ Point on graph

To find the *slope* of the graph when $x = -2$, evaluate the derivative, $f'(x) = 2x$, at $x = -2$.

$m = f'(-2) = -4$ Slope of graph at $(-2, 4)$

Now, using the point-slope form of the equation of a line, you can write

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 4 = -4[x - (-2)]$ Substitute for y_1, m, and x_1.

$y = -4x - 4$. Simplify.

(See Figure 2.17.)

Acknowledgments

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If you have suggestions for improving this text, please feel free to write to us. Over the past 25 years we have received many useful comments from both instructors and students, and we value these very much.

Ron Larson

Robert P. Hostetler

Bruce H. Edwards

Supplements

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Website (*college.hmco.com*)

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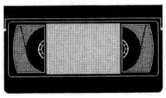

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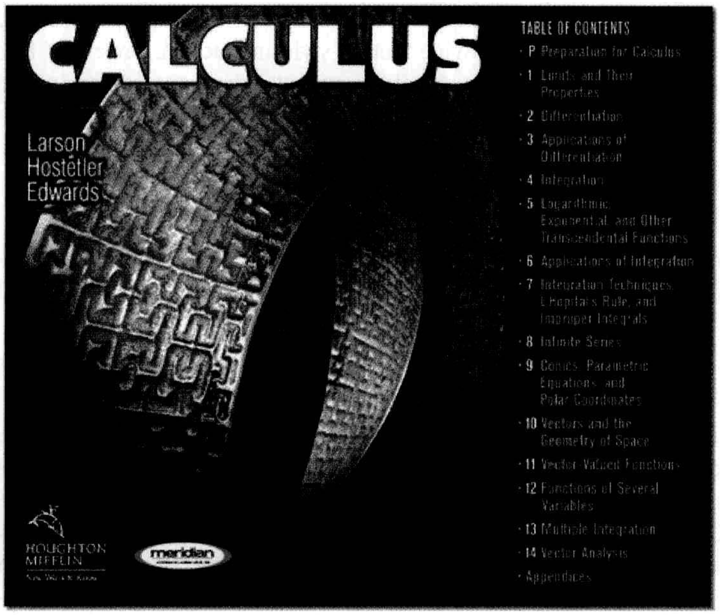

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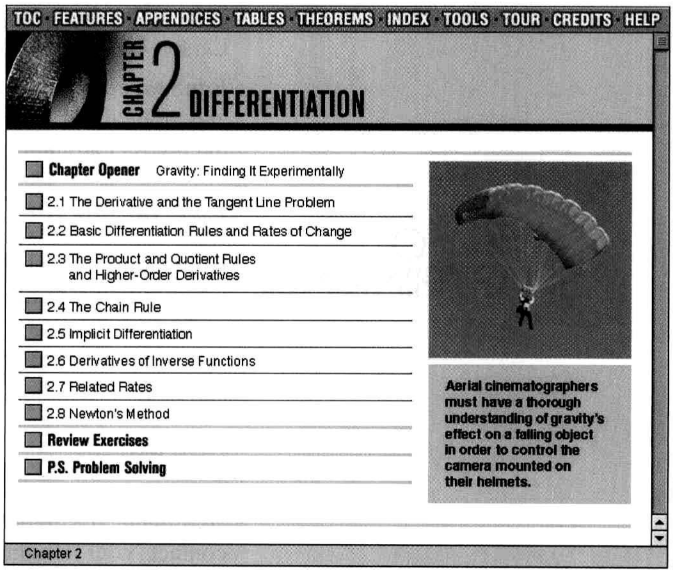

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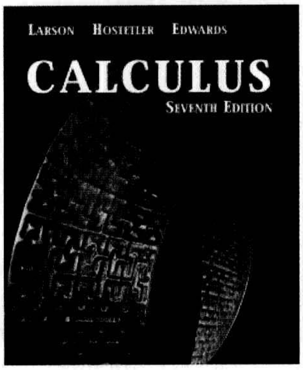

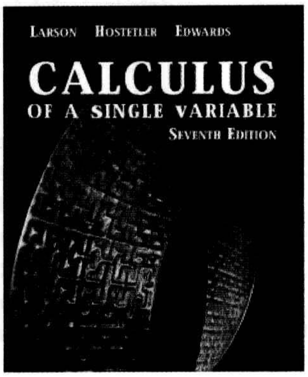

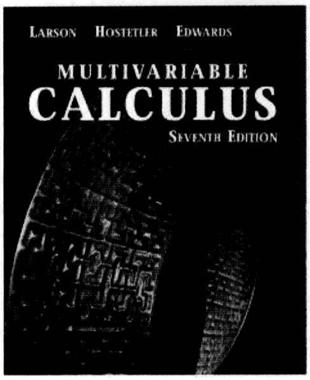

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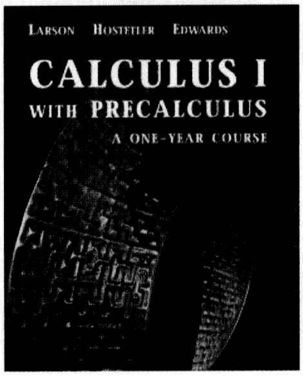

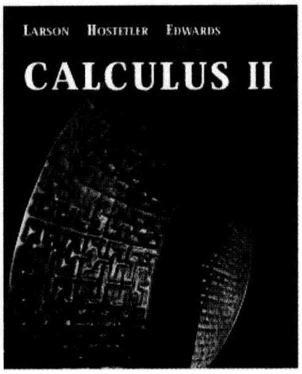

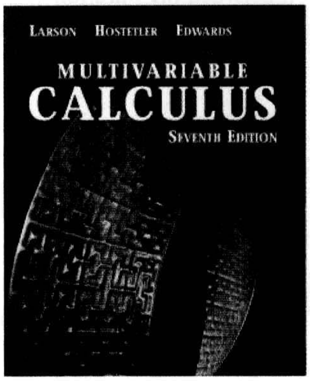

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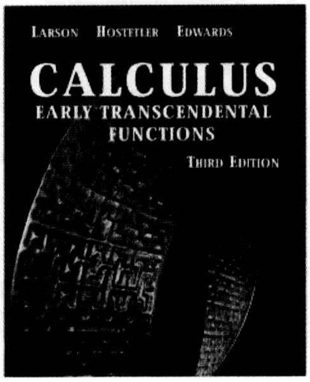

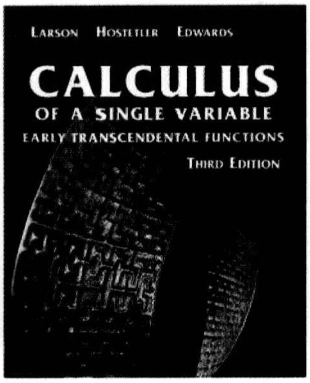

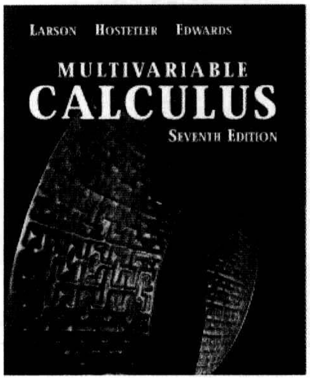

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