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APPLIED GEOMETRIC PROGRAMMING

Applied Geometric Programming

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Applied Geometric Programming

Board of Advisors, Engineering

To

Lloyd Carlton Stearman
1898 - 1975

without whose biplane this book

would have been finished much sooner

Preface

In this text, we have sought to collect and organize most of the important writings dealing with the application of geometric programming to real-world problems. These include research papers which have appeared in journals of mathematics, optimization, and engineering, plus reports of successful applications of the technique which have been published in books, company reports, and technical journals. The primary purpose of this text is to introduce the entire modern theory of geometric programming at an applied level for the first time. The principal mathematics required are elementary linear algebra and a first course in advanced calculus.

Representing as it does one of the most powerful tools yet developed in optimization theory, geometric programming naturally holds great interest for the practical professions of operations research, management science, economics, and mathematics. The greatest impact of geometric programming is being felt in the area of engineering design, and many examples of successful applications in this area are described in the text. It is not surprising that this new technique holds such great interest for engineers, since the principal motivation of the engineering profession has been to find better designs and design methods. Although optimization methods have enjoyed wide application in planning and scheduling operations for existing manufacturing systems, their use in the design of new systems is only just beginning. The principal reason for this lag is that the optimization methods most widely used are not well suited for design problems, which typically contain highly nonlinear constraints. All this is beginning to change now that geometric programming, with its rapid and elegant solution techniques, has been made available to the designer. One of the basic strengths of this new method lies in its power to transform a mathematical programming problem having nonlinear inequality constraints into an equivalent mathematical program constrained only by linear equalities. In practice, this means that more realistic models without questionable linearizations can be used with no increase in computational effort. Another important attribute of geometric programming is the uncovering of invariant physical, geometric, and technological considerations, which often do not depend on such transient economic conditions as prices. The presence of invariance properties is one well known to engineering estimators--designs may change, but cost ratios often remain relatively constant. Of particular interest to the design engineer will be Chapters 3, 5, 8 and 11; production engineers and management will want to include Chapter 4 as well.

Of striking importance in geometric programming is the fundamental approach to analyzing problems. One looks for the distribution of the unknown optimum cost among the various components of a system <u>before</u> optimizing the cost itself. More remarkable, this distribution is sometimes unaffected by changes in the cost coefficients of the individual terms. This separation of technological effects, as reflected by the exponents on the design variables, from the economic effects, as measured by the coefficients, is one of the attractive features of this approach. In addition, the minimum cost can be found before solving for the optimal values of the design variables which will produce that minimum cost. This makes possible a check on the economic feasibility of a project before going ahead with detailed policy or design considerations. Finally, the method provides an important insight into the effect of changes in the values of the cost coefficients on this optimal distribution (as noted above, for some problems, these changes have little or no effect).

Numerous other advantages to the use of geometric programming for practical problem-solving will become apparent to the reader, especially when he reaches the final chapter on selected applications. We hope that we will succeed in alerting practitioners from many disciplines to new possibilities for developing better designs and more efficient operations of existing systems. Written in what we believe to be straightforward and unpretentious style and notation, this book is intended to promote the widespread use and understanding of this most remarkable and powerful optimization method.

We are indebted to many people for their part in making this book possible. In particular, we wish to express our appreciation to R.E.D. Woolsey for writing Chapter 4, to Glenn Staats for material from his doctoral dissertation used in Chapter 6, to G. V. Reklaitis for writing Chapter 10, to Ron S. Dembo for his major contribution to Chapter 11, and most especially to Douglass J. Wilde. Dr. Wilde not only read the manuscript in detail and provided many corrections, but also was responsible for the original development of constrained derivatives, a fundamental concept in mathematical programming. Furthermore, the derivations of the geometric programming algorithms given in

Chapters 3 and 5 are basically unchanged from those originally set down by Doug Wilde in the book "Foundations of Optimization" written with the senior author in 1967.

We offer our sincere gratitude to M. Avriel, M. J. Rijckaert, P. G. Petropoulis, Franklin E. Grange, Ury Passy, Gary A. Kochenberger, P. A. Beck and J. G. Ecker for allowing us to extract material from their publications in this field. Of course, we are forever in the debt of Richard Duffin, Elmor Peterson, and Clarence Zener for their original development of geometric programming, including the general duality theory and the fundamental transformations for expressing optimization problems in the proper posynomial form.

We also thank the many students at The University of Texas and at Purdue who used rough drafts of several chapters in courses taught by the authors. Finally, we are grateful for the excellent typing and proof-reading assistance of Mary Lou Hibbs, Suzi Patterson and Julie Underhill.

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1 Preliminaries

1.1 INTRODUCTION

Geometric programming is a relatively new technique, developed for solving algebraic nonlinear programming problems subject to linear or nonlinear constraints. Geometric programming algorithms have recently been improved so that they now provide a powerful tool for solving nonlinear programming problems in general, and engineering design problems in particular. Geometric programming first emerged in 1961 when Clarence Zener, then Director of Science at Westinghouse Corporation, discovered that many engineering design problems consisting of a sum of component costs could sometimes be minimized almost by inspection under suitable conditions [1]. Although Dr. Zener discovered this remarkable fact through the process of engineering observation and inquiry, he soon realized that such an observation should have roots in a deeper mathematical theory. Coincidentally, Richard Duffin, then Professor of Mathematics at Carnegie-Mellon University, was engaged in the development of a duality theory with direct applications to nonlinear programming problems. Professor Duffin learned of Dr. Zener's work while visiting Westinghouse Corporation and soon solidified Dr. Zener's discovery mathematically through application of his own recently