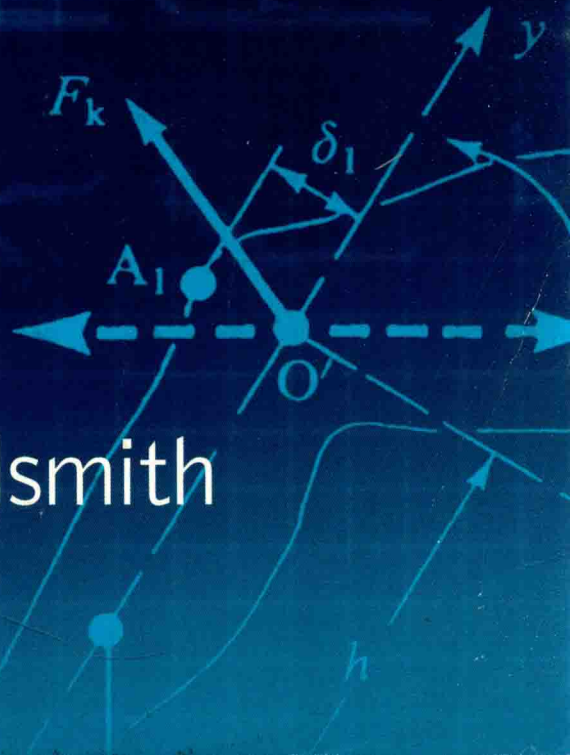


OXFORD

# Introduction to Bioengineering



Edited by  
S.A. Berger, W. Goldsmith  
and E.R. Lewis

# Introduction to Bioengineering

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# Introduction to Bioengineering

Dr. A. J. Smith

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# PREFACE

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There are many definitions of bioengineering. Other labels, such as medical engineering, biomechanics, bioelectronics, etc., each clearly refer only to part of the subject. In the use of the term bioengineering in this book we exclude genetic engineering; that is, the systematic design of phenotypes by manipulation of genotypes. Bioengineering here is taken to be the application of the concepts and methods of the physical sciences and mathematics in an engineering approach to problems in the life sciences. Most often, but not exclusively, bioengineering is concerned with human problems. Thus, while traditional engineering is the application of these sciences and mathematics to the design and analysis of inanimate, manufactured objects and structures, bioengineering may be viewed as the application of these disciplines to the study of living structures and organisms. The intent of such studies is to understand the physical processes and engineering aspects of performance under both normal and abnormal conditions, and to design, develop and use diagnostic or artificial devices meant to measure, improve, safeguard or replace life functions.

The material in this book originated in a course first introduced at the University of California at Berkeley over twenty years ago. From the beginning, it was recognized that it would be impossible to cover the entire field of bioengineering in a single course or book. Even when the text was scaled down to what was considered to be of broadest interest, it was felt that coverage of all the material was beyond the expertise of any one individual and several instructors were enlisted to do the teaching. This had the advantage of exposing the students to a wide range of bioengineering topics. However, this teaching arrangement may not be followed by other institutions. In fact, a principal reason for writing this book was to provide sufficient material to make it possible for a single instructor to teach such a course successfully.

Students in this course are expected to have had basic university-level training in physics, chemistry and mathematics. In writing this book, we had the same expectation of other users. Prior training in basic biology (including organ-systems physiology and anatomy) will enhance the value of this book to the reader. No separate chapters on anatomy and physiology are presented. Where appropriate, sections of the book include relevant physiological and anatomical background material.

The text contains contributions from instructors in mechanical, electrical, chemical, and nuclear engineering as well as from orthopedics and human biodynamics. The topics mirror the fundamental engineering science courses taught in the several engineering areas, usually at the intermediate university level, but as applied to problems in the biological world. Nevertheless, the basic principles of engineering science are presented, albeit in abbreviated form, so that the students specializing in fields not closely related to any of the subjects will be able to grasp the essence of that particular topic and be able to integrate it with one or more additional subjects necessary for the comprehension of a particular bioengineering problem. The order of presentation consists of basic engineering science subjects, followed by discussion of applications. Linear transforms, a topic applicable to a number of chapters, have been included as an appendix.

The basic mechanics of solids is presented first as the cornerstone upon whose principles many subsequent topics are built. This is followed by chapters on fluid mechanics, mass transfer, and heat transfer, which are components of mechanical and chemical engineering programs. The focus then switches, with emphasis on modeling, to systems and circuits—using theories derived largely from electrical engineering science but which apply many of the principles of mechanics. Next come two chapters on biomaterials, which lean heavily on the mechanics section. The text then covers a group of topics whose content might appear to be more directly related to the common perception of bioengineering, but which could not be reasonably presented without the underpinning of the earlier, more basic science subjects. These include human locomotion, electrophoretic separations, medical imaging, and radiation applied to diagnostics and treatment. These topics are included to give the student a flavor of some of the more important and currently very active areas of research. Brief treatments of bioinstrumentation, a broad and wide-ranging subject in its own right, are integrated in some of the chapters where appropriate.

Since the course attracts students from outside engineering, particularly from the life sciences, the chapters are presented at the simplest possible level consistent with rigor. It has been our experience that the student who has not taken the first-level engineering mechanics and electrical circuit courses usually must exert extra effort entailing outside reading; a substantial bibliography for this purpose and for providing exposure to more advanced subjects within each area is included.

For a single-semester course, there will have to be much culling of material, but we see this as an asset, giving the instructor leeway in the material he or she chooses to cover or emphasize. Those bioengineering programs that offer multiple specialized courses may find that some of the chapters are extensive enough to serve alone, or perhaps with some supplementation, as texts for those courses. That may be the case for the chapters on biomechanics, fluid mechanics, mass transfer, systems, and networks. Alternatively, the book may be used as a self-study text and reference for readers who have had formal training in the physical sciences or engineering and who wish to learn something about or review the application of these subjects to the life sciences.

Experience has shown that successful mastery of the material demands the solution of a significant number of homework problems, requiring a substantial effort on the part of both student and instructor (or assistant). This often takes the form of a one-on-one discussion or alternative presentations involving the basic approaches for this problem-solving procedure, particularly for those students that have not been previously exposed to the segment of engineering science that applies to the topic currently being covered.

Some of the chapters include sample problems to assist the student in developing the necessary skills. Instructors may wish to supplement the problems presented in the text with their own, or else choose questions adapted from the very wide literature that exists for this subject. Audiovisual aids or brief experimental demonstrations, particularly if they involve student participation, have also assisted in stimulating the interest of the audience. Each chapter also includes reference lists of books and papers relevant to the text material and for supplementary reading. These can also serve as a basis for further study in each area.

The authors would like to acknowledge and express their appreciation to Drs J. M. Kabo, Jack Winters, R. Peterson, D. Lee, Kathy Cortopassi, Steve Moore and, in particular, Kurt Eto for their invaluable contributions to this volume during the course of their participation as graduate instructors. The course has been substantially enriched by their suggestions, comments, and scrutiny of both the text and the problems.

*Berkeley, California  
October, 1995*

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# BIOMECHANICS OF SOLIDS\*

Werner Goldsmith



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## Symbols

|                 |                                |                                      |   |              |   |
|-----------------|--------------------------------|--------------------------------------|---|--------------|---|
| $a, \mathbf{a}$ | acceleration                   | $f$                                  | friction coefficient                        | $I'_k$       | area moment of inertia                          |
| $A$             | area                           | $f_{ij}$                             | internal force                              | $J'$         | polar area moment of inertia                    |
| $A'$            | projected area                 | $\mathbf{F}$                         | force                                       | $k$          | spring constant                                 |
| $b$             | damping parameter, $c/2m$      | $g$                                  | acceleration of gravity                     | $K$          | universal constant of gravitation; bulk modulus |
| $c$             | damping coefficient; clearance | $G$                                  | shear modulus                               | $L$          | length  |
| $C_D$           | drag coefficient               | $h$                                  | height                                      | $L^*$        | current length                                  |
| $C_{ijkl}$      | elastic constants              | $\mathbf{H}$                         | angular momentum                            | $\mathbf{L}$ | linear momentum                                 |
| $d$             | distance                       | $H(\cdot)$                           | Heaviside step function                     | $m$          | mass  |
| $D$             | drag                           | $\mathbf{i}, \mathbf{j}, \mathbf{k}$ | unit vectors in directions $x, y$ , and $z$ | $M$          | magnification factor                            |
| $e$             | coefficient of restitution     | $I_{ii}$                             | moment of inertia about axis $ii$           | $M_0$        | moment about 0                                  |
| $\hat{e}$       | unit vector                    | $I_{ij}$                             | product of inertia about axes $i$ and $j$   | $N$          | normal force                                    |
| $e_i$           | principal strain               |                                      |   | $q$          | damped frequency                                |
| $E$             | energy; Young's modulus        |                                      |   | $r, R$       | radius; radius distance                         |

\* The author is indebted to Dr Kurt Eto for his invaluable assistance in the production of the text and figures from computer diskettes.

|                 |  |                         |   |             |  |
|-----------------|--|-------------------------|---|-------------|--|
| $r$             | displacement   |                         | axes  | $\mu$       | absolute viscosity                                       |
| Re              | real part  | $\alpha$                | angular acceleration  | $\nu$       | Poisson's ratio; frequency ratio                         |
| $s$             | distance   | $\alpha, \beta$         | shear angles  | $\xi$       | damping parameter; eigenvalue                            |
| $s_{ij}$        | stress deviator  | $\alpha, \beta, \gamma$ | direction cosines   | $\rho$      | density  |
| $S$             | mean stress  | $\gamma_{ij}$           | total shear angle   | $\rho$      | displacement   |
| $t$             | time; thickness  | $\delta$                | extension; logarithmic  | $\sigma_i$  | principal stress   |
| $T$             | kinetic energy; torque   |                         | decrement; delta function   | $\tau$      | stress; period   |
| $TR$            | transmissibility   | $\delta_{ij}$           | Kronecker delta ( $\delta_{ij} = 1$ ,<br>$i = j$ ; $\delta_{ij} = 0$ , $i \neq j$ ) | $\tau_{ij}$ | stress on plane perpendicular to $i$<br>in direction $j$ |
| $v, \mathbf{v}$ | velocity   | $\Delta$                | difference  | $\phi$      | spherical coordinate; phase angle;<br>angle of twist     |
| $V$             | potential energy; volume;<br>shear force                       | $\epsilon_{ij}$         | strain on plane perpendicular<br>to $i$ in direction $j$                            | $\phi_{ij}$ | relaxation function                                      |
| $w$             | load per unit length   | $\theta$                | angle between vectors;<br>angular displacement                                      | $\psi_{ij}$ | creep function   |
| $W$             | weight; work   |                         |   | $\omega$    | angular velocity   |
| $x, y, z$       | rectangular coordinates;<br>positions along coordinate<br>axes | $\lambda$               | Lamé constant   | $\omega$    | natural frequency; angular speed                         |
| $X, Y, Z$       | body forces along the $x, y, z$                                | $\lambda_i$             | principal moment of inertia   | $\Omega$    | angular velocity   |
|                 |  | $\Lambda$               | extension ratio   | $\Omega$    | forcing frequency  |

## Subscripts

|   |                    |                |                                   |                   |                                    |
|---|--------------------|----------------|-----------------------------------|-------------------|------------------------------------|
| a | axial              | i              | inside                            | $R, \phi, \theta$ | along spherical coordinate<br>axes |
| c | approach; critical | max            | maximum                           | rot               | rotational                         |
| d | dynamic            | n              | normal                            | t                 | tangential, transverse             |
| e | earth              | o              | outside                           | tr                | translational                      |
| f | frictional; final  | p              | particular                        | $x, y, z$         | along the $x, y, z$ directions     |
| G | mass center        | r              | radial; along $r$ ; restitution   | 0                 | initial                            |
| h | homogeneous        | $r, \theta, z$ | along cylindrical coordinate axes |                   |                                    |

## Prologue

Biomechanics describes that discipline which applies the science of mechanics to biological systems, i.e. those encompassing both the animal and the vegetable kingdom. Mechanics, in turn, is the field that describes the response of bodies—discrete or continuous, rigid or deformable, and solid, liquid, or gaseous—to the action of forces and/or couples. When attention is focused on the fluid phase, this domain is also frequently designated as physiological fluid mechanics. However, a parallel term, 'solid biomechanics', which might describe the use for biological systems of the field of 'applied mechanics' (which, by custom, is primarily concerned with solid bodies), is not usually employed. In practice, the greatest utility of biomechanics is in relation to humans, who represent the most important and interesting member of

the animal kingdom (Gans 1974; Aerts 1992; Alexander 1992). Nevertheless, similar studies for other species are also highly relevant and are beginning to draw increased attention (Webb and Weihs 1983). The agricultural field, which is just now emerging as a significant component of biomechanics, presents an array of diverse and challenging questions where mechanics can play a critical role (Phipps 1983; Mohsenin 1986; Reznicek 1988; Niklas 1992).

Even when restricted to human activities, such an enormous variety of problems have come within the purview of biomechanics that it is impossible to completely catalog all subjects that comprise its sphere. However, several broad sub-specialties of the field have emerged that can be separately categorized. These

include, but are not limited to: kinesiology, the science of human motion and locomotion (Hinson 1981; Hay and Reid 1988; Vaughan *et al.* 1987; Adrian and Cooper 1989; Rasch 1989; Winter 1990, 1991; LeVeau 1992); the mechanical, structural and geometric properties of the human body and its individual components (Yamada 1970; Evans 1973; Easterby *et al.* 1982; Gray 1984; Silver 1987; Hirokawa 1993); human achievement; sports mechanics (Plagenhoef 1971; Miller and Nelson 1973; Vaughan 1989; Hay 1993); exercise and leisure time activities (A. T. Johnson 1991); orthopedics (Frost 1973; Mow and Hayes 1991; Niwa *et al.* 1992), applications of which will be further discussed in subsequent chapters; clinical, diagnostic and surgical procedures, including component replacement (Ghista 1981; Dowson and Wright 1981; Marcotte 1990); and the effect of environment, such as vibrations (Lippert 1963). For example, specific achievements in the area of orthopedics have included an increased understanding of the function of bones, muscles, ligaments, and tendons (Mow and Hayes 1991; Schafer 1987); this has embraced stress growth and the interaction of stress and piezoelectric behavior in bones. These efforts have led to an improvement or even elimination of traumatic conditions such as scoliosis or bone healing. They have also resulted in the production of substantially more effective prosthetic and orthotic devices, including joint and whole hip replacement, as well as in the restoration of normal joint lubrication and other corrective processes (Journal of Bone and Joint Surgery, Inc.). A special challenge has been the development of biocompatible implantable materials and the satisfaction of the necessary interface conditions. A major triumph is represented by the success of organ replacement, where, however, much more needs to be done. Advanced surgical procedures such as arthroscopy, microsurgery, and novel methods of disk repair using suction have evolved, including nerve control replacement, that permit handicapped persons to at least partly regain normal functioning. In other instances, commands for the operation of assist devices such as wheelchairs have been communicated by breathing processes, representing a highly successful example of rehabilitation methods.

Substantial progress has been made in an understanding of traumatic conditions, their diagnosis and correction. This has included the domain of head and neck injury (Gurdjian 1970) as well as damage to other areas of the body (Pergamon Press 1969–; Aldman and Chapon 1983; Gozna and Harrington 1982; Nahum and Melvin 1993), such as the abdominal region, organs, and

extremities. Studies have been conducted to attempt to delineate tolerance levels and, very importantly, to develop devices providing maximum protection for hazardous environments. Controls have been provided to ensure occupational health and safety in the workplace and elsewhere (Chaffin and Andersson 1991). However, one of the most critical areas where further study is needed is in the safe operation of vehicles, whether travelling on highways, in the sky, or on water.

The discipline of biomechanics incorporates a broad array of experimental techniques, frequently of a novel character. It also represents biological systems by mathematical models of their anatomy, physiology, and function (Morecki 1987), and examines their behavior and assesses their response to load (King and Mertz 1973; Fung 1993); this may require solution of numerical programs executed on high-speed computers. A satisfactory correlation of the experimental and theoretical results provides a certain degree of confidence in the measuring technique and the analytical representation, but is no assurance of precise predictability in view of the enormous variations in the biological properties of the human race and other species. The study of the field of biomechanics can provide a better understanding of the manner, requirements and limitations of all forms of human physical endeavors. It will also permit, in many cases, a reasonable quantitative estimate of the consequence of specified mechanical actions. In addition, its pursuit presents an enormous intellectual challenge for an expansion of our knowledge of human, animal and plant properties, functions and growth that will lead to an improved quality of life.

Biomechanics of solids rests upon an engineering science foundation encompassing well-recognized disciplines portrayed by standard courses in an engineering curriculum. A major subdivision involves mechanics of particles and rigid bodies that include courses in statics, dynamics and vibrations. When objects can no longer be idealized as being rigid—muscles being an outstanding example—the study concerns the mechanics of deformable or continuous media. Here, courses have long been developed in the strength of materials (a term that really signifies the reaction of deformable materials to loading), continuum mechanics, elasticity, viscoelasticity, and rheology. The first category is analytically represented by ordinary differential equations, while response of continuous media to loads must be described by partial differential equations. As in all disciplines, the degree of difficulty in obtaining an engineering solution in biomechanics depends upon the degree of approximation



embedded in a description of the material properties, geometry, deformations, and processes incorporated in a model of the system. An excellent example of this evolutionary process of modeling may be found in an analysis of the pole vault, which is described, in successive stages of complexity, by (a) Hay (1968), (b) W. Johnson *et al.* (1975), (c) Hubbell (1980), and (d) Walker and Kirmser (1982).

The material in this section is divided into the basic science text, which presents the fundamental analytical formulations, applications involving example problems using these relations drawn from the field of biomechanics, and problems to be solved. In general, the topical

treatment is not exhaustive, being limited to concepts and the simplest forms of their analytical representation; some of the material on continuum mechanics can be omitted without loss of continuity. More comprehensive or advanced descriptions of the various subjects, as well as supplementary information concerning geometric and mechanical properties of humans and animals can be found in the bibliography cited at the end of the text. The level of the subject is designed for college students in their senior year or at the graduate level, in the biological or physical sciences, in mathematics or in any engineering discipline with a solid foundation in basic physics courses and two years of college mathematics.

## 1.1 Statics of particles and rigid bodies

### 1.1.1 Introduction

The objective of this discipline is the study of the behavior of solid objects under the application of loads. The phenomena occurring in such processes are ordinarily first observed experimentally (although they may also be hypothesized) and then described by a set of principles (laws and axioms) that predict the behavior of a system that is subjected to such loads. The system is represented by a model (with different degrees of idealization) that is amenable to quantitative analysis. The mathematical tools for this process utilize vector (and tensor) operations, ordinary and partial differential equations, matrix algebra, integral representations, statistical and probabilistic concepts, operator theory, special functions, and many other well-known analytical methods. The present scope will include only a limited number of these fields, primarily covering the first two categories. A very important engineering function is the judgment as to whether the analytical predictions for a given model fit the observed behavior of its counterpart in the real world with a sufficient degree of approximation for the purposes of the user, taking full account of possible experimental errors incurred in the testing and data collection process.

### 1.1.2 Fundamental concepts

A rigid body is defined as a solid object all of whose infinitesimal constituent components (particles) remain at

the same distance from any other specified particle regardless of what external loads may be applied. A deformable body, on the other hand, experiences changes in the distance separating any two given particles under the action of such loads. Solid objects are those whose configuration and relative particle position remain unchanged when moved to a different location or orientation in the absence of applied external loads except gravity. The response of rigid bodies to loads is described by ordinary differential equations, whereas that of deformable bodies is expressed by partial differential equations.

The events with which mechanics is concerned take place in a four-dimensional manifold involving the three-dimensional region of space and time. Dimensions in this space are measured by comparison to a standard reference length, while time, which measures a succession of occurrences, is recorded by a standard clock. Spatial locations are oriented with respect to a coordinate system; the axes of such a system are generally orthogonal. The most common is the rectangular Cartesian system  $xyz$ . Matter occupies space and reacts to the application of loads. Matter may be idealized as a particle which occupies no space but has mass; by a system of particles enveloped by a closed surface defining the volume in which the particles move; or represented by a rigid or, alternatively, by a deformable body of finite (or, in the limit, of infinite) extent. Mass is the property associated with a particle or a rigid body defining its resistance to changes in its motion when the