

SECOND EDITION

System Dynamics

WILLIAM J. PALM III

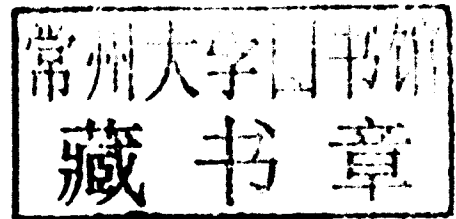
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System Dynamics

Second Edition

William J. Palm III
University of Rhode Island



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To my wife, Mary Louise; and to my children, Aileene, Bill, and Andrew.

PREFACE

System dynamics deals with mathematical modeling and analysis of devices and processes for the purpose of understanding their time-dependent behavior. While other subjects, such as Newtonian dynamics and electrical circuit theory, also deal with time-dependent behavior, system dynamics emphasizes methods for handling applications containing multiple types of components and processes such as electromechanical devices, electrohydraulic devices, and fluid-thermal processes. Because the goal of system dynamics is to understand the time-dependent behavior of a system of interconnected devices and processes as a whole, the modeling and analysis methods used in system dynamics must be properly selected to reveal how the connections between the system elements affect its overall behavior. Because systems of interconnected elements often require a control system to work properly, control system design is a major application area in system dynamics.

TEXT PHILOSOPHY

This text is an introduction to system dynamics and is suitable for such courses commonly found in engineering curricula. It is assumed that the student has a background in elementary differential and integral calculus and college physics (dynamics, mechanics of materials, thermodynamics, and electrical circuits). A previous course in differential equations is desirable but not necessary, as the required material on differential equations, as well as Laplace transforms and matrices, is developed in the text.

The decision to write a textbook often comes from the author's desire to improve on available texts. The decisions as to what topics to include and what approach to take emerge from the author's teaching experiences that give insight as to what is needed for students to master the subject. This text is based on the author's thirty-seven years of experience in teaching system dynamics.

This experience shows that typical students in a system dynamics course are not yet comfortable with applying the relevant concepts from earlier courses in dynamics and differential equations. Therefore, this text reviews and reinforces these important topics early on. Students often lack sufficient physical insight to relate the mathematical results to applications. The text therefore uses everyday illustrations of system dynamics to help students to understand the material and its relevance.

If laboratory sessions accompany the system dynamics course, many of the text's examples can be used as the basis for experiments. The text is also a suitable reference on hardware and on parameter estimation methods.

MATLAB[®] AND SIMULINK^{®1}

MATLAB and Simulink are used to illustrate how modern computer tools can be applied in system dynamics.² MATLAB was chosen because it is the most widely used program in system dynamics courses and by practitioners in the field. Simulink,

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²The programs in this text will work with the following software versions, or higher versions: Version 6 of MATLAB, Version 5 of Simulink, and Version 5 of the Control Systems Toolbox.

which is based on MATLAB and uses a diagram-based interface, is increasing in popularity because of its power and ease of use. In fact, students convinced the author to use Simulink after they discovered it on their own and learned how easy it is to use! It provides a useful and motivational tool.

It is, however, not necessary to cover MATLAB or Simulink in order to use the text, and it is shown how to do this later in the Preface.

CORE MATERIAL FOR SYSTEM DYNAMICS

This text has been designed to accommodate a variety of courses in system dynamics. The core material is in Chapters 1 through 6 and Chapters 8 and 9.

Chapter 1 introduces the basic terminology of system dynamics, covers commonly used functions, and reviews the two systems of units used in the text: British Engineering (FPS) units and SI units. These are the unit systems most commonly used in system dynamics applications. The examples and homework problems employ both sets of units so that the student will become comfortable with both. Chapter 1 also introduces methods for parameter estimation. These methods are particularly useful for obtaining spring constants and damping coefficients. The chapter then illustrates how MATLAB can be used for this purpose.

Chapter 2 covers rigid-body dynamics, including planar motion. Using the models developed in Chapter 2, Chapter 3 reviews solution methods for linear ordinary differential equations where either there is no forcing function (the homogeneous case) or where the forcing function is a constant. The chapter then develops the Laplace transform method for solving differential equations and applies it to equations having step, ramp, sine, impulse, and other types of forcing functions. It also introduces transfer function models.

Chapter 4 covers modeling of mechanical systems having stiffness and damping, and it applies the analytical methods developed in Chapter 3 to solve the models.

Chapter 5 develops the state-variable model, which is useful for certain analytical techniques as well as for numerical solutions. The optional sections of this chapter introduce Simulink, which is based on diagram descriptions, and apply the chapter's concepts using MATLAB.

Chapter 6 treats modeling of electric circuits, operational amplifiers, electro-mechanical devices, sensors, and electroacoustic devices. It also discusses how motor parameters can be obtained, and it shows how to analyze motor performance.

Chapters 8 and 9 cover analysis methods in the frequency domain and the time domain, respectively. Chapter 8 demonstrates the usefulness of the transfer function for understanding and analyzing a system's frequency response. It introduces Bode plots and shows how they are sketched and interpreted to obtain information about time constants, resonant frequencies, and bandwidth.

Chapter 9 integrates the modeling and analysis techniques of earlier chapters with an emphasis on understanding system behavior in the time domain, using step, ramp, and impulse functions primarily. The chapter covers step response specifications such as maximum overshoot, peak time, delay time, rise time, and settling time. Block diagram models are graphical representations of system structure. Chapter 9 introduces these models as preparation for Chapter 10, which deals with control systems.

ALTERNATIVE COURSES IN SYSTEM DYNAMICS

The choice of remaining topics depends partly on the desired course emphasis and partly on whether the course is a quarter or semester course.

Some courses omit fluid and thermal systems, which are covered in Chapter 7. This chapter can be skipped if necessary because only some examples in the remaining chapters, and not the theory and methods, depend on it. Part I of the chapter covers fluid systems. Part II covers thermal systems. These two parts are independent of each other. A background in fluid mechanics or heat transfer is not required to understand this chapter, but students should have had elementary thermodynamics before covering the material on pneumatic systems in Section 7.5.

Chapters 10, 11, and 12 deal with a major application of system dynamics, namely, control systems. Chapter 10 is an introduction to feedback control systems, including the PID control algorithm applied to first- and second-order plants. Chapter 11 deals with control systems in more depth and includes design methods based on the root locus plot and practical topics such as compensation, controller tuning, actuator saturation, reset wind-up, and state-variable feedback, with emphasis on motion control systems. Chapter 12 covers series compensation methods and design with the Bode plot.

Chapter 13 covers another major application area, vibrations. Important practical applications covered are vibration isolators, vibration absorbers, modes, and suspension system design.

At the author's institution, the system dynamics course is a junior course required for mechanical engineering majors. It covers Chapters 1 through 10, with some optional sections omitted. This optional material is then covered in a senior elective course in control systems, which also covers Chapters 11 and 12.

GLOSSARY AND APPENDICES

There is a glossary containing the definitions of important terms, four appendices, and an index. Appendices C and D are on the text website.

Appendix A is a collection of tables of MATLAB commands and functions, organized by category. The purpose of each command and function is briefly described in the tables.

Appendix B is a brief summary of the Fourier series, which is used to represent a periodic function as a series consisting of a constant plus a sum of sine terms and cosine terms. It provides the background for some applications of the material in Chapter 8.

Appendix C is a self-contained introduction to MATLAB, and it should be read first by anyone unfamiliar with MATLAB if they intend to cover the MATLAB and Simulink sections. It also provides a useful review for those students having prior experience with MATLAB.

Appendix D covers basic numerical methods, such as the Runge-Kutta algorithms, that form the basis for the differential equation solvers of MATLAB. It is not necessary to master this material to use the MATLAB solvers, but the appendix provides a background for the interested reader.

CHAPTER FORMAT

The format of each chapter follows the same pattern, which is

1. Chapter outline
2. Chapter objectives
3. Chapter sections
4. MATLAB sections (in most chapters)
5. Simulink section (in most chapters)

6. Chapter review
7. References
8. Problems

This structure has been designed partly to accommodate those courses that do not cover MATLAB and/or Simulink, by placing the optional MATLAB and Simulink material at the end of the chapter. Note that coverage of Simulink requires that the chapter's MATLAB sections also be covered. Because the chapter problems are arranged according to the chapter section whose concepts they illustrate, all problems requiring MATLAB and/or Simulink have been placed in separate, identifiable groups.

OPTIONAL TOPICS

In addition to the optional chapters (7, 10, 11, 12, and 13), some chapters have sections dealing with material other than MATLAB and Simulink that can be omitted without affecting understanding of the core material in subsequent chapters. All such optional material has been placed in sections near the end of the chapter. This optional material includes:

1. Function discovery, parameter estimation, and system identification techniques (Sections 1.4, 1.5, 8.5, and 9.4)
2. General theory of partial fraction expansion (Section 3.5)
3. Impulse response (Sections 3.6 and 4.6)
4. Motor performance (Section 6.5)
5. Sensors and electroacoustic devices (Section 6.6)

DISTINGUISHING FEATURES

The following are considered to be the major distinguishing features of the text.

1. **MATLAB.** Stand-alone sections in most chapters provide concise summaries and illustrations of MATLAB features relevant to the chapter's topics.
2. **Simulink.** Stand-alone sections in chapters 5 through 12 provide extensive Simulink coverage not found in most system dynamics texts.
3. **Parameter estimation.** Coverage of function discovery, parameter estimation, and system identification techniques is given in Sections 1.4, 1.5, 8.5, and 9.4. Students are uneasy when they are given parameter values such as spring stiffness and damping coefficients in examples and homework problems, because they want to know how they will obtain such values in practice. These sections show how this is done.
4. **Motor performance evaluation.** Section 6.5 discusses the effect of motor dynamics on practical considerations for motor and amplifier applications, such as motion profiles and the required peak and rated continuous current and torque, and maximum required voltage and motor speed. These considerations offer excellent examples of practical applications of system dynamics, but are not discussed in most system dynamics texts.
5. **System dynamics in everyday life.** Commonly found illustrations of system dynamics are important for helping students to understand the material and its relevance. This text provides examples drawn from objects encountered in everyday life. These examples include a storm door closer, fluid flow from a

bottle, shock absorbers and suspension springs, motors, systems with gearing, chain drives, belt drives, a backhoe, a water tower, and cooling of liquid in a cup.

6. **Theme applications.** Two common applications provide themes for examples and problems throughout the text. These are motion control systems such as a conveyor system and a robot arm, and vehicle suspension systems.

WEBSITE

The publisher maintains a website for this text at www.mhhe.com/palm. An on-line instructors manual is available at this site. It contains solutions to the problems and other pedagogical aids, and is accessible to instructors who have adopted the text for their course. The site is also home to the text Appendices C & D.

ELECTRONIC TEXTBOOK OPTION

This text is offered through CourseSmart for both instructors and students. CourseSmart is an online resource where students can purchase access to this and other McGraw-Hill textbooks in a digital format. Through their browsers, students can access the complete text online for almost half the cost of a traditional text. Purchasing the eTextbook also allows students to take advantage of CourseSmart's web tools for learning, which include full text search, notes and highlighting, and email tools for sharing notes between classmates. To learn more about CourseSmart options, contact your sales representative or visit www.CourseSmart.com.

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The University of Rhode Island provided an atmosphere that encourages teaching excellence, course development, and writing, and for that I am grateful.

Finally, I thank my wife, Mary Louise, and my children, Aileene, Bill, and Andrew, for their support, patience, and understanding.

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Kingston, Rhode Island

August 2008

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During his thirty-seven years as a faculty member, he has taught nineteen courses. One of these is a junior system dynamics course, which he developed. He has authored nine textbooks dealing with modeling and simulation, system dynamics, control systems, vibrations, and MATLAB. These include *Introduction to MATLAB 7 for Engineers* (McGraw-Hill, 2005) and *A Concise Introduction to MATLAB* (McGraw-Hill, 2008). He wrote a chapter on control systems in the *Mechanical Engineers' Handbook* third edition, (M. Kutz, ed., Wiley, 2006), and was a special contributor to the fifth editions of *Statics* and *Dynamics*, both by J. L. Meriam and L. G. Kraige (Wiley, 2002).

Professor Palm's research and industrial experience are in control systems, robotics, vibrations, and system modeling. He was the Director of the Robotics Research Center at the University of Rhode Island from 1985 to 1993, and is the co-holder of a patent for a robot hand. He served as Acting Department Chair from 2002 to 2003. His industrial experience is in automated manufacturing; modeling and simulation of naval systems, including underwater vehicles and tracking systems; and design of control systems for underwater vehicle engine test facilities.

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CHAPTER 1

Introduction

CHAPTER OUTLINE

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Parameter Estimation 15
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CHAPTER OBJECTIVES

When you have finished this chapter, you should be able to

1. Define the basic terminology of system dynamics.
2. Apply the basic steps used for engineering problem solving.
3. Apply the necessary steps for developing a computer solution.
4. Use units in both the FPS and the SI systems.
5. Develop linear models from given algebraic expressions.
6. Identify the algebraic form and obtain the coefficient values of a model, given a set of data.
7. Apply MATLAB to the methods of this chapter.

This chapter introduces the basic terminology of system dynamics, which includes the notions of *system*, *static* and *dynamic elements*, *input*, and *output*. Because we will use both the foot-pound-second (FPS) and the metric (SI) systems of units, the chapter introduces these two systems. Developing mathematical models of input-output relations is essential to the applications of system dynamics. Therefore, we begin our study by introducing some basic methods for developing algebraic models of static elements. We show how to use the methods of function identification and parameter estimation to develop models from data, and how to fit models to scattered data by using the least-squares method. We then show how to apply MATLAB for this purpose.

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Although Sections 1.4, 1.5, and 1.6 provide the foundation for understanding how to develop models of static elements, coverage of these sections is not required to understand the methods of the remaining chapters, because the appropriate models will be supplied in the examples and chapter problems and thus need not be derived. ■

1.1 INTRODUCTION TO SYSTEM DYNAMICS

This text is an introduction to system dynamics. We presume that the reader has some background in calculus (specifically, differentiation and integration of functions of a single variable) and in physics (specifically, free body diagrams, Newton's laws of motion for a particle, and elementary dc electricity). In this section we establish some basic terminology and discuss the meaning of the topic "system dynamics," its methodology, and its applications.

SYSTEMS

The meaning of the term *system* has become somewhat vague because of overuse. The original meaning of the term is a combination of elements intended to act together to accomplish an objective. For example, a link in a bicycle chain is usually not considered to be a system. However, when it is used with other links to form a chain, it becomes part of a system. The objective for the chain is to transmit force. When the chain is combined with gears, wheels, crank, handlebars, and other elements, it becomes part of a larger system whose purpose is to transport a person.

The system designer must focus on how all the elements act together to achieve the system's intended purpose, keeping in mind other important factors such as safety, cost, and so forth. Thus, the system designer often cannot afford to spend time on the details of designing the system elements. For example, our bicycle designer might not have time to study the metallurgy involved with link design; that is the role of the chain designer. All the systems designer needs to know about the chain is its strength, its weight, and its cost, because these are the factors that influence its role in the system.

With this "systems point of view," we focus on how connections between the elements influence the overall behavior of the system. This means that sometimes we must accept a less-detailed description of the operation of the individual elements to achieve an overall understanding of the system's performance.

INPUT AND OUTPUT

Like the term "system," the meanings of *input* and *output* have become less precise. Nowadays, for example, a factory manager will call a meeting to seek "input," meaning opinions or data, from the employees, and the manager may refer to the products manufactured in the factory as its "output." However, in the system dynamics meaning of the terms, an *input* is a *cause*; an *output* is an *effect* due to the input. Thus, one input to the bicycle is the force applied to the pedal. One resulting output is the acceleration of the bike. Another input is the angle of the front wheel; the output is the direction of the bike's path of travel.

The behavior of a system element is specified by its *input-output relation*, which is a description of how the output is affected by the input. The input-output relation expresses the cause-and-effect behavior of the element. Such a description, which is represented graphically by the diagram in Figure 1.1.1, can be in the form of a table of numbers, a graph, or a mathematical relation. For example, a force f applied to a

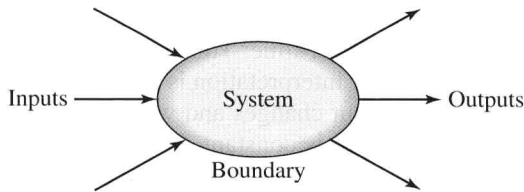


Figure 1.1.1 A system input-output diagram, showing the system boundary.

particle of mass m causes an acceleration a of the particle. The input-output or causal relation is, from Newton's second law, $a = f/m$. The input is f and the output is a .

The input-output relations for the elements in the system provide a means of specifying the connections between the elements. When connected together to form a system, the inputs to some elements will be the outputs from other elements.

The inputs and outputs of a system are determined by the selection of the system's boundary (see Figure 1.1.1). Any causes acting on the system from the world external to this boundary are considered to be system inputs. Similarly, a system's outputs are the outputs from any one or more of the system elements that act on the world outside the system boundary. If we take the bike to be the system, one system input would be the pedal force; another input is the force of gravity acting on the bike. The outputs may be taken to be the bike's position, velocity, and acceleration. Usually, our choices for system outputs are a subset of the possible outputs and are the variables in which we are interested. For example, a performance analysis of the bike would normally focus on the acceleration or velocity, but not on the bike's position.

Sometimes input-output relations are reversible, sometimes not. For example, we can apply a current as input to a resistor and consider the resulting voltage drop to be the output ($v = iR$). Or we can apply a voltage to produce a current through the resistor ($i = v/R$). However, acceleration is the cause of a change in velocity, but not vice versa. If we integrate acceleration a over time, we obtain velocity v ; that is $v = \int a \, dt$. Whenever an output of an element is the time integral of the input and the direction of the cause-effect relation is not reversible, we say that the element exhibits *integral causality*. We will see that integral causality constitutes a basic form of causality for all physical systems.

Similar statements can be made about the relation between velocity and displacement. Integration of velocity produces displacement x : $x = \int v \, dt$. Velocity is the cause of displacement, but not vice versa.

Note that the mathematical relations describing integral causality can be reversed; for example, we may write $a = dv/dt$, but this does not mean that the cause-and-effect relation can be reversed.

STATIC AND DYNAMIC ELEMENTS

When the present value of an element's output depends only on the present value of its input, we say the element is a *static* element. For example, the current flowing through a resistor depends only on the present value of the applied voltage. The resistor is thus a static element. However, because no physical element can respond instantaneously, the concept of a static element is an approximation. It is widely used, however, because it results in a simpler mathematical representation; that is, an algebraic representation rather than one involving differential equations.

If an element's present output depends on past inputs, we say it is a *dynamic element*. For example, the present position of a bike depends on what its velocity has been from the start.

In popular usage, the terms static and dynamic distinguish situations in which no change occurs from those that are subject to changes over time. This usage conforms to the preceding definitions of these terms if the proper interpretation is made. A static element's output can change with time only if the input changes and will not change if the input is constant or absent. However, if the input is constant or removed from a dynamic element, its output can still change. For example, if we stop pedaling, the bike's displacement will continue to change because of its momentum, which is due to past inputs.

A *dynamic system* is one whose present output depends on past inputs. A *static system* is one whose output at any given time depends only on the input at that time. A static system contains all static elements. Any system that contains at least one dynamic element must be a dynamic system. *System dynamics*, then, is the study of systems that contain dynamic elements.

MODELING OF SYSTEMS

Table 1.1.1 contains a summary of the methodology that has been tried and tested by the engineering profession for many years. These steps describe a general problem-solving procedure. Simplifying the problem sufficiently and applying the appropriate fundamental principles is called *modeling*, and the resulting mathematical description is called a *mathematical model*, or just a *model*. When the modeling has been finished, we need to solve the mathematical model to obtain the required answer. If the model is highly detailed, we may need to solve it with a computer program.

The form of a mathematical model depends on its purpose. For example, design of electrical equipment requires more than a knowledge of electrical principles. An electric circuit can be damaged if its mounting board experiences vibration. In this case, its force-deflection properties must be modeled. In addition, resistors generate heat, and a thermal model is required to describe this process. Thus, we see that devices

Table 1.1.1 Steps in engineering problem solving.

<div>1. Understand the purpose of the problem.</div> <div>2. Collect the known information. Realize that some of it might turn out to be not needed.</div> <div>3. Determine what information you must find.</div> <div>4. Simplify the problem only enough to obtain the required information. State any assumptions you make.</div> <div>5. Draw a sketch and label any necessary variables.</div> <div>6. Determine what fundamental principles are applicable.</div> <div>7. Think generally about your proposed solution approach and consider other approaches before proceeding with the details.</div> <div>8. Label each step in the solution process.</div> <div>9. If you use a program to solve the problem, hand check the results using a simple version of the problem. Checking the dimensions and units, and printing the results of intermediate steps in the calculation sequence can uncover mistakes.</div> <div>10. Perform a "reality check" on your answer. Does it make sense? Estimate the range of the expected result and compare it with your answer. Do not state the answer with greater precision than is justified by any of the following:</div>	
<div>a. The precision of the given information.</div> <div>b. The simplifying assumptions.</div> <div>c. The requirements of the problem.</div>	<div>Interpret the mathematics. If the mathematics produces multiple answers, do not discard some of them without considering what they mean. The mathematics might be trying to tell you something, and you might miss an opportunity to discover more about the problem.</div>