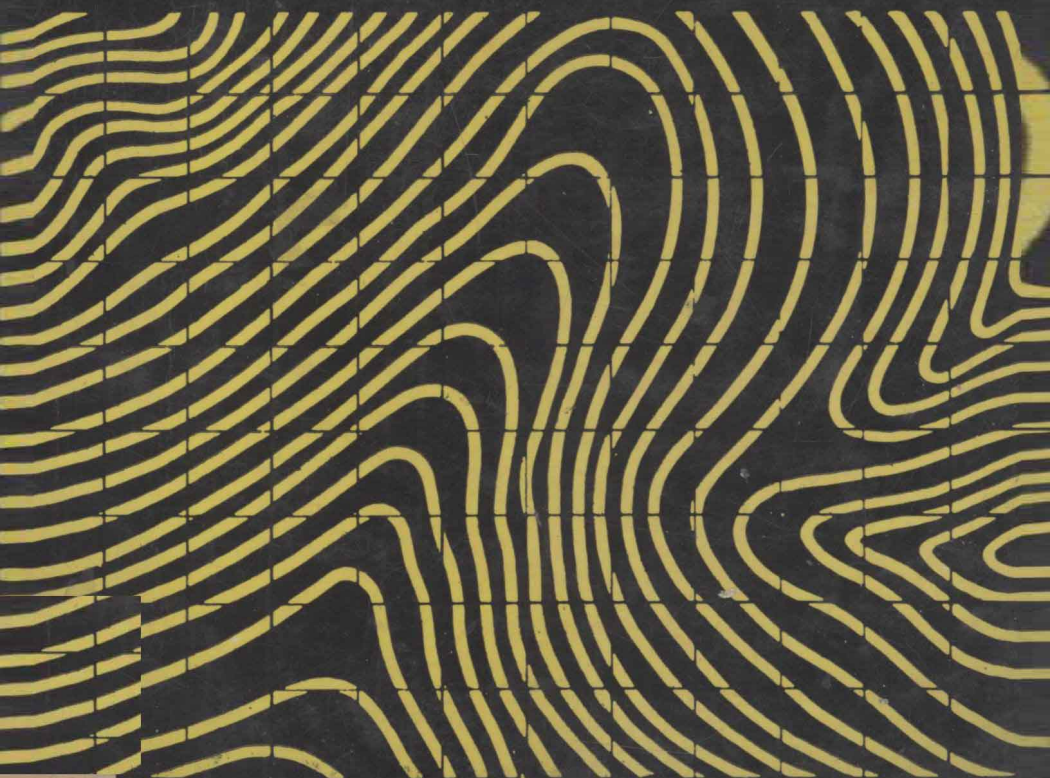


# Plane and Geodetic Surveying For Engineers

by Late DAVID CLARK

## HIGHER SURVEYING

Vol 2



4th edition, revised by JE Jackson

# PLANE AND GEODETIC SURVEYING

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## VOLUME TWO HIGHER SURVEYING

SIXTH EDITION REVISED AND  
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**PLANE AND GEODETIC SURVEYING**  
**VOLUME TWO**

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## PREFACE TO THE SIXTH EDITION

Readers acquainted with the fifth edition of this volume will immediately notice that this new edition has far fewer pages. Of course, successive editions of a work such as this generally tend to get bigger, by inclusion of material needed to cover modern developments in the subject, and by the natural reluctance to discard any text that could possibly still be relevant.

But nowadays, even staying the same size means becoming much more expensive, and a considerable reduction of wordage seemed highly desirable. This has been achieved by using more methods than just the removal of obsolete material, and some of the reduction has been possible through developments in the instruments and techniques of the subject itself. Overlapping with subjects treated in Vol. I has been avoided as far as possible, but subjects new in the present Volume are introduced as if new to the reader, so the text may seem very elementary in places.

Much of the matter formerly in Chapter VIII now seems irrelevant or inappropriate in this particular publication: the modern surveyor is likely to do his reconnaissance by examining air photographs or even by inspecting the area from an aircraft! Descriptive detail about particular instruments has been kept to a minimum; either it would be a repetition of old information already available at many sources, or the developments in instrumentation are going on so rapidly that any constructional details now described may be out of date before the words can be printed: the automatic recording of observations is on the way in; new EDM devices seem to be arriving on the market at the rate of two or three a year! Descriptions of purely arithmetic processes have been omitted wherever possible; this applies particularly to the formal solution of sets of simultaneous equations—an operation that can hardly be described as surveying: in any case, computing procedures are largely determined by the type of calculating equipment that is available to the surveyor, and here too the rate of appearance of new models is quite hectic. On the other hand, the modern surveyor may have very little computing to do himself, if his work is so organised.

In this new edition there are no references to photographic surveying, air survey, or photogrammetry. These are subjects that

must either be omitted altogether or take up a considerable amount of space if treated adequately. There are now many excellent books devoted solely to these subjects.

In view of the above remarks, it is not surprising that this edition is completely newly written—no text has been directly transferred from the fifth edition. Only the general arrangement has been followed as closely as possible. However, the opportunity has been taken to attempt to get away from the somewhat stereotyped style of presentation that has developed over the years and is particularly noticeable in writings on field astronomy. It is hoped that this change will be acceptable and even pleasant to the student reader, but any comments about it will be noted with interest and no malice.

I am very grateful to numerous friends and colleagues whose opinions I have sought on many technical matters, and especially to Mr. Peter Dale who also has read through some of the typed drafts.

J. E. JACKSON

CAMBRIDGE  
*October 1973*

FIELD ASTRONOMY—INTRODUCTORY

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**1.01 Latitude, Longitude, and Azimuth.** Surveying on land is the measurement of angles, distances, and heights, and it results in the description of the relative positions of points either by the angles and lengths in geometrical figures, or as coordinates of positions in a reference system: the coordinate systems in use are spheroid latitude and longitude as described in Chapter VI, and plane rectangular coordinates in linear measure in a defined system of map projection as described in Chapter IX.

In Field Astronomy, the quantities observed, the other data used in the work, and the results of the calculations, are entirely angles. The observer's object is to obtain astronomical latitude, longitude and bearings at points on the Earth. It must be emphasised that astronomical observation cannot directly give any information about linear distances. Astronomical terminology sometimes gives unfamiliar meanings to common words, and one example is the expression 'astronomical position'; this simply means 'astronomical latitude and longitude', and must not be thought of as giving positional information in any of the systems mentioned in the preceding paragraph.

On the other hand, astronomical values are absolute and in no way arbitrary, in the sense that at any point the determinable astronomical quantities are real and unique: any two observers at the same point will get the same results, within the limits of their observational errors, and the results will not have to be referred to any arbitrary system, except that longitudes must be reckoned from some adopted zero meridian, often the meridian of the old Observatory at Greenwich, or perhaps the meridian of one of the points included in the survey.

The navigator at sea or in the air, if he cannot connect his position to land by direct observation or by radio navigation aids, observes latitude and longitude, and describes them as his 'position'. By calling one minute of latitude a 'nautical mile' the men of the sea have added another bit of unfamiliar usage!

Of course it is true that differences of astronomical values give a good idea of relative linear position on the Earth: for instance, two points differing by  $1^\circ$  of latitude and of the same longitude will be about 110 km apart. This quantity comes, however, from our knowledge of the size of the Earth and could not be found from

astronomical observations alone. The same two astronomical 'positions' on the Moon would be about 30 km apart!

**1.02 Uses of Field Astronomy.** Because astronomical observation produces results that are always expressed in angle measure, its function in relation to Surveying and Geodesy must be carefully understood.

(1) In earlier times, explorers travelling in unmapped country often carried simple surveying instruments and made observations of latitude and bearing as they went along; observations of longitude were much more difficult and were liable to large errors. However, the information they obtained enabled them to plot their journeys, necessarily on rather small scale, and they could interpret their astronomical results on the assumption that they were on a simple geometrical surface such as a sphere of radius 6000 km. Salient topographical features could be added to these route surveys. Referring to the astronomical results, the explorer could direct his travel so that he arrived more or less accurately at the place where he intended to terminate his journey.

(2) In a more systematic manner, if astronomical latitudes and longitudes are observed at a number of points distributed over an area, they can be plotted on a network of lines suitably drawn on paper to represent meridians and parallels. The 'positions' so fixed can be used as control for the survey of detail by simple or graphical methods such as compass traversing and plane tabling. A considerable amount of mapping has in the past been done on this basis, and on small scales such as 1/200 000.

(3) The construction of charts of islands, soundings, etc. in oceanic regions is technically the same process as mentioned in the preceding paragraph. Astronomical 'positions' are observed on the islands or by standard navigational astronomy on the ships. Methods of navigational astronomy are introduced in Section 2.27.

(4) Bearings observed by simple methods may be used to check the accumulation of directional errors in ordinary surveys, especially in traverses composed of a large number of short lines.

(5) It is desirable that the plans of any large survey should be correctly oriented in relation to a north line. If the survey cannot be connected to existing reference points such as the national system, one or more astronomical observations will provide the orientation. Moreover, in extensive topographical mapping it is necessary to provide a datum of latitudes and longitudes so that the graticule of meridians and parallels can be drawn on the maps. See Section 8.02.

(6) Precise astronomical observations have essential parts to play in geodetic surveying. In combination with the linear and

angular measurements of triangulation and traverse surveys, the astronomical results are used in calculating the size and shape of the Earth, and in determining the ellipsoidal surfaces to be used as datum surfaces for extended mapping systems. See Section 8.03.

**1.03 Celestial Sphere.** In field astronomy, use may be made of the Sun, Moon, and Planets, as well as the Stars. However, the stars are so numerous and make such excellent signals on which to direct the telescope, that the most precise results are obtainable from them. Moreover, stellar astronomy is easier to understand and simpler to compute, therefore consideration of other objects mentioned will be deferred till after the use of the stars has been discussed.

That stellar astronomy is comparatively simple is due mostly to the great distances of the stars. One effect of this is that the stars' own movements about in space are almost imperceptible, the familiar patterns of the constellations have remained the same for centuries, and we can identify each star by its position in the pattern. Another effect is that the directions of the stars as seen from the Earth are practically independent of the movements of the Earth or the position of an observer on the Earth: even the nearest star is so far away that the directions to it from two diametrically opposite points on the Earth will differ by less than 0.000 1 second. Because of these circumstances, it is permissible and convenient to regard the stars as point sources of light fixed on the inside of a colossal hollow sphere, the *celestial sphere*, having a radius effectively infinite, and the Earth at its centre.

Because there are so many stars, distributed all over the sky, the observer can choose those that are in the positions to give optimum accuracy of the calculated results.

**1.04 Apparent Movement of Stars.** Anyone who watches the stars on successive nights will become familiar with the pattern of the constellations and will also note that they pass steadily across the sky, night after night, each star apparently tracing out the same path each time. This phenomenon is of course simply an apparent movement due to the actual steady rotation of the Earth about its axis. An observer in middle northern latitudes, looking towards the northern parts of the sky, will see that the stars move round in circular concentric paths as shown in Fig. 1; the centre P of the tracks is of course a point where the Earth's axis of rotation meets the celestial sphere. For the present we can assume that the axis of rotation maintains a fixed direction in space (see Section 1.11); it is a particular diameter of the celestial sphere and it determines the positions of two *celestial poles*.

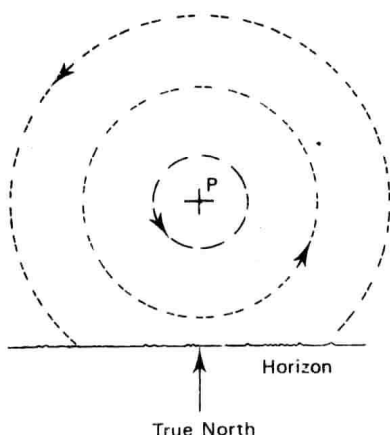


FIG. 1. NORTH POLAR SKY.

**1.05 True North: Azimuth: Transit.** As seen from a point on Earth in the northern hemisphere, the horizontal direction of the pole *P* in Fig. 1 is the definition of *true or astronomical north*. If a telescope of a theodolite could be set on point *P* (in fact there is no star exactly at this point), and then brought down to horizontal, it would be set in the direction of true north; then if the instrument is swung to the right about its vertical axis the angle turned is the *true or astronomical bearing*. Astronomical bearing is usually called *azimuth*, and this word will henceforth be used when the astronomical bearing is intended. It is usually numbered clockwise round to  $360^\circ$ .

If the angle turned is exactly  $180^\circ$  the telescope will be directed at azimuth  $180^\circ$ . The observer (still at a middle northern latitude!) will now note that the stars' tracks in this part of the sky are somewhat as shown in Fig. 2. Each star appears into view at a particular point on the horizon, rises, goes down, then disappears at its

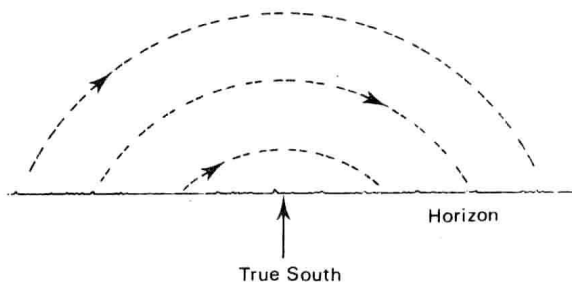


FIG. 2. TRANSIT OF STARS.



appropriate point on the horizon. When at exactly  $180^\circ$  azimuth, or due South, the star is at its greatest altitude and is said then to be at *transit*. For an observer in the southern hemisphere, the figures will be much the same; the arrows will point the other way, and the words 'south' and 'north' will be interchanged.

**1.06 Basic Geometry.** In astronomical observations for the purposes already mentioned, the observer is concerned with three directions, indicated in Fig. 3. We may suppose point O to be the

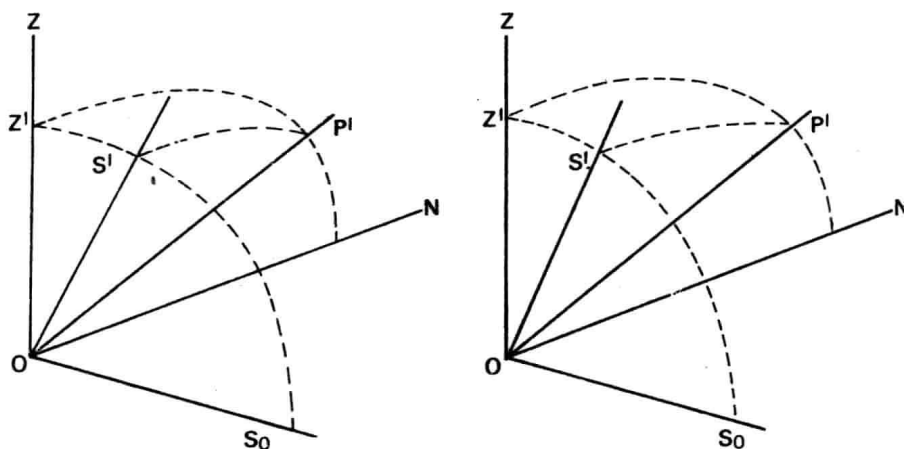


FIG. 3. ASTRONOMICAL GEOMETRY.  
(this diagram is a stereoscopic pair)

centre of the theodolite, the point where the telescope axis meets the transit axis. OZ is the direction of the vertical and is the axis of rotation of the instrument, assuming that it has been properly 'levelled'. The point where OZ meets the celestial sphere is the observer's *zenith*. The line OP' is the direction of the pole of the sky and is therefore parallel to the Earth's axis of rotation; it is the direction from the observer to point P in Fig. 1. These two directions at O are fixed in relation to the Earth, one by the Earth's gravity and the other by the direction of the Earth's axis. The plane defined by the lines OZ and OP' is the meridian plane of point O; a star at transit is in this plane. The third line OS' is a typical direction from the instrument to a star; this is not only different for each star, but for any one star it is continuously changing on account of the apparent movement of the stars across the sky. As the celestial sphere apparently rotates, the line OS' for a particular star will appear to rotate steadily about OP.

**1.07 Latitude: Declination.** Now the angle ZOP' is the angle between the axis of the Earth and the vertical at O; this is the