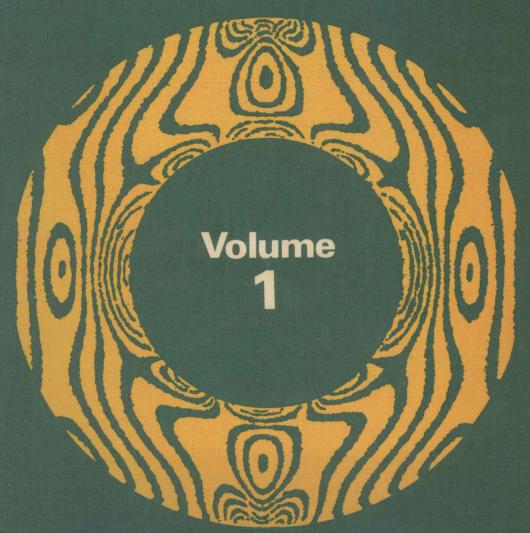
International Series on Materials Science and Technology Volume 19

Mechanics of **Materials**

An Introduction to the Mechanics of Elastic and Plastic Deformation of Solids and Structural Components

E.J.HEARN City of Birmingham Polytechnic, England



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MECHANICS OF MATERIALS

An Introduction to the Mechanics of Elastic and Plastic Deformation of Solids and Structural Components

In Two Volumes

VOLUME I

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ROBERT MAXWELL
Publisher at Pergamon Press

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INTRODUCTION

In recent years there have been significant changes in educational philosophies, teaching methods and individual subject disciplines. In particular, the subject of strength of materials now appears under numerous titles including solid mechanics, engineering science, mechanical technology, stress analysis and the now more accepted mechanics of materials. A careful analysis of the syllabuses offered under these titles, particularly in British polytechnics, universities and service colleges, reveals substantial overlap in terms of the topics covered in any particular year of engineering or combined studies courses; the advent of modular treatment of courses inevitably also produces variations in topics offered at any given level. Thus, whilst this volume is principally intended to cover at least the first two years of conventional degree courses in mechanics of materials, the material covered allows for the overlaps noted above and provides a substantial lead-in to final-year studies. Indeed, in many of the courses examined, the subject coverage of the text is entirely adequate for the final year, particularly of unclassified degree options. The text should also prove valuable to students following Higher National or T.E.C.† certificate and diploma courses and any courses leading to C.E.I.‡ exemption.

The study of mechanics of materials is the study of the behaviour of solid bodies under load. The way in which they react to applied forces, the deflections resulting and the stresses and strains set up within the bodies, are all considered in an attempt to provide sufficient knowledge to enable any component to be designed such that it will not fail within its service life. Typical components considered in detail in this text include beams, shafts, cylinders, struts, diaphragms and springs and, in most simple loading cases, theoretical expressions are derived to cover the mechanical behaviour of these components. Inevitably, however, these expressions rely on certain basic assumptions which are not always completely achieved, and no text would be complete without reference to the importance of experimental testing techniques. Chapter 21 in Volume 2 is therefore devoted to an introduction to the major techniques of experimental stress and strain measurement in use today, and it is suggested that this should be reinforced by further reading of the specialist texts noted in the Bibliography at the end of the chapter. Similarly, additional reference should be made to specialist texts for detailed treatment of other associated topics such as materials testing (including creep, fatigue and fracture mechanics) and finite element techniques of stress and strain analysis which are considered to be outside the scope of this text.

Each chapter of the book (except Chapter 21) contains a summary of essential formulae which are developed within the chapter and a large number of worked examples. The examples have been selected to provide progression in terms of complexity of problem and to illustrate the logical way in which the solution to a difficult problem can be developed. Graphical solutions have been introduced where appropriate. In order to provide clarity of working in the

[†] Technician Education Council.

[‡] Council of Engineering Institutions.

worked examples there is inevitably more detailed explanation of individual steps than would be expected in the model answer to an examination problem.

All chapters (again with the exception of Chapter 21) conclude with an extensive list of problems for solution by students together with answers. These have been collected from various sources and include questions from past examination papers in imperial units which have been converted to the equivalent SI values. Each problem is graded according to its degree of difficulty as follows:

A Relatively easy problem of an introductory nature.

A/B Generally suitable for first-year studies.

B Generally suitable for second-year studies.

C More difficult problems.

Gratitude is expressed to the following examination boards, universities and colleges who have kindly given permission for questions to be reproduced:

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In all, the text contains 113 worked examples and more than 400 problems for solution, and whilst it is hoped that no errors are present it is perhaps inevitable that some errors will be detected. In this event any comment, criticism or correction will be gratefully acknowledged.

The symbols and abbreviations throughout the text are in accordance with the latest recommendations of BS 1991 and PD 5686^{\dagger} .

As mentioned above, graphical methods of solution have been introduced where appropriate since it is the author's experience that these are more readily accepted and understood by students than some of the more involved analytical procedures; substantial time saving can also result. Extensive use has also been made of diagrams throughout the text since in the words of the old adage "a single diagram is worth 1000 words".

Finally, the author is indebted to all those who have assisted in the production of this volume, and particularly to the Editor, Professor H.G. Hopkins, for his valuable guidance and encouragement, to Mr. R. Brettell and Mr. R.J. Phelps for their work in checking, editing and proof reading, and to Mrs. J. Beard for typing the manuscript. Thanks also go to the publishers for their advice and assistance, especially in the preparation of the diagrams and in editing.

August 1975 E.J. HEARN

[†] Relevant Standards for use in Great Britain: BS 1991; PD 5686: Other useful SI Guides: The International System of Units, N.P.L. Ministry of Technology, H.M.S.O. (Britain). Mechty, The International System of Units (Physical Constants and Conversion Factors), NASA, No. SP-7012, 3rd edn. 1973 (U.S.A.) Metric Practice Guide, A.S.T.M. Standard E380-72 (U.S.A.).

NOTATION

Quantity	Symbol	SI Unit
Angle	$\alpha, \beta, \theta, \gamma, \phi$	rad (radian)
Length	L,s	m (metre)
26	_,_	mm (millimetre)
Area	A	m²
Volume	V	m³
Time	t	s (second)
Angular velocity	ω	rad/s
Velocity	v	m/s
Weight	W	N (newton)
Mass	m	kg (kilogram)
Density	ρ	kg/m³
Force	F or P or W	N
Moment	M	Nm
Pressure	P	Pa (Pascal)
		N/m ²
		$bar (= 10^5 \text{ N/m}^2)$
Stress	σ	N/m ²
Strain	ϵ	-
Shear stress	au	N/m ²
Shear strain	γ	_
Young's modulus	E	N/m^2
Shear modulus	G	N/m^2
Bulk modulus	K	N/m ²
Poisson's ratio	ν	 -
Modular ratio	m	 -
Power	_	W (watt)
Coefficient of linear expansion	α	m/m °C
Coefficient of friction	μ	-
Second moment of area	I	m ⁴
Polar moment of area	J	m ⁴
Product moment of area	I_{xy}	m ⁴
Temperature	T	°C
Direction cosines	l, m, n	
Principal stresses	σ_1 , σ_2 , σ_3	N/m ²
Principal strains	ϵ_1 , ϵ_2 , ϵ_3	
Maximum shear stress	$ au_{ ext{max}}$	N/m^2
Octahedral stress	$\sigma_{ m oct}$	N/m^2

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Notation

Quantity	Symbol	SI Unit
Deviatoric stress	σ'	N/m^2
Deviatoric strain	$\frac{\epsilon'}{\sigma}$	_
Hydrostatic or mean stress	$\overline{\sigma}$	N/m^2
Volumetric strain	Δ	_
Stress concentration factor	K	=
Strain energy	U	J
Displacement	δ	m
Deflection	δ or <i>y</i>	m
Radius of curvature	ρ	m
Photoelastic material fringe value	f	N/m²/fringe/m
Number of fringes	n	-
Body force	X, Y, Z	N/m^3
	F_{R} , F_{θ} , F_{Z}	
Radius of gyration	\boldsymbol{k}	m
Slenderness ratio	L/k	_
Gravitational acceleration	g	m/s ²
Cartesian coordinates	x, y, z	_
Cylindrical coordinates	r , θ , z	_
Eccentricity	e	m
Number of coils or leaves of spring	n	-

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