



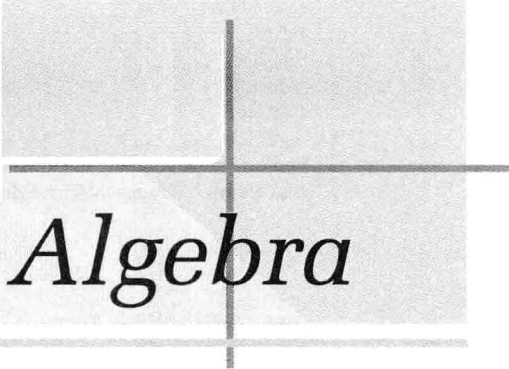
HUBBARD ROBINSON

College ALGEBRA

VISUALIZING

and

DETERMINING SOLUTIONS



College Algebra

Visualizing and Determining Solutions

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College Algebra

Visualizing and Determining Solutions

PREFACE

College Algebra: Visualizing and Determining Solutions is written for students who are meeting their core curriculum requirements in mathematics. The content includes the topics found in any mainstream college algebra course. Moreover, this text provides excellent preparation for such additional courses as statistics and business calculus.

The Approach

A major goal of *College Algebra: Visualizing and Determining Solutions* is to engage the students by promoting their active participation in the study of mathematics. The pedagogical features of the book are designed to achieve that goal.

Technology The use of a graphing calculator is integrated throughout the text. Rather than compartmentalizing technology in special boxes or marginal activities, we employ the graphing calculator as an integral part of the presentation and as an essential aspect of the learning process.


Although solutions of equations and properties of functions, for example, can be determined with a graphing calculator, we prefer not to treat graphing and algebraic methods as equivalent. Rather, our focus is on the mathematics and the algebraic methods, with the technology serving as a guide to the development of concepts and as a visual aid for reaching conclusions. Therefore, when both approaches are presented, the graphing method leads to the algebraic method rather than one approach being an alternative to the other. In short, first we see, then we do.

Exploration We believe that “discovery learning” is primarily a classroom activity. Students are best able to explore and discover when they are guided by the teaching skills of the instructor. Throughout this text, we have included classroom activities under the heading “Exploration.” With leading questions and, usually, the visual assistance of the graphing calculator, the student is guided from a concrete set of circumstances to a general conclusion. All the questions are open-ended, thereby providing maximum flexibility in the use of these activities.

Developing the Concept A parallel feature called “Developing the Concept” usually follows each “Exploration” or, less often, is a stand-alone activity. This feature is one that students are better able to use on their own. Structured like an example, “Developing the Concept” guides the student with leading questions, and answers them as well. The combination of exploring and developing the concept is a major effort to promote students’ interaction and involvement in their own learning.

Communication Skills Increased emphasis has been placed on writing and speaking in mathematics. When a student says, “I know the answer, I just don’t know how to say it,” the student probably does not know the answer.

To help with this, we have included two features that are specifically designed to improve the student's ability to communicate about mathematics. One, "Speaking the Language," precedes the exercise sets (except in sections that are dedicated exclusively to applications) and helps the student to practice the use of vocabulary in context.

The other main communication feature is the generous number of writing exercises, which are identified with a  icon. Although the answers to such writing exercises can often vary, a sample answer is given in the Answers section at the back of the book. Rather than assigning writing exercises, some instructors may prefer to use them as the basis of class discussion. Thus, writing exercises can promote both written and oral communication development. These exercises truly set this text apart from many others.

Exercises Most exercise sets are divided by the headings "Concepts and Skills," "Concept Extension," and "Applications." Sometimes the applications exercises have subheads to indicate the type of application that is included. All of these headings have been designed to assist the instructor in the preparation of assignments.

Exercises included in the "Concepts and Skills" are representative of the basic material of the section. Nearly all are illustrated by text examples.

Most exercise sets include a number of "Concept Extension" exercises designed to go somewhat beyond the basic concepts and skills discussed in the text section. These exercises are designed to synthesize concepts and to use previously learned skills in a new context.

Most sections contain numerous real-life and real-data applications, with source acknowledgments. Several sections are devoted exclusively to such exercises. These include a wide variety of subject areas and are designed to promote a greater understanding of the relevancy of mathematics.


Other Features

Definitions and Rules All definitions, rules, properties, and procedures are highlighted in colored boxes for easy reference.

Examples Numerous titled examples provide immediate illustrations of the concepts and techniques discussed in the sections. Comments helpful to the student are often included in the detailed solutions.

Graphs Visualization is provided by a large number of traditional coordinate plane graphs and calculator graphs that appear in both the exposition and the exercises. Calculator graphs are designed to resemble the displays of most calculator models.

Quick Reference With the exception of applications sections, a "Quick Reference" appears at the end of each section. These are detailed summaries of the rules, definitions, properties, and procedures discussed in the section. All are grouped by subsection and can be used by the student for reference and review.

Calculator Guide When a graphing calculator function is first introduced in the text, a key word  icon appears along with a brief description of the

function. Students can reference these key words in Appendix B, which is a calculator keystroke guide for the TI-83 Plus calculator.

Calculator Programs Appendix C contains sample calculator programs. Although these programs are written for the TI-83 Plus, the algorithms are described in detail, and they can be used to program any calculator model.

A Final Word

The methods and approaches used in *College Algebra: Visualizing and Determining Solutions* are based on our many years of classroom teaching experience and are time-tested for their effectiveness in promoting student success. We are pleased to share our experiences with you.

Supplements

Student Solutions Manual This manual includes solutions to the odd-numbered exercises in the text.

Graphing Calculator Guide The Graphing Calculator Keystroke Guide includes information for the TI-82, TI-83, and TI-83 Plus. A Key Word icon in the text alerts students to specific keystroke information in this supplement. The sequence of the Key Word explanations in the Guide corresponds to the sequence in the main text.

Complete Solutions Guide The Complete Solutions Guide includes detailed solutions to all Preamble Exercises, end-of-section exercises, and Chapter Review Exercises, including those in Chapter R. The Guide also includes answers to the Speaking the Language exercises, which appear before the section exercises in each chapter.

Test Item File This printed manual contains multiple-choice, open-ended, and writing questions. There is a set of questions for each section of the book.

Computerized Test Bank Available for both Windows and Macintosh.

Tutorial Software This tutorial software is algorithmically driven and interactive. Lessons and problems are presented in a lively manner. It includes student-friendly features such as: animated solution steps, extensive hints, and a glossary of key terms and definitions.

Videotapes Prepared by Dana Mosely, the videotapes provide a thorough review of concepts and worked-out examples to reinforce lessons within the text.

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OPENING FEATURES

Chapter Opener

Each chapter begins with a short introduction to a real-data application. Connecting mathematics with students' view of the world leads students to a better understanding of the practical nature of the discipline. The chapter opener also includes a helpful overview of the topics that will be covered in the chapter and a list of the section titles.

112 ■ CHAPTER 2 Basic Functions and Graphs

2.1 Relations and Functions

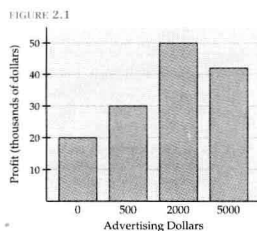
Relations ■ Functions ■ Function Notation and Evaluation
■ Domain and Range ■ Piecewise Functions ■ Applications

In many real-world situations, the value of one quantity depends on the value of another. For example, the federal tax rate depends on your income, the distance that you can drive depends on your speed, and your success in a college course depends on the number of hours that you study.

Being able to describe how one variable affects another is an important step in analyzing a problem. In this section, we use a *function* to describe the association of two sets of numbers. The topic of functions is central to the study of mathematics.

Relations

The bar graph in Figure 2.1 shows how advertising affects the profits of a small business.



The graph shows that advertising does increase profits, but that beyond a certain level, the cost of too much advertising actually decreases profits. The advertising costs and corresponding profits shown in the bar graph can be written as a set of ordered pairs.

$$A = \{(0, 20,000), (500, 30,000), (2000, 50,000), (5000, 42,000)\}$$

Such a set is called a **relation**.

Definition of a Relation

A **relation** is a set of ordered pairs. The **domain** of a relation is the set of all first coordinates of the ordered pairs. The **range** is the set of all second coordinates.

Chapter 2

Basic Functions and Graphs

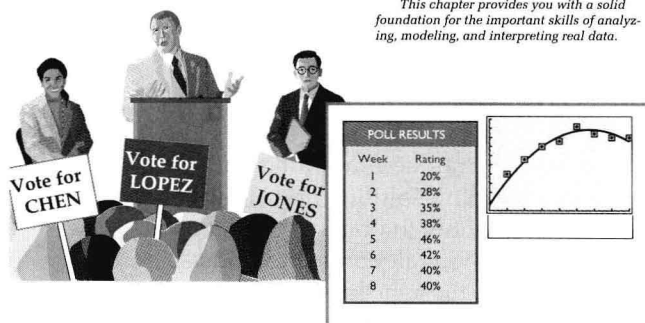
- 2.1 Relations and Functions
- 2.2 Linear Functions
- 2.3 Linear Regression Models
- 2.4 Topics in Analytic Geometry
- 2.5 Quadratic Functions
- 2.6 Quadratic Regression Models
- 2.7 Graphs of Basic Functions
- 2.8 Transformations

A politician's "approval rating" can change quickly and dramatically. The rating can be affected by such factors as the state of the economy, the conduct of foreign affairs, and the politician's stand on domestic issues.

By periodically polling small samples of people, political advisors can estimate the trend in a candidate's support among voters. Polling data can be modeled with a regression equation whose graph best fits the data, and this model can then be used to predict the level of support that a candidate can expect to receive.

In this chapter, we introduce the essential concept of a function, and we investigate the properties of the graphs of a variety of functions. In particular, you will learn how real data can be modeled with functions and how to judge the most appropriate model to use. Although many different models are available, our emphasis in this chapter is on linear and quadratic regression models.

This chapter provides you with a solid foundation for the important skills of analyzing, modeling, and interpreting real data.



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Section Opener

Each section begins with a list of subsection titles, thus providing a brief outline of the material that follows.

VISUALIZING AND DETERMINING SOLUTIONS

These headings appear in examples in which a graphing solution approach is considered.

Visualizing the Solution

This first portion shows students how to depict the problem using a graphing calculator and how to estimate a solution from this visualization (often highlighting any potential pitfalls from using graphing technology).

Determining the Solution

The algebraic solution follows the graphing solution and shows students how to obtain an exact solution with algebraic methods. Together, these headings remind students that graphical approaches are used to best advantage when coupled with algebraic approaches—a “see first, then do” logical progression.

Domain and Range

Because a function is a particular kind of relation, the definitions of domain and range of a function are the same as those for relations. An unspecified domain of a function is assumed to be the largest subset of real numbers for which the function is defined.

The graph of a function provides a visual way of estimating the domain and range. However, most graphs extend beyond the viewing window, so you must make sure that your window settings are such that you can see the important features of the graph.

EXAMPLE 7 Domains and Ranges

Use graphing and algebraic methods to determine the domain and range of each of the given functions.

(a) $f(x) = 3\sqrt{12 - x}$ (b) $g(x) = x^4 - 3x^3 + 2x - 1$

SOLUTION

(a) VISUALIZING THE DOMAIN AND RANGE

Figure 2.6 shows the graph of $f(x) = 3\sqrt{12 - x}$ with the tracing cursor on the right endpoint of the graph.

The domain consists of the x -coordinates of all the points of the graph. The largest x -coordinate is 12 at the endpoint, and the graph appears to extend to the left forever. Thus the estimated domain is $(-\infty, 12]$.

The range of the function consists of the y -coordinates of the points of the graph. The smallest y -coordinate is 0 at the endpoint, and the graph appears to rise forever. The estimated range is $[0, \infty)$.

DETERMINING THE DOMAIN AND RANGE

The function $f(x) = 3\sqrt{12 - x}$ is defined only for those values of x for which the radicand $12 - x$ is nonnegative.

$$\begin{aligned} 12 - x &\geq 0 \\ -x &\geq -12 \\ x &\leq 12 \end{aligned}$$

The domain is $(-\infty, 12]$.

Because $\sqrt{12 - x}$ is nonnegative, $f(x) \geq 0$. Thus the range is $[0, \infty)$.

(b) VISUALIZING THE DOMAIN AND RANGE

Figure 2.7 shows the customized window for viewing the important features of the graph of function g . The graph extends to the left and right without end, so the domain appears to be \mathbb{R} .

The tracing cursor is on the lowest point of the graph, where the y -coordinate is approximately -5.1 , and the graph extends upward without end. The estimated range is $[-5.1, \infty)$.

FIGURE 2.6

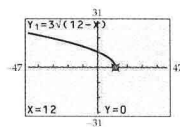
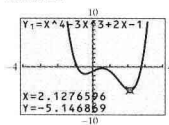


FIGURE 2.7



REAL-DATA AND REAL-LIFE APPLICATIONS

SECTION 2.6 Quadratic Regression Models ■ 185

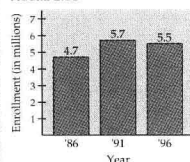
Real-Life Applications

Because the vertex of a vertical parabola is the highest or lowest point of the curve, we sometimes make use of a quadratic regression equation to estimate or predict a maximum or minimum value. "The regression equation has the form $y = ax^2 + bx + c$. Knowing a and b allows us to determine the coordinates of the vertex."

EXAMPLE 2 Enrollment at Two-Year Colleges

Figure 2.53 shows the fall enrollments (in millions) at two-year colleges for selected years in the period 1986–1996. (Source: National Center for Educational Statistics.)

FIGURE 2.53



- Produce a quadratic regression equation to model the given data.
- According to your regression equation, in what year did enrollment peak?

SOLUTION

- We enter ordered pairs of the form (year, enrollment) in the statistics list. For convenience, we let the first coordinate be the number of years since 1986:

$$(0, 4.7), (5, 5.7), (10, 5.5)$$

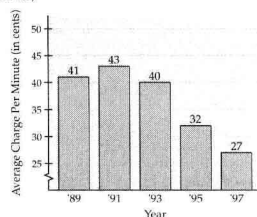
Then we select the quadratic regression option and obtain the equation $y = -0.024x^2 + 0.32x + 4.7$.

- Note that $a = -0.024$ and $b = 0.32$. Because $a < 0$, the parabola opens downward and the function has a maximum (peak) value at the vertex. The first coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{0.32}{2(-0.024)} = 6.6$$

Thus we estimate the peak year to be 7 years after 1986, or 1993.

- Cellular Phone Rates** In the early 1990s, only two cellular telephone carriers were allowed in each U.S. city. As shown in the bar graph, deregulation has increased competition and lowered rates. (Source: Herschel Shostack Associates.)



Applications

We have included numerous real-life and real-data applications, with source acknowledgments. These include a wide variety of subject areas and are designed to promote a greater understanding of the relevancy of mathematics.

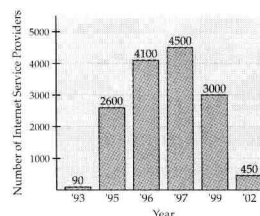
Some sections are devoted exclusively to examples and exercises involving real-life applications. Other sections contain dedicated blocks of such problems.

- If the data can be modeled by some quadratic function $f(x) = ax^2 + bx + c$, what is your prediction about the sign of a ?
- Just by examining the bar graph, estimate the vertex of the graph of f .
- Letting x represent the number of years since 1985, produce a quadratic regression equation of the data. Produce the graph of the equation and trace it to estimate the vertex. What do the coordinates of that point represent?

- Carbon Dioxide Emissions** In 1983, the atmosphere in the United States was polluted by 1.2 billion tons of carbon dioxide emissions. By 1995, the emissions had reached 1.45 billion tons. (Source: U.S. Department of Energy.) The goal set by environmentalists for the year 2010 is to reduce emissions to 1.38 billion tons.

- Letting x represent the number of years since 1980, write a quadratic regression equation to model the emissions.
- What emissions level does your model project for the year 2005?

- Internet Service Providers** The bar graph shows the number (and projected number) of Internet service providers through the year 2002. (Source: Gartner Group.)



- Use a scatterplot of the data to help you decide what type of model is reasonable.
- Let x represent the number of years since 1990 and determine a quadratic regression equation to model the data. (Round coefficients to the nearest integer.)

DISCOVERY LEARNING

Exploration

This pedagogical device provides opportunities for students to become active participants by exploring and discovering mathematical concepts. Classroom activities are accompanied by leading questions and, usually, the visual assistance of the graphing calculator. The student is guided from a concrete set of circumstances to a generalization. All the questions are open-ended, thereby providing maximum flexibility in the use of these activities as in-class discussion starters.

Developing the Concept

This is a parallel feature that immediately follows Exploration and that provides a similar scenario, but this feature also provides answers to leading questions. Because the features are separate and use slightly different scenarios, the Exploration is intact as a discovery learning exercise. The instructor can thus progress through the Exploration and Developing the Concept, or the Exploration can be bypassed entirely, with Developing the Concept serving as the sole instructional vehicle. Sometimes, Developing the Concept features appear by themselves to lead the reader to an understanding of the formula or theoretical concept.

WRITING AND SPEAKING IN MATHEMATICS

Increased emphasis has been placed on communicating in mathematics.

Speaking the Language

This feature precedes the exercise sets (except in sections that are dedicated exclusively to applications) and helps students think and communicate in the language of mathematics by reinforcing vocabulary and contextual meanings. Speaking the Language answers can be found in the Complete Solutions Guide.

Writing Exercises

The generous number of writing exercises are identified with a pencil icon. Although the answers to such writing exercises can often vary widely, a sample answer is given in the Answers section at the back of the book. Rather than assigning writing exercises, some instructors may prefer to use them as the basis of class discussion. Thus, writing exercises can promote both written and oral communication development.

DISCOVERY

Distortions of the Graph of $f(x) = \sqrt{x}$

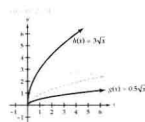
Consider functions of the form $y = c\sqrt{x}$.

- For several values of $c > 1$, produce the graph of the function in the same coordinate system with $f(x) = \sqrt{x}$. What appears to be the effect of c on the graph?
- Use several values of c , $0 < c < 1$, and repeat the experiment in part (a). What appears to be the effect of c on the graph?
- Are your observations in parts (a) and (b) consistent with the results for parabolas of the form $y = ax^2$?

DEVELOPING THE CONCEPT

Distortions of the Graph of $f(x) = \sqrt{x}$

Figure 2.70 shows some distortions (rescaled transformations) of the graph of the square root function, $f(x) = \sqrt{x}$.



Note that $g(x) = 0.5\sqrt{x} = 0.5 \cdot f(x)$ and that the graph of g rises more gradually than the graph of f . We also observe that $h(x) = 3\sqrt{x} = 3 \cdot f(x)$ and that the graph of h rises more rapidly than the graph of f .

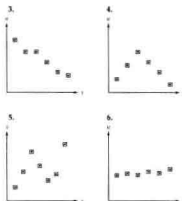
These observations support the following generalizations:

2.3 Speaking the Language

- A graph that consists of plotted points is called a(n) _____.
- Determining the line or curve that is the best fit of given data is called _____.
- The equation of a line that best fits linearly related data is called a linear equation.
- A numerical measure of how well an equation fits given data is called the _____.

2.3 EXERCISES

- Suppose that a scatterplot consists of three points that are not points of the same line. What is the maximum number of these points that the linear regression line could contain? Why?
- What do we mean when we say that data are linearly related?
- In Exercises 3–6, judge whether a linear model would be appropriate for the given scatterplot.
- (a) Write the equation of the line that contains the points $P(-2, 5)$ and $Q(6, -3)$.
- (b) Use your calculator to produce a linear regression equation for the line in part (a).
- (c) Compare your results in parts (a) and (b).
- (d) Why is $r = -1$ rather than -17 ?
- (a) Write the equation of the line that contains the points $A(0, 5)$ and $B(8, 20)$.
- (b) Use your calculator to produce a linear regression equation for the line in part (a).
- (c) Compare your results in parts (a) and (b).
- (d) Why is $r = -1$ rather than -17 ?



In Exercises 9 and 10, a table of values of a function is given.

- By examining the changes in the x -values and the corresponding changes in the y -values, decide whether the function could be linear. If so, what is the function?
- If you were to produce a linear regression equation, what would be the value of r ? How do you know?
- Verify your answers to parts (a) and (b) by producing the linear regression equation with your calculator.

9. x	0	5	10	15	20
y	4	11	18	25	32

10. x	-3.0	-1.0	3.0	11.0	27.0
y	3.6	-1.8	12.6	-34.2	-77.4

CALCULATOR FEATURES

Key Words

Appearing at the initial point of use, Key Words briefly describe a pertinent graphing calculator function and alert students to related detailed coverage, including keystrokes, found in Appendix B and the accompanying Graphing Calculator Guide supplement. Selected keys from the TI-83 Plus are found inside the front cover.

The sequence of Key Words in the main text matches the sequence of Key Word explanations in the Graphing Calculator Keystroke Guide.

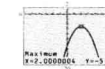
Graphs

Visualization is provided by a large number of traditional coordinate plane graphs and calculator graphs that appear in both the exposition and the exercises. Calculator graphs are designed to resemble the displays of most calculator models.

Maximum (Minimum) You can move your tracing cursor to the vicinity of the vertex of a displayed parabola and have your calculator display the coordinates of the vertex.

Figure 2.42 shows a calculator's estimate of the maximum of the function in Example 2(a).

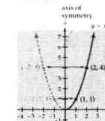
FIGURE 2.42



Because the second coordinate of a vertex represents the minimum or maximum value of a function, it can be used to determine the range of the function. For example, the vertex of the graph in Figure 2.42 is $(1/2, -5)$. Because the graph opens downward, the maximum value of the function is -5 . Thus the range of the function is $(-\infty, -5]$.

The second-quadrant portion of the graph of $y = -x^2$ is a mirror image of the first-quadrant portion. (See Figure 2.43.) We say that the graph is symmetric with respect to the y -axis. The line $x = 0$ (y -axis) is the **axis of symmetry** for the function $y = -x^2$.

FIGURE 2.43



• **NOTE** A vertical shift or a change in the width of a parabola does not change the axis of symmetry.

When a parabola is shifted to the left or right, the axis of symmetry is also shifted to the left or right. Thus the axis of symmetry of $y = a(x - h)^2 + k$ is the vertical line containing the vertex (h, k) , and its equation is $x = h$.

PEDAGOGICAL SUPPORT

The figure suggests that the constant term b is the second coordinate of the y -intercept of the line. We can show that this is true by using the fact that the first coordinate of a y -intercept is 0.

$$\begin{aligned} f(0) &= ax + b \\ f(0) &= a(0) + b \quad \text{For the } y\text{-intercept, } x = 0. \\ &= b \end{aligned}$$

Thus the y -intercept of the graph of $f(x) = ax + b$ is $(0, b)$.

Because the numbers a and b describe specific properties of a line, we give a special name to the equation $y = ax + b$.

The Slope-Intercept Model of an Equation of a Line
The slope-intercept model of an equation of a line is $y = ax + b$, where a is the slope of the line and $(0, b)$ is the y -intercept.

Use advantage of the slope-intercept model is that the slope and y -intercept of a line can be determined directly from the equation.

EXAMPLE 3 Determining Slope and y -Intercept from an Equation

Determine the slope and the y -intercept of each line whose equation is given.

- (a) $y = \frac{3}{5}x + 4$
(b) $2x + 3y = 6$

SOLUTION

- (a) The equation is in the slope-intercept form, where $a = \frac{3}{5}$ and $b = 4$. Thus the slope is $\frac{3}{5}$, and the y -intercept is $(0, 4)$.

- (b) $2x + 3y = 6$ Write the equation in the slope-intercept form.
 $3y = -2x + 6$ Subtract $2x$ from both sides.
 $y = -\frac{2}{3}x + 2$ Divide both sides by 3.

The slope is $-\frac{2}{3}$, and the y -intercept is $(0, 2)$.

If we know the slope and one point of a line, we can sketch the graph of the line.

Definitions, Properties, and Rules

All definitions, rules, properties, and procedures are highlighted in colored boxes for easy reference.

Examples/Solutions

All sections contain numerous, titled examples, many with multiple parts graded by difficulty. These Examples illustrate concepts, procedures, and techniques, and they reinforce the reasoning and critical thinking needed for problem solving. Detailed solutions include comments helpful to the student that justify the steps taken, explain their purpose, and identify properties and rules.

Notes (see sample page above)

Special remarks and cautionary notes offering additional insight appear throughout the text.

END-OF-SECTION FEATURES

A typical section ends with Quick Reference, Speaking the Language (see Writing and Speaking in Mathematics), and Section Exercises.

2.2 Quick Reference

- Graphs of Linear Functions**
- A linear function is a function that can be written in the form $f(x) = ax + b$, where a and b are real numbers and $x \in \mathbb{R}$. The domain and range are both \mathbb{R} . The graph is a line.
 - A constant function has the form $f(x) = b$. The domain is \mathbb{R} , and the range is $\{b\}$.
- Slope of a Line**
- The slope m of a line containing $P(x_1, y_1)$ and $Q(x_2, y_2)$ is $m = \frac{y_2 - y_1}{x_2 - x_1}$, $x_1 \neq x_2$.
 - If $m > 0$, the line rises from left to right.
 - If $m = 0$, the line falls from left to right.
 - If $m < 0$, the line is horizontal.
 - If m is undefined, the line is vertical.
 - For a linear function $f(x) = ax + b$, the coefficient of x is the slope.
 - To determine a y -intercept of a graph, replace x with 0 and solve for y .
 - To determine a x -intercept, replace y with 0 and solve for x .
 - For a linear function $f(x) = ax + b$, the constant term b is the y -intercept.
 - The slope-intercept model of an equation of a line is $y = mx + b$, where m is the slope of the line and b is the y -intercept.

Quick Reference

Quick Reference appears at the end of all sections except those dealing exclusively with applications. These detailed summaries of the important rules, properties, and procedures are grouped by subsection for a handy reference and review tool.

2.2 Exercises

Concepts and Skills

- Suppose that P and Q are points of a line. What do you know about the slope of the line if:
 - (a) the first coordinates of P and Q are the same?
 - (b) the second coordinates of P and Q are the same?
- The graph of a constant function $f(x) = b$ is a line. What is the slope of the line? Why?
- In Exercises 3–8, determine the slope of a line containing the given points.
 3. $(4, -1), (-2, 2)$
 4. $(-3, -1), (-1, 3)$
 5. $(-5, 3), (-2, 3)$
 6. $(1, 4), (1, -3)$
 7. $(5, -6), (3, -1)$
 8. $(4, -1), (7, -9)$
- In Exercises 9–12, determine the intercepts of the line.
 9. $y + 9 - 2x = 0$
 10. $x = b - y$
 11. $4x - y = 12$
 12. $2y - 5x = 10$

- In Exercises 13–18, write a linear function $f(x) = mx + b$ that is defined by the equation. Determine the slope and y -intercept of the graph of the equation.
13. $0 = x - y - 5$
 14. $8y = 4x + 7$
 15. $4x + 5y = 3$
 16. $10x + 25y = 2$
 17. $1 = 2y = 0$
 18. $2y + 3 = -b$
- In Exercises 19 and 20, use the same coordinate system to sketch the graphs of the lines containing the given point P with the given slope. Give the coordinates of two other points of each line.
19. $P(3, -2)$ (a) $m = 2$ (b) $m = -\frac{3}{2}$
 20. $P(1, 4, 3)$ (a) $m = 0$ (b) $m = -2$
- (c) Undefined slope
- (d) $m = -1$
21. Compute the ranges of a linear function and a constant function.

- In Exercises 27–30, write a linear function f that satisfies the given conditions. [Hint: If $f(a) = b$, then (a, b) is a point of the line.]
27. $f(4) = 3$, slope $\frac{1}{2}$
 28. $f(3) = -2$, slope -1
 29. $f(0) = -3$, slope 0
 30. $f(-2) = 1$, slope $-\frac{3}{2}$

- In Exercises 31–34, write an equation of the line containing the given points.
31. $(-4, 3), (1, 0)$
 32. $(5, 0), (0, -4)$
 33. $(-2, 7), (6, 7)$
 34. $(4, -2), (4, 1)$

- In Exercises 35–38, write a linear function f that satisfies the given conditions. [Hint: If $f(a) = b$, then (a, b) is a point of the line.]
35. $f(2) = -5, f(1) = 1$
 36. $f(0) = 0, f(3) = 7$
 37. $f(1.5) = -3, f(0.5) = -1$
 38. $f(1.2) = 7, f(1) = 3.5$

Concept Extension

39. Points are collinear if they are points of the same line.
- (a) Plot $P(-6, -5)$, $Q(0, 0)$, and $R(8, 4)$. Do the points appear to be collinear?
 - (b) By calculating slopes, determine whether the points are collinear.
40. Another model for a linear equation in two variables is $\frac{x}{a} + \frac{y}{b} = 1$. What properties of the graph of the equation are represented by a and b ?

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In Exercises 41–44, determine c such that the line containing the points has the given slope.

41. $(3, -1), (7, c)$, $m = 1$
 42. $(-4, 10), (c, 1)$, $m = -\frac{3}{2}$
 43. $(c, 0), (-4, 1)$, m is undefined
 44. $(2, c), (-4, 4)$, $m = 0$
- If g is a linear function, what is the relationship between the rate of change of g and the slope of the graph of g ?
45. If f is a constant function, what is the rate of change of f ? Why?

In Exercises 47–50, suppose that $P(a, b)$ is a point of the line with the given slope. For the given increase/decrease of one variable, determine the corresponding increase/decrease of the other variable.

47. $m = \frac{3}{4}$, b increased by 6
48. $m = \frac{2}{3}$, b increased by 4
49. $m = 2$, a decreased by 12
50. $m = -3$, b decreased by 12

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56. Suppose that $A(-4, 1)$ and $R(0, 3)$ are points of a line. Use the point-slope model and the two-point model in Exercise 35 to show that the same slope-intercept forms of the equations of the line are obtained with either model.

Applications

57. **Senior Citizen Population** In 1995 there were 1 million Americans who were 85 or older. That number is projected to reach 8.1 million by the year 2030. (Source: U.S. Bureau of Census.) Letting x represent the number of years since 1995, write a linear function to model the population of senior citizens.

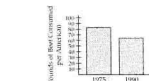


58. **Senior Citizen Population** In 1995 the percentage of the population who were 85 or older was 1.4%. That percentage is projected to reach 2.4% by the year 2030. (Source: U.S. Bureau of Census.) Letting x represent the number of years since 1995, write a linear function to model the percentage of the population who are senior citizens.



59. **Beef Consumption** In 1975 each American consumed an average of 83 pounds of beef annually. By 1990 the average had fallen to 64 pounds annually. (Source: U.S. Department of Agriculture.) Letting x represent the number of

years since 1970, write a linear function to model the average annual beef consumption.



60. **Coffee Consumption** From 1970 to 1995, the average number of pounds of coffee used per person declined from 13.6 pounds to 8.2 pounds. (Source: U.S. Department of Agriculture.) Letting x represent the number of years since 1970, write a linear function to model the average annual coffee consumption.



61. **Maintenance Agreement** For a fixed annual charge of \$300, a heating and air conditioning company will provide service to residential customers with no hourly charge. A second plan has an annual charge of \$100 and an hourly rate of \$40. Let x represent the number of service hours.

- (a) Write a function f for the annual cost under the first plan. What is such a function called?

- (b) Write a function g for the annual cost under the second plan. What is the significance of the slope and y -intercept of the graph of g ?

62. **Depreciation** An office copier had an original cost of \$4000. Using the "straight line" depreciation method, the accounting department valued the three-year-old copier at \$2400. Let x represent the age (in years) of the copier.

Exercises

Most exercises are divided by the subheadings Concepts and Skills, Concept Extension, and Applications. Sometimes the application exercises have subheadings to indicate the type of application that is included. Exercises included in Concepts and Skills are representative of the basic material in the section. Nearly all are illustrated by text examples.

The Concept Extension exercises are designed to go somewhat beyond the basic concepts and skills discussed in the text section. These exercises are designed to synthesize concepts and to use previously learned skills in a new context. The answers to the odd-numbered exercises are included at the back of the text.

END-OF-CHAPTER FEATURES

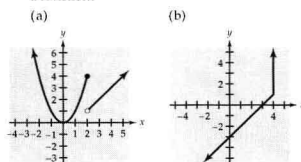
Review Exercises

These appear at the end of each chapter, organized by section. The answers to the odd-numbered exercises are included at the back of the text.

Chapter Review Exercises

Section 2.1

1. The graph of a relation is given. Is the relation a function?



In Exercises 2 and 3, determine whether the given equation defines a function.

2. $x^2 = \sqrt{y^2 - 25}$ 3. $|x^2 - 3| = y - 1$

In Exercises 4–6, evaluate the given function and simplify.

4. $f(x) = 2x - x^2$
(a) $f\left(\frac{1}{2}\right)$ (b) $f(-3)$ (c) $f(t - 1)$

5. $g(t) = \frac{t + 8}{t^2 - 16}$
(a) $g(0)$ (b) $g(4)$ (c) $g(2a)$

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6. $f(x) = \begin{cases} 2 - |x| & x \leq 4 \\ 3x - 10 & x > 4 \end{cases}$

(a) $f(-5)$ (b) $f(4)$ (c) $f(6)$

In Exercises 7 and 8, produce a graph of the given function. Use the graph to estimate the domain and range of the function.

7. $h(x) = |x - 8| - |x|$ 8. $g(x) = \sqrt[3]{x^2 - 8}$

In Exercises 9 and 10, determine the domain of the given function.

9. $r(x) = \frac{x + 2}{5x - x^2}$ 10. $s(t) = \sqrt{t^2 + 1}$

11. Which point(s) of the graph of f corresponds to the value(s) of x for which $f(x) = 0$?

12. Use a graph of $f(x) = \frac{x}{x - 2}$ to estimate the value of x so that (a) $f(x) = -4$ and (b) $f(x) = 2.25$.

Section 2.2

13. Determine the slope of the line containing $(7, -2)$ and $(5, 1)$.

14. Write the equation $2y + x = 14$ in slope-intercept form. What are the slope and y -intercept of the graph of the equation?

In Exercises 15 and 16, write an equation of the line that satisfies the given conditions.

15. Contains $A(1, -3)$, $m = -4$
16. Contains $P(-4, 2)$ and $Q(-4, 0)$

In Exercises 17 and 18, write a linear function satisfying the given conditions.

17. $f(0) = 0$, $m = -2$
18. $f(-3) = 6$, $f(-1) = 6$

19. Determine the intercepts of the line $3y - x = 15$.

20. Write a linear function that intersects the graph of $f(x) = \sqrt{x - 3}$ where $x = 4$ and $x = 12$.

21. Use graphs of linear functions to estimate the solutions of each equation or inequality. Then solve algebraically.

(a) $1 - 3x = x + 13$ (b) $1 - 3x < x + 3$

22. **Military Aircraft Losses** The number of military aircraft lost in crashes decreased from 132 in 1989 to 54 in 1997. (Source: Department of Defense.) Let t represent the number of years since 1985 and write a linear function $A(t)$ to model the data. According to the model, what was the number of aircraft losses in 1987?

Section 2.3

23. **Sport Utility Vehicles** The table shows the percentage of vehicles sold for over \$28,000 that were sport utility vehicles. (Source: Mercedes-Benz.)

Year	Percentage
1993	33%
1995	38%
1997	46%

Use a calculator to determine a linear regression equation to model the data. (Let t represent the number of years since 1990.) According to the model, in what year will the percentage exceed 60%?

24. Suppose that a table of values for a function is given. Explain how to decide if the function could be linear.

25. **Women's Earnings** The accompanying table shows women's annual earnings as a percentage of men's annual earnings for the period 1980–1992. (Source: Department of Labor, Women's Bureau.)

Year	Percentage
1980	60.2
1982	61.7
1984	63.7
1986	64.3
1988	66.0
1990	71.6
1992	70.6

- (a) Letting x represent the number of years since 1980, produce a linear regression equation to model the data.

- (b) Produce a scatterplot of the data and the graph of the equation in part (a). How well does the graph appear to fit the data?

- (c) Use the model to estimate the year in which women's and men's annual earnings are projected to be equal.

26. In Exercise 25, what is the correlation coefficient? What does this value indicate about the quality of the linear regression equation as a model of the data?

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