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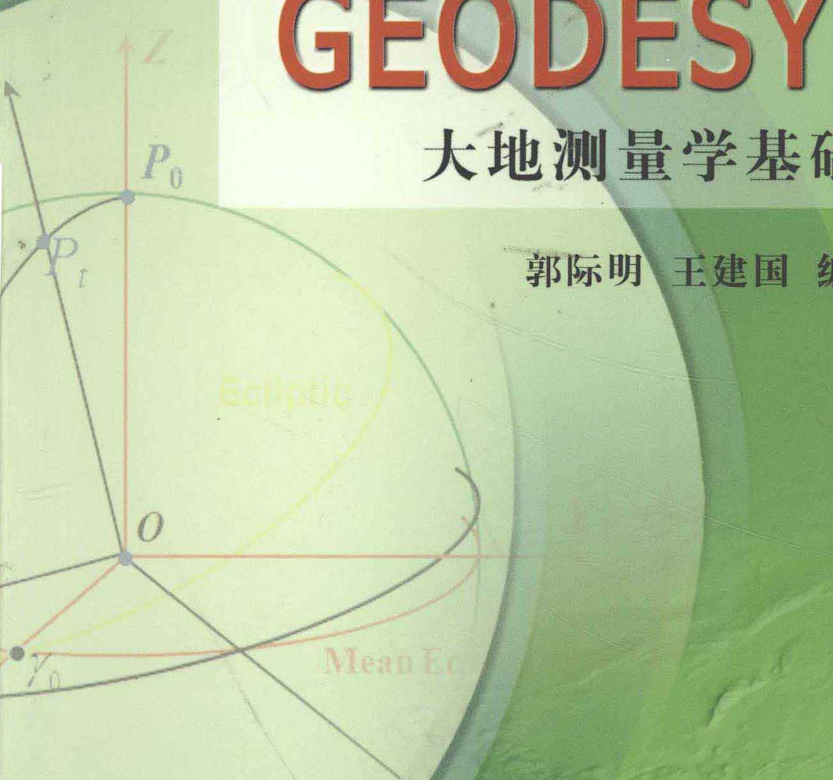
FOUNDATION OF GEODESY

大地测量学基础（英文版）



郭际明 王建国 编著

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Abstract

This book centers on the basic theory and techniques of geodesy. It starts with the definition and classification of geodesy and its historical development. Then, the Earth and its motions are described followed by the time systems and coordinate reference systems together with ellipsoid, geoid and geodetic datum in detail. As the third part, the gravity field related theory is presented as a help for data observation and reduction. Subsequently, geodetic calculations based on the ellipsoid are discussed such as the meridians, parallels, prime verticals, the reduction of the measurements, the geodesic differential equations, and the direct and inverse problem of geodesic. Map projections are then discussed inclusive of the geometric distortions caused by the projection and the mathematic method for Gauss-Krüger projection. The horizontal positioning techniques, vertical positioning techniques and GNSS three dimensional positioning techniques come after. As the last part, the horizontal and vertical geodetic control networks as well as ITRF series, IGS data and products are outlined.

Foreword

Foundation of Geodesy is one of the key courses for undergraduate students with geodesy and geomatics as major. This book is funded by the "Eleventh Five-Year Publishing Project" of the Ministry of Education of China. It is suitable as the textbook for undergraduate students or as a reference book for engineers in geodesy and geomatics.

Many people helped us compose this book, we hereby would like to gratefully acknowledge the contributions of Prof. Xu Aigong of Liaoning Technical University, Prof. Spiros Pagiatagis of York University, Prof. Kong Xiangyuan of Wuhan University, Prof. Liu Zongquan of Wuhan University.

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December 2010

Contents

Chapter 1 Introduction

Geodesy has a long history. It's changing with the improvement of the knowledge of human about the Earth and the development of the positioning technology. The definition, classification, task and historical development of geodesy are briefly introduced in this chapter.

1.1 Definition and Classification of Geodesy

Geodesy is the science of measuring and mapping the Earth's surface by F. Helmert (1843 - 1917). It is the scientific discipline that deals with the size, shape and gravitational field of the Earth, as well as positioning theory and technique on the Earth surface in a reference system. It is a branch of Earth sciences.

Geodesy may be divided into geometric geodesy, physical geodesy, and space geodesy. Geometric geodesy is the method to locate and map objects on the Earth surface. The national geodetic control network is set up based on geometric geodesy. Physical geodesy is responsible for the determination of the gravity field of the Earth. Space geodesy is about the theory and technology to observe the Earth with the help of satellite positioning system (e. g. GPS, GLONASS, Galelio, Compass), VLBI and SLR etc.

Geodesy plays a key role in Geomatics. The terrestrial reference frame in geodesy is the base for engineering surveying, marine surveying, photogrammetry, mapping and geographic information system (GIS).

Both of mathematics and computer science and engineering provide important support for geodesy. The geodetic calculation relies on geometry and algebra theory, and other mathematic branches while the network adjustments is based on Least Squares (LS) theory and statistics. Nowadays, the realization of the geodetic algorithms has closely connected to computer science and engineering, which has revolutionarily changed the way how to perform geodetic computation to process, store and manage the geospatial data. Internet also has its significant influence on geodesy. It makes the information exchange and communication easier than ever before. The traditional large scale geodetic control network has almost entirely been replaced by active control system through GPS technology. For example, the National Continuous Operating Reference Stations System using GNSS has become a regular form of geodetic control network in China, and many other countries worldwide.

1.2 Task of Geodesy

The major task of geodesy is to determine the size, shape and gravity field of the Earth as well as to locate and map the objects near the Earth surface.

Here the size, shape of the Earth, we mean the physical and the mathematical surface of the Earth. The physical surface of the Earth is the border between the solid or fluid masses and the atmosphere. The irregular surface of the solid Earth is incapable of being represented by a simple mathematical model. Therefore, it is described point wise by the use of coordinates of the geodetic control points. On the other hand, the ocean surface takes about 70% of the total Earth's surface and an equipotential (level) surface of the Earth's gravity field that best fit the mean sea level (MSL) in the ocean can be used to represent the figure of the Earth. This level surface is called geoid.

A defined mathematic model of the Earth is required for the computations in geometric geodesy. Because of its simple equation, a rotational ellipsoid flattened at the poles is better suited as a geodetic reference surface than the geoid. The optimal ellipsoid approximating the geoid is called as mean Earth ellipsoid. Figure 1.1 shows the relationship among the surfaces related to geodesy.

The body of the Earth and its gravity field are subject to temporal variations of secular, periodic, and singular nature, which can occur globally, regionally, and locally. The geodetic measurements and evaluation techniques can detect these changes. Geodesy contributes to the investigation of the dynamics of the terrestrial body. The figure of the Earth and the external gravity field are accordingly conceived as time dependent variables.

The objectives of geodesy can be summarized as follows:

(1) To determine the shape of the Earth and its external gravity field as well as their changes with time, to study the crust deformation, and to observe the polar motion and monitoring the ocean surface.

(2) To set up the geodetic coordinate system and to maintain the national horizontal geodetic control network and the vertical leveling network.

(3) To study the observation methods for geodetic instruments such as total stations, levels,

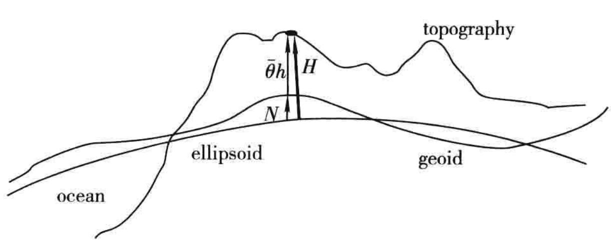

Figure 1.1 Earth's surface and reference surfaces

GPS, VLBI etc. and to perform the data processing for distances, directions, height differences, GPS baselines.

(4) To describe the mathematic models for the geodetic calculation on the Earth ellipsoidal surface and the projection from ellipsoid surface to plane.

1.3 Historical Development of Geodesy

The shape and size of the figure of the Earth had been raised in antiquity. We may classify the historical development of geodesy according to the knowledge of the figure of the Earth. There are three stages for the model of the Earth. They are spherical Earth model, Earth ellipsoid model, and Earth geoid model.

1.3.1 Spherical Earth Model

The spherical Earth model is solved by Eratosthenes (276 – 194 B. C.), an ancient Greek mathematician. He deduced from measurements a radius for the Earth under the assumption of a spherical Earth. The principle of the arc measurement method developed by him was still applied in modern ages (Figure 1.2). From geodetic measurements, the length ΔG of a meridian arc can be determined, and astronomical observations can furnish the associated central angle γ. The radius of the Earth is then given by

$$R = \frac{\Delta G}{\gamma} \tag{1.1}$$

Eratosthenes found that at the time of the summer solstice, the rays of the sun projected vertically into a well in Syene, whereas in Alexandria, roughly on the same meridian, they formed an angle with the direction of the plumb line. From the length of the shadow of a vertical staff produced in a hemispherical shell, he determined this angle as 1/50 of a complete circle, i. e. $\gamma = 7° 12'$. The distance from Syene to Alexandria was measured to be 5000 stadia using Egyptian step counters. With the length of an Egyptian stadium as 157. 5 m, the Earth radius can be obtained as 6267 km (Torge, 2001).

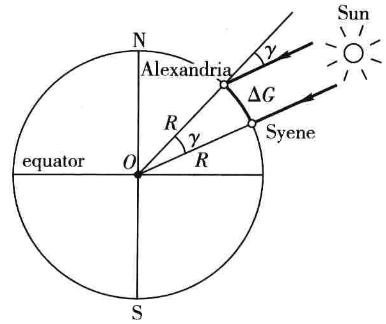

Figure 1.2 Arc measurement of Eratosthenes

1.3.2 Ellipsoidal Earth Model

Newton (1687) obtained a rotational Earth ellipsoid based on the assumption of as an equilibrium figure for a homogeneous, fluid, rotating Earth. The flattening α is defined as

$$\alpha = \frac{a - b}{a} \tag{1.2}$$

Here, a is the semimajor axis, b is the semiminor axis(Figure 1.3).

With the ellipsoidal Earth model, the arcs along a meridian with respect to the same angle will have their different lengths from the equator toward the pole. If more than two arcs are measured, the ellipsoidal parameters a and f can be solved.

The flattening f was also achieved by A. C. Clairaut (1713 - 1765) from the gravity measurements. Clairaut's theorem is noted as Equation (1.3), (1.4) and (1.5)

$$\gamma_\varphi = \gamma_e(1 + \beta \cdot \sin^2\varphi) \tag{1.3}$$

$$\beta = \frac{5}{2}q - f \tag{1.4}$$

$$q = \frac{\omega^2 a}{\gamma_e} \tag{1.5}$$

wherein γ_φ is the gravity at a point with latitude φ,β is called as gravity flattening, ω is the angular velocity of the Earth, a is the semimajor axis of the Earth, q is the ratio of the centrifugal force to the gravitation on the equator. The geometric flattening f could be determined from Equation (1.4) if γ_p, the gravity at the pole and γ_e, the gravity on the equator, are available to calculate the gravity flattening as follows

$$\beta = \frac{\gamma_p - \gamma_e}{\gamma_e}$$

1.3.3 Earth's Geoid Model

With the improvement in accuracy of measurements, the deviation of the physical plumb line from the ellipsoidal normal can no longer be ignored. In the field, the geodetic instruments are operated according to the plumb line, while the geodetic calculation is based on the ellipsoid surface and the normal. The raw measurements need to be corrected first for the vertical

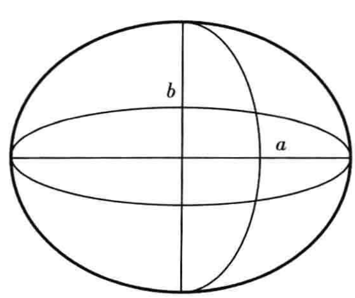

Figure 1.3 The Earth ellipsoid

Figure 1.4 The Geoid

deflection which is related to the geoid. With the development in Global Navigation Satellite System (GNSS), the geoid has increased with the transformation of geodetic height of GNSS to normal height again. Figure 1.4 shows the typical figure of a Geoid.

Review Questions

1. What is geodesy?
2. Classify the different geodetic areas.
3. Summarize the tasks in geodesy.
4. Why model the Earth as an ellipsoid?
5. Why a geoid is needed?

Chapter 2 Coordinate Systems and Time Systems

The coordinates system in geodesy is to provide a uniform geometric reference for the reduction of measurements and to denote the calculated result. As the Earth is changing all the time, the spatial position in geodesy is time dependent. Both the coordinate system and the time system are related to the motions of the earth.

2.1 Earth and Its Motions

It is known that the Earth undergoes the following kinds of motions simultaneously (Vanicek and Krakiwsky, 1986):

(1) It moves with our galaxy in respect to other galaxies.

(2) It circulates with the solar system within our galaxy.

(3) It revolves around the sun, together with other planets.

(4) It rotates (spins) around its instantaneous axis of rotation.

Of these motions, the first two are of importance to astronomers in studying galactic and intergalactic phenomena. For geodesy, we are generally interested in the last two motions. (3) and (4) are also called the annual motion and the diurnal motion, respectively. The related concepts include precession, nutation and polar motion.

2.1.1 Earth's Annual Motion

For the annual motion (Figure 2.1), the Earth and other celestial bodies can be regarded as point mass, compared with the dimensions of the solar system. Under these conditions, Kepler's three laws of planetary motion can be used to describe the orbit of the Earth around the Sun:

(1) The orbit of any planet is an ellipse and the sun stands in one of its foci.

(2) A planet moves along its orbit with a constant area velocity.

(3) The ratios of squares of orbital periods (T) of the planets are the same as the ratios of cubes of lengths of major semi-axes (a) of their orbits

$$T^2/a^3 = C \tag{2.1}$$

The plane of the Earth's orbit is called the ecliptic. After Kepler's second law, the Earth moves faster when it is closer to the Sun and slower when it is farther away. It completes one revolution in one sidereal year. In reality, the presence of other planets and the Moon influence the shape of the Earth's orbit, so it is not exactly elliptical and not even planar. These perturbations are, however, very small compared with the orbit's dimensions and, for many

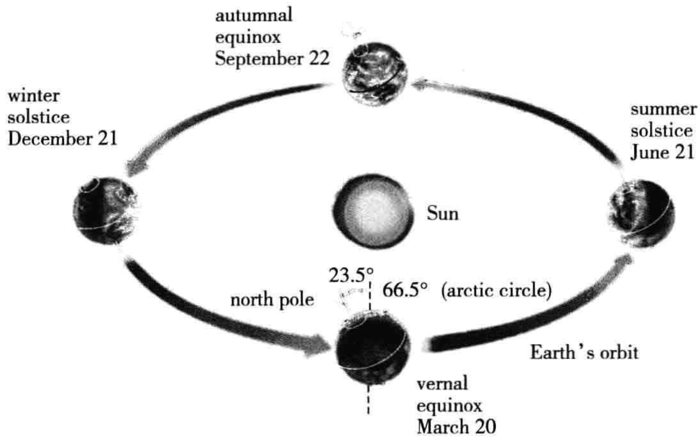

Figure 2.1 Annual motion of the Earth

practical purposes, can be neglected.

2.1.2 Earth's Precession and Nutation

The direction of the spin axis of the Earth is changing in space due to the force applied on the equatorial bulge of the Earth by the Sun and the Moon. The total motion is composed of a mean secular component (precession) and a periodic component (nutation) (Figure 2.2). Precession is the motion of the spin axis of the Earth around the ecliptic north pole clockwise which forms a cone in a period of 25800 years. The cone angle is about 23.5°. The nutation has a period of 18.6 years, a maximum amplitude of 9.21″ and a minimum amplitude 6.86″.

When only the influence of precession is considered, the position and orientation of the equatorial plane are called mean equator and the correspondent equinox is the mean equinox. When nutation is taken into account, they are called true equator and true equinox. The mean equinox at reference epoch T_0 is denoted as γ_0, the mean equinox at time t is denoted as γ_t, the true equinox at time t is denoted as γ_i.

The precession can be divided into three components: ζ, θ, z (Figure 2.3), which can be calculated by Equation (2.2) for IAU2000A (IERS Technical Note No. 32).

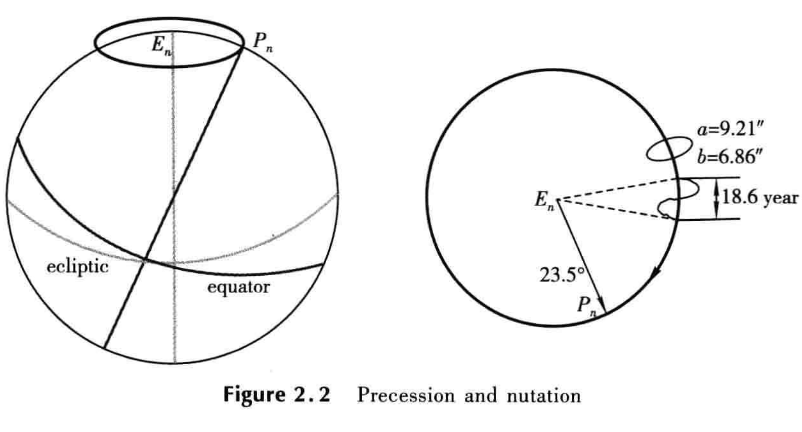

Figure 2.2 Precession and nutation

$$\zeta = 2.5976176'' + 2306.0809506''t + 0.3019015''t^2 + 0.0179663''t^3 - 0.0000327''t^4 - 0.0000002''t^5$$

$$\theta = 2004.1917476''t - 0.4269353''t^2 - 0.0418251''t^3 - 0.0000601''t^4 - 0.0000001''t^5$$

$$z = 2.5976176'' + 2306.0803226''t + 1.094779t^2 + 0.0182273''t^3 + 0.000047''t^4 - 0.0000003''t^5 \quad (2.2)$$

where $t = TT - TT_0$ is counted in Julian centuries of 36525 days and TT_0 = Julian Date 2451545.0 (2000 - 01 - 01, 12:00:00).

The transformation from the mean equator and equinox to the instantaneous true equator and equinox for a given observation epoch is performed with the nutation components $\varepsilon, \Delta\psi, \varepsilon + \Delta\varepsilon$, wherein ε is the obliquity of the ecliptic, $\Delta\psi$ is the nutation in longitude (counted in the ecliptic) and $\Delta\varepsilon$ is the nutation in obliquity (Figure 2.4). The Equation for IAU 2000A is given in (2.3) (IERS Technical Note No. 32).

$$\varepsilon = \varepsilon_0 - 46.84024''t - 0.00059''t^2 + 0.001813''t^3$$

$$\Delta\psi = \sum_i [(A_i + A_i't)\sin V + (A_i'' + A_i'''t)\cos V] \quad (2.3)$$

$$\Delta\varepsilon = \sum_i [(B_i + B_i't)\cos V + (B_i'' + B_i'''t)\sin V]$$

wherein

$$V = \sum_{j=1}^{5} N_j F_j$$

$$\varepsilon_0 = 84381.448''$$

$A_i, A_i', A_i'', A_i''', B_i, B_i', B_i'', B_i'''$ are the known coefficients which can be obtained from ftp://tai.bipm.org/iers/convupdt/chapter5/tab5.3a.txt and ftp://tai.bipm.org/iers/convupdt/chapter5/tab5.3b.txt.

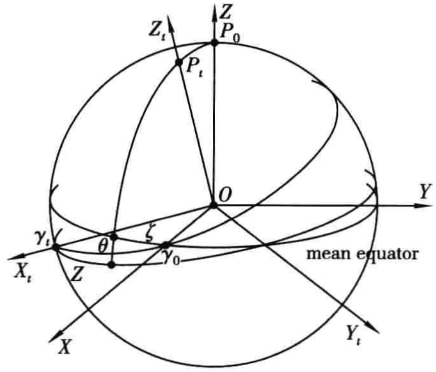

Figure 2.3 Precession components ζ, θ, z

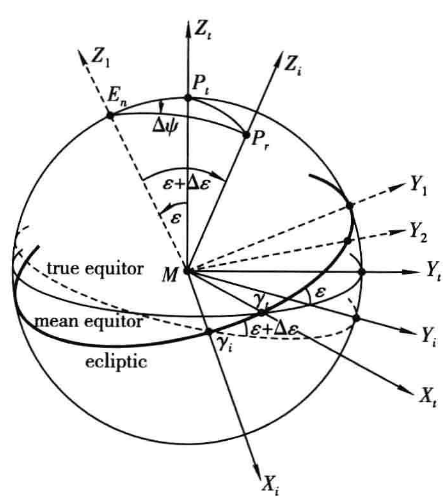

Figure 2.4 Nutation components

$N_j (j = 1, 2, 3, 4, 5)$ are the integer coefficients for F_j as defined below

$$
\begin{aligned}
F_1 = l &= \text{mean anomaly of the Moon} \\
&= 134.96340251° + 1717915923.2178''t + 31.8792''t^2 + 0.051635''t^3 - 0.00024470''t^4 \\
F_2 = l' &= \text{mean anomaly of the Sun} \\
&= 357.52910918° + 129596581.0481''t - 0.5532''t^2 + 0.000136''t^3 - 0.00001149''t^4 \\
F_3 = F &= L - \Omega \\
&= 93.27209062° + 1739527262.8478''t - 12.7512''t^2 - 0.001037''t^3 + 0.00000417''t^4 \\
F_4 = D &= \text{mean elongation of the Moon from the Sun} \\
&= 297.85019547° + 1602961601.2090''t - 6.3706''t^2 + 0.006593''t^3 - 0.00003169''t^4 \\
F_5 = \Omega &= \text{mean longitude of the lunar ascending node} \\
&= 125.04455501° - 6962890.2665''t + 7.4722''t^2 + 0.007702''t^3 - 0.00005939''t^4
\end{aligned}
\tag{2.4}
$$

where L is the mean longitude of the Moon.

2.1.3 Earth's Polar Motion

Polar motion of the earth is the movement of Earth's rotation axis across its surface. This is measured with respect to a reference frame in which the solid Earth is fixed (a so-called Earth-centered, Earth-fixed or ECEF reference frame). It consists of two quasi-periodic components and a gradual drift, mostly in the direction of the 80th meridian west, of the Earth's instantaneous rotational axis or North pole, from a conventionally defined reference axis, the CIO (Conventional International Origin). The offsets of the instantaneous North pole away from the reference pole can be described by a rectangular coordinates x (towards Greenwich) and y (towards 90 degree west), and the origin is at the reference pole corresponded to a conventional time epoch, shown in Figure 2.5.

The two periodic parts are a more or less circular motion called Chandler wobble with a period of about 435 days, and a yearly circular motion. There is also a slow drift which is less well known. These motions are illustrated in Figure 2.6.

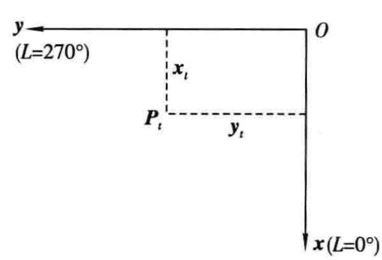

Figure 2.5 Polar coordinates

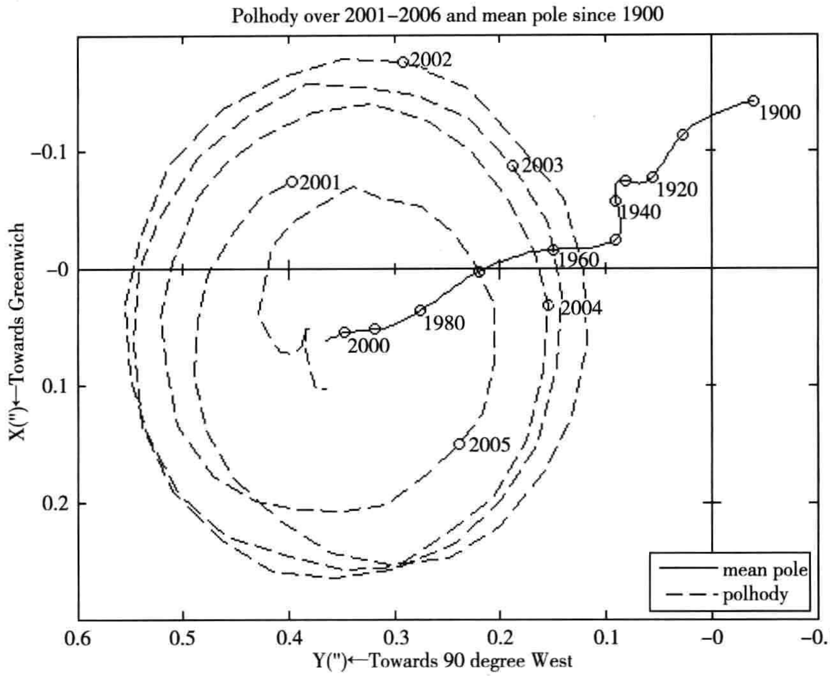

Figure 2.6 Polar motion, 2001 – 2006 (Solid line: mean pole displacement, 1900 – 2006).

2.2 Time Systems

The spatial position in geodesy is time dependent. Time scales and timing methods are of importance to geodesy. Generally, we have the following time definitions.

(1) Universal time (UT) is defined by the Earth spin period referenced to the Sun. It is counted from zero o'clock at midnight in the unit of the mean solar day, which is the interval between two transits of the fictitious sun through the meridian. UT0 is the rotational time of a particular place of observation. UT1 is computed by correcting UT0 for the effect of polar motion on the longitude of the observing site. UT2 is computed by correcting UT1 for annual and semiannual variations in the earth's rotation.

(2) Sidereal time (ST) is counted by the Earth spin period referenced to a point nearly fixed, normally the vernal equinox, with respect to the stars. Thus, it is defined as the hour angle of the vernal equinox. Greenwich mean sidereal time (GMST) is the hour angle of the average position of the vernal equinox, neglecting short term motions of the equinox due to nutation, referenced to the Greenwich mean meridian. Greenwich apparent sidereal time (GAST) is Greenwich mean sidereal time (GMST) corrected for the shift in the position of the vernal equinox due to nutation. Local mean sidereal time (LMST) is GMST plus the observer's longitude measured positive to the east of Greenwich. Local apparent sidereal time (LAST) is the hour angle of the true vernal equinox referenced to the local meridian.