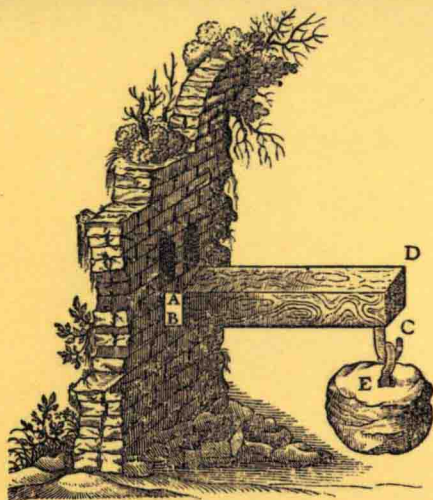


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A HISTORY OF  
THE THEORY  
OF ELASTICITY  
AND OF  
THE STRENGTH  
OF MATERIALS

VOLUME 2: PART 1  
SAINT-VENANT TO LORD KELVIN (1)

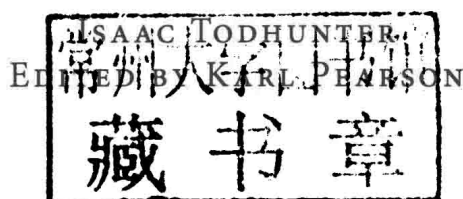
ISAAC TODHUNTER  
EDITED BY KARL PEARSON



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# A History of the Theory of Elasticity and of the Strength of Materials

VOLUME 2: PART 1  
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**CAMBRIDGE**  
UNIVERSITY PRESS

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## **A History of the Theory of Elasticity and of the Strength of Materials**

A distinguished mathematician and notable university teacher, Isaac Todhunter (1820–84) became known for the successful textbooks he produced as well as for a work ethic that was extraordinary, even by Victorian standards. A scholar who read all the major European languages, Todhunter was an open-minded man who admired George Boole and helped introduce the moral science examination at Cambridge. His many gifts enabled him to produce the histories of mathematical subjects which form his lasting memorial. First published between 1886 and 1893, the present work was the last of these. Edited and completed after Todhunter's death by Karl Pearson (1857–1936), another extraordinary man who pioneered modern statistics, these volumes trace the mathematical understanding of elasticity from the seventeenth to the late nineteenth century. Volume 2 (1893) was split into two parts. Part 1 includes the work of Saint-Venant from 1850 to 1886.

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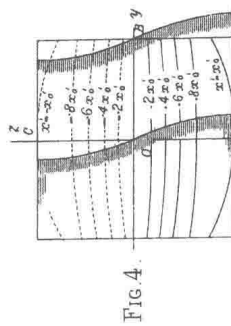
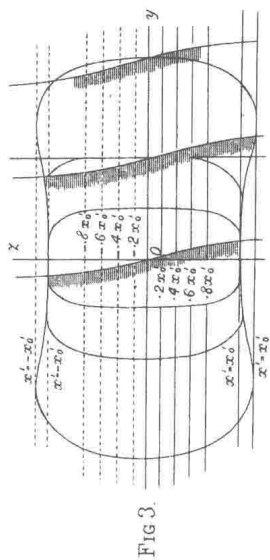
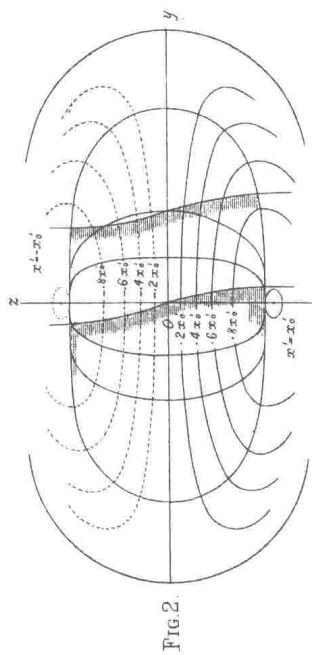
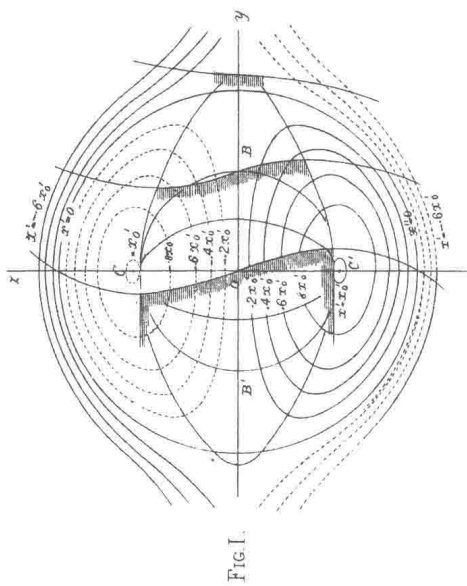
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# A HISTORY OF THE THEORY OF ELASTICITY

AND OF  
THE STRENGTH OF MATERIALS

*FROM GALILEI TO THE PRESENT TIME.*

BY THE LATE  
ISAAC TODHUNTER, D.Sc., F.R.S.

EDITED AND COMPLETED  
FOR THE SYNDICS OF THE UNIVERSITY PRESS

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VOL. II. SAINT-VENANT TO LORD KELVIN.  
PART I.

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1893.

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AT THE UNIVERSITY PRESS.

TO

LORD KELVIN P.R.S.

WHOSE RESEARCHES HAVE SO LARGELY  
CONTRIBUTED TO THE RECENT PROGRESS  
OF THE SCIENCE OF ELASTICITY  
THE EDITOR DEDICATES HIS LABOUR  
ON THE PRESENT VOLUME.

Man hat aber erst angefangen die Gesetze der Elasticität in ihrem ganzen Umfange zu studiren ; bei jedem Schritte stösst man in diesen Untersuchungen auf neue Eigenschaften der elastischen Körper ; je weiter man vorgeht desto mehr Verwickelung. Bei solchen Umständen ist wohl in diesem Augenblick keine völlig abgeschlossene Arbeit über irgend eine Eigenschaft der elastischen Körper möglich.

*Kupffer.*

I cannot doubt but that these things, which now seem to us so mysterious, will be no mysteries at all ; that the scales will fall from our eyes ; that we shall learn to look on things in a different way—when that which is now a difficulty will be the only common-sense and intelligible way of looking at the subject.

*Lord Kelvin.*

Works of this nature form, as it were, the principal fund of the science property of mankind, the *interest* of which we may turn to further profit. We might compare them to a capital invested in land. Like the soil, of which landed property consists, the knowledge stored up in these catalogues, lexicons, etc., may have but slender attractions for the vulgar, the man unacquainted with the subject can have no idea of the labour and cost at which the soil has been prepared ; the work of the husbandman appears to him terribly toilsome, tedious and clumsy. But although the work of the lexicographer and physical science cataloguer calls for the same painful and persevering industry as the labour of the husbandman, we must not therefore hastily assume that the work itself is of an inferior character, or that it is as dry and mechanical as it at first appears when we have the catalogue or lexicon ready printed before us. For it is necessary in such compilations that all the isolated facts should be selected by careful observation, and afterwards tested and compared with one another, the essential sifted from the unessential,—and all this it is plain, he only can efficiently accomplish who has clearly conceived the end and aim of his work, and the scope and method of the branch of science which it concerns ; but for such an one each minute detail will have its own peculiar interest from its position in relation to the whole science of which it is a part. Were it not so, such work would indeed be the worst kind of mental drudgery it were possible to conceive.

*von Helmholtz.*

## PREFACE.

NINE years have elapsed since the manuscript of the earlier part of this History was placed in my hands; seven years since the first volume was published<sup>1</sup>. Some words of apology are needful for this delay. Interest in my subject and a desire to complete without breach of continuity a work which I had commenced led me to persist in the task of editing even after I had recognised how little prompt execution of that task was compatible with the large demands which the work of a London teacher makes upon limited physical strength. Rapid and efficient fulfilment needed the single-hearted devotion of one to whom this History would have been the first and not a secondary duty. To complete the work, as I could have wished it completed, would have needed the undivided energies, the fresh and undisturbed intellectual power of several years' labour. As it is the Editor has failed to fulfil the promise made on the title-page and bring the History down "to the present time." The Second Volume carries the analysis of individual memoirs completely to the year 1860, but beyond that year the work of certain elasticians only has been dealt with up to the present date. These elasticians, however, —Saint-Venant and Boussinesq, Rankine and Lord Kelvin, F. Neumann, Kirchhoff and Clebsch—are those upon whose researches the modern science of elasticity rests. It may be safely

<sup>1</sup> Chapter X. of the present volume appeared in 1889 as an extract entitled: *The Elastic Researches of Barré de Saint-Venant*.

said that without a thorough study of their writings, it is impossible to be an accomplished elastician, or to follow without great difficulty the drift of modern elastical research. Their memoirs and treatises form the frame, which the Editor had hoped he might be able to fill up by briefer accounts of the discoveries due to, perhaps, less distinguished but none the less useful workers in the same field. This process of filling up is only completed for the years 1850-60, but the Editor ventures to think that the reader of his Chapter XI. will be surprised at the wealth of material, theoretical, technical and physical, which was brought to light in that decade. Many facts have been discovered, more, perhaps, rediscovered since 1860, but till the last few years it may be doubted whether any period has been more fruitful of genuine progress in the science of elasticity than these ten years.

The number of the memoirs included in this volume by no means measures the work of preparation it has involved. The study and analysis of many memoirs not included in its contents had to be undertaken. But the chief task has been the verification of the analysis of all the more important mathematical memoirs. In some cases the whole of this analysis has been undertaken *de novo*, occasionally with different results. As examples of this I may cite Resal's researches on the figure of the earth, the whole of Winkler's work on the strained form of the links of chains, and Lord Kelvin's analysis of the strains produced by the tides in an elastic earth. In all the work of verification, not only of others' analysis but of my own, I have had the most self-sacrificing and devoted assistance from Mr. C. Chree of King's College, Cambridge. Without his aid not only would this volume have been much longer delayed, but I veritably shudder to think of the blunders which would certainly have escaped my unaided revision. My thanks are due to him, not as to a mere friendly proof-reader, but as to one whose cooperation in the task of editing has given the volume the major portion of any freedom from error it may possess. I trust that many serious

errors may not still remain to be found, but in a work of reference like the present errors and misinterpretations of a writer's meaning are sure to occur. I can only hope that my criticisms, especially when they deal with the work of living men of science, will be received in the spirit in which they were written; namely, in that spirit the sole motive of which is the impersonal one of attaining truth and eliminating error. A somewhat lengthy list of additions and corrections to the first volume is issued with this, and I should be glad of any suggestions or emendations of the present volume which my readers may care to send me, and which might be issued with later copies<sup>1</sup>.

Of others besides Mr Chree who have helped me in the work of revision, I must refer in the first place to M. Flamant, Professeur à l'École des Ponts et Chaussées, whose help especially in the chapters devoted to Saint-Venant and Boussinesq has been very considerable. To my colleagues Professors G. Carey Foster and T. G. Bonney, and to Mr W. H. Macaulay of King's College, Cambridge, I am indebted for assistance in special points. To Lord Kelvin I owe a number of corrections in Chapter XIV. In several instances I had misunderstood or misinterpreted passages in his papers. He has enabled me to express something of the gratitude which I among other elasticians feel to him for his contributions to our science, by accepting the dedication of the present volume.

The editorial preponderance in this volume—the articles due to Dr Todhunter<sup>2</sup> are practically confined to a few dealing with

<sup>1</sup> Mr A. E. H. Love in his *Treatise on the Theory of Elasticity*, Vol. I. § 107, refers to certain terms in Saint-Venant's theory of flexure which are discussed in Art. 96 of the present volume as expressing only a "rigid-body rotation" and states that they "need not therefore be considered." It seems to have escaped Mr Love that Saint-Venant's theory allows for what experimentally is easily demonstrated to exist, namely, a small but finite change of direction in the central line of a bar under flexure either at a section where a load is applied or at a built-in end. The terms referred to do not therefore correspond to a "rigid-body rotation," and the deflections as given by Mr Love are really measured from a line, *i.e.* the tangent at a load or at the built-in end, the position of which he has not determined.

<sup>2</sup> Articles due to the Editor have their numbers enclosed in square brackets.



Clebsch's *Treatise*—arises chiefly from two causes. In the first place Dr Todhunter omitted all memoirs dealing with the physical or technical branches of our subject, and more than a third of the present volume will be found to deal with physical or technical problems. In the second place a still larger portion of the work falls beyond the period to which Dr Todhunter had carried his researches. On this point I may, perhaps, be permitted to refer to the remarks I have made in the preface to *The Elastical Researches of Barré de Saint-Venant*, and content myself here with citing from them the following words :

...it has seemed to me that the best memorial to the first Cambridge historian of mathematics would be that the last history bearing his name should have the widest possible sphere of usefulness. That usefulness will, I am firmly convinced, be best obtained by its comprehensive character, by its attempt to be a *Repertorium* of elasticity rather than an *Historique Abrégé* of its purely mathematical side.

For the Index to the present volume I alone am responsible. In a work of this comprehensive character a complete and systematic index is a first necessity. To prepare it is a duty which experience has taught me no one can fulfil so efficiently as the writer of a book.

Lastly, I have to express the great sense of the indebtedness I feel to the Syndics of the University Press for the patience with which they have submitted to the delay in the publication of this History, and the kindness with which they have permitted these volumes to grow so much beyond my original estimate. Should the reader complain that the work after all remains a fragment, then the blame must fall on the shoulders of the Editor, who much underestimated the extent of his material and overestimated his own powers, when he reported to the Syndics nine years ago on the original manuscript.

KARL PEARSON.

UNIVERSITY COLLEGE, LONDON.

June 7, 1893.