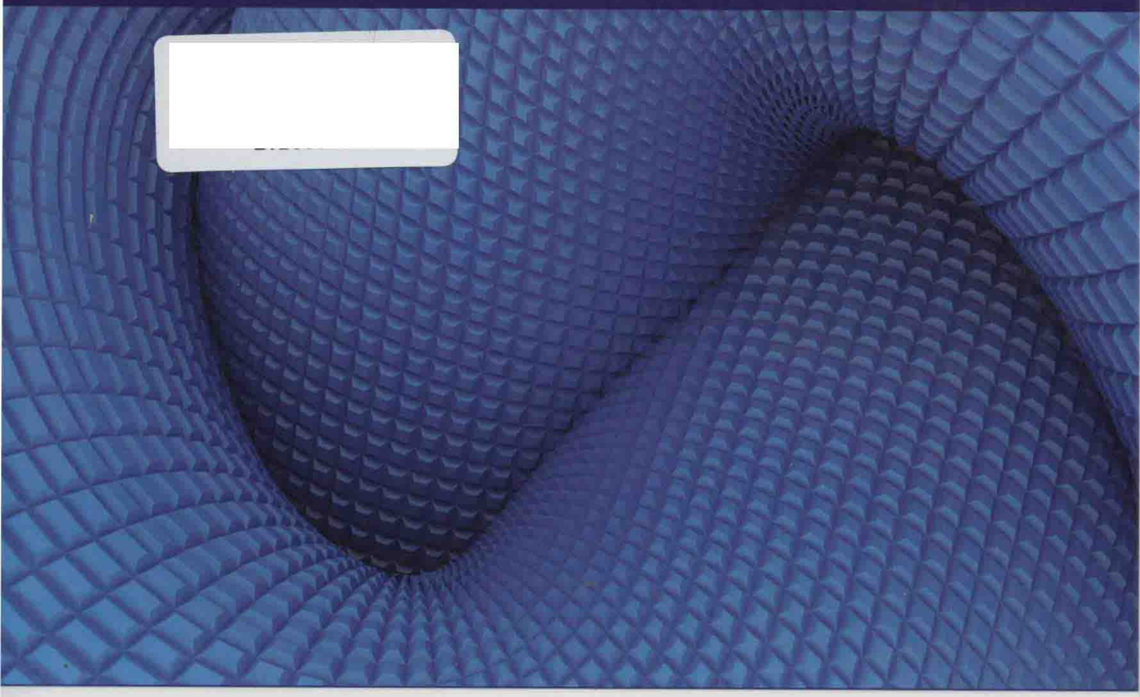

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STRUCTURES ON COMPLEX MANIFOLDS

DEDICATED TO MY BELOVED PARENTS

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PREFACE

There are two types of well-known structures on manifolds, namely almost Hermitian structures and almost contact metric structures. Kaehler, nearly Kaehler, complex space forms, generalized complex space forms are certain particular cases of almost Hermitian manifolds. On the other hand, Sasakian and Kenmotsu structures are two well-known classes of almost contact metric structures on a manifold.

The theory of submanifolds of manifolds equipped with certain structures, such as Kaehler, nearly Kaehler, quaternion is one of the most interesting topics in differential geometry. According to the behavior of the tangent bundle of a submanifold under the action of $(1, 1)$ structure tensor field of the ambient manifold, there are two well-known classes of submanifolds, namely invariant submanifolds and anti-invariant submanifolds. In the first case, the tangent space of the submanifold remains invariant under the action of the $(1, 1)$ structure tensor field, whereas in the second case, it is mapped into the normal space.

The study of differential geometry of CR-submanifolds as a generalization of invariant and anti-invariant submanifolds of an almost Hermitian manifold was initiated by A. Bejancu and was followed by several geometers. CR-submanifolds have good interactions with other parts of mathematics and substantial applications in (pseudo) conformal mappings and relativity.

The present book STRUCTURES ON COMPLEX MANIFOLDS is the outcome of the research which was carried out by author towards the fulfillment for the award of D. Phil. Degree from Department of Mathematics, University of Allahabad, Allahabad. All the Chapters (except IV and VIII) of this book are the part of my thesis entitled, A STUDY OF GEOMETRY OF COMPLEX MANIFOLDS AND THEIR SUBMANIFOLDS.

This book is divided into eight chapters and each chapter contains several sections. The decimal notation has been employed for numbering the equations. References to the equations, definitions and theorems are of the form (C.S.E), where C, S and E stand for the corresponding chapter, section and equation respectively. The numbers in square bracket [ ] refer to the references given at the end.

The first chapter is introductory in nature and opens with brief account of differentiable manifolds, differentiable functions, tangent vectors, vector fields, linear connections, Riemannian manifolds, almost complex manifolds, almost Hermitian manifolds, Kaehler and nearly Kaehler manifolds, submanifolds, generalized complex space forms, quaternion space forms.

In 1990, B.Y. Chen introduced the notion of slant submanifolds as a generalization of invariant and anti-invariant submanifolds of an almost Hermitian manifold. One of the basic problems in submanifold theory is to find simple relationships between the main extrinsic invariants and the main intrinsic invariants of submanifolds. Chen established relationship between Ricci curvature and squared mean curvature for submanifolds in Riemannian space form. In chapter-II, the inequalities between the Ricci curvature and the squared mean curvature and also between k-Ricci curvature and scalar curvature for slant, bi-slant and semi-slant submanifolds in generalized complex space forms are established.

The Riemannian invariants of a Riemannian manifold are the intrinsic characteristics of the Rimannian manifold. In 1990, B.Y. Chen established some inequalities involving an invariant, called Chen invariant and squared mean curvature. Similar inequalities for slant submanifolds in complex space forms have been studied by A. Oiaga and I. Mihai. Several geometers [41], [43] established contact version of these inequalities. The chapter-III deals with B.Y. Chen inequalities for bi-slant submanifolds in generalized complex space forms.

In 1974, S. Ishihara introduced the notion of quaternion Kaehler manifold. Afterwards, M. Barros, B.Y. Chen and F. Urbano studied quaternion CR-submanifolds of quaternion Kaehler manifolds. The chapter-IV deals with totally real warped product submanifolds in generelized complex space forms. Here, we establish an inequality between the warping function and the squared mean curvature for totally real warped product submanifolds in generalized complex space forms.

The chapter-V is devoted to obtain sharp estimate of the Ricci curvature of quaternion slant, bi-slant and semi-slant submanifolds in a quaternion space form, in terms of the squared mean curvature. Invariant and anti-invariant submanifolds are base submanifolds which have been extensively investigated in the last decades from different points of view. In 2001, B.Y. Chen introduced a series of Kaehlerian invariants and also discussed their applications in Kaehlerian geometry. The chapter-VI deals with some inequalities involving these invariants for invariant submanifolds in quaternion space forms.

CR-submanifolds are the generalization of invariant and anti-invariant submanifolds. In 1995, I. Mihai introduced the notion of generalized CR-submanifolds of Kaehler manifold. This class contains both CR-submanifolds and slant submanifolds.The chapter-VII is concerned with the study of generalized CR-submanifolds of nearly Kaehler manifolds. The integrability conditions of the distributions involved in the definition of such submanifolds are investigated. Geometry of leaves of these distributions are also studied. The chapter-VIII tells about the study of semi-slant submanifolds of Kaehler product manifolds where we find some basic results related to the geometry of semi-slant submanifolds of Kaehler product manifolds.

Finally, I would like express my sincere gratitude to my supervisor Dr. S.S. Shukla for his ture guidence during research. I am grateful to Prof. Ramji Lal, Prof. B. Rai, Prof. M. Tiwari and Prof. K.K. Azad, Prof. P.N. Pandey and all the teachers, research colleagues and the whole staff of the Department of Mathematics, University of Allahabad, Allahabad.

Lastly but heartly I am indebt to my parents, elder sisters Suman Rao, Rekha Rao, sweet younger sister Poonam Rao, lovely brothers Pankaj Rao, Alok Rao and Virendra Rao whose patience and enlightenment have given the way to complete this work.

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CHAPTER-I

INTRODUCTION

1.1. Differentiable Manifolds

A locally Euclidean space of dimension n is a Hausdorff topological space for which each point has a neighbourhood homeomorphic to an open subset of $\mathbb{R}^n$, where $\mathbb{R}^n$ is an n-dimensional Euclidean space. Roughly speaking, a differentiable manifold is a topological space with a structure which helps us to define differentiable functions on it, just as a topological structure on a set is designed to define continuous functions on that set. We can also look a differentiable manifold as a Hausdorff topological space which can be obtained by gluing together open subsets of some Euclidean space in a nice way, for example, the surface of a ball or a torus covered with small paper disks pasted together on overlaps without making any crease or fold.

Let M be a Hausdorff topological space. A local chart on M is a pair (U, ϕ), where U is an open subset of M and ϕ is a homeomorphism from U onto an open subset of $\mathbb{R}^n$. If $p \in U$ and if we denote by $x^i(p) = u^i(\phi(p))$, that is, the i^{th} coordinate of the point $\phi(p) \in \mathbb{R}^n$, then the numbers $x^i(p)$ are called local coordinates of p. Thus a local chart (U, ϕ) on M gives local coordinates of each point $p \in U$. So, a local chart (U, ϕ) on M is also called a local coordinate system on M.

An n-dimensional differentiable structure on M is a collection of local charts $\{(U_\alpha, \phi_\alpha) : \alpha \in \wedge\}$ such that

(i) $\cup_{\alpha\in\wedge} U_\alpha = M$;

(ii) $\phi_\alpha \circ \phi_\beta^{-1}$ are C^∞-functions, $\forall\ \alpha,\ \beta \in \wedge$;

(iii) The collection is maximal with respect to (i) and (ii) i.e., if (U, ϕ) is a local chart on M such that $\phi \circ \phi_\alpha^{-1}$ and $\phi_\alpha \circ \phi^{-1}$ are C^∞-maps, $\forall\ \alpha \in \wedge$, then (U, ϕ) must lie in above collection.

An n-dimensional differentiable manifold (or C^∞-manifold) of dimension n is a Hausdorff topological space together with a differentiable structure of dimension n.

1.2. Differentiable Functions

Let G be an open subset of M, then a function $f : G \to \mathbb{R}$ is said to be differentiable or C^∞ if $f \circ \phi^{-1}$ is C^∞ on $\phi(G \cap U)$ for every local chart (U, ϕ) on M.

Let M be a smooth n-dimensional manifold. Let a point $p \in M$ and $C^\infty(p)$ denote the set of all real valued functions that are C^∞ in some neighbourhood of p.

A tangent vector X_p at a point $p \in M$ is a mapping

$$X : C^\infty(p) \to \mathbb{R} \text{ such that}$$

(i) $X_p f \in \mathbb{R}, \quad \forall\ f \in C^\infty(p)$;

(ii)$X_p(\alpha f + \beta g) = \alpha(X_p f) + \beta(X_p g), \quad \alpha,\ \beta \in \mathbb{R}, \quad f, g \in C^\infty(p)$;

(iii)$X_p(fg) = g(p)(X_p f) + f(p)(X_p g)$.