Introduction to Particle Technology

# Introduction to Particle Technology

**Martin Rhodes** 

Monash University, Australia

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## Introduction to Particle Technology

### **Preface**

#### Particle Technology

Particle technology is a term used to refer to the science and technology related to the handling and processing of particles and powders. Particle technology is also often described as powder technology, particle science and powder science. Powders and particles are commonly referred to as bulk solids, particulate solids and granular solids. Today particle technology includes the study of liquid drops, emulsions and bubbles as well as solid particles. In this book only solid particles are covered and the terms particles, powder and particulate solids will be used interchangeably.

The discipline of particle technology now includes topics as diverse as the formation of aerosols and the design of bucket elevators, crystallisation and pneumatic transport, slurry filtration and silo design. A knowledge of particle technology may be used in the oil industry to design the catalytic cracking reactor which produces gasoline from oil or it may be used in forensic science to link the accused with the scene of the crime. Ignorance of particle technology may result in lost production, poor product quality, risk to health, dust explosion or storage silo collapse.

### Objective

The objective of this textbook is to introduce the subject of particle technology to students studying degree courses in disciplines requiring knowledge of the processing and handling of particles and powders. Although the primary target readership is amongst students of chemical engineering, the material included should form the basis of courses on particle technology for students studying other disciplines including mechanical engineering, civil engineering, applied chemistry, pharmaceutics, metallurgy and minerals engineering.

A number of key topics in particle technology are studied giving the fundamental science involved and linking this, wherever possible, to industrial practice. The coverage of each topic is intended to be exemplary rather than

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exhaustive. This is not intended to be a text on unit operations in powder technology for chemical engineers. Readers wishing to know more about the industrial practice and equipment for handling and processing are referred to the various handbooks of powder technology which are available.

The topics included have been selected to give coverage of broad areas within particle technology: characterisation (size analysis), processing (fluidized beds, granulation), particle formation (granulation, size reduction), fluidparticle-separation (filtration, settling, gas cyclones), safety (dust explosions), transport (pneumatic transport and standpipes). The health hazards of fine particles or dusts are not covered. This is not to suggest in any way that this topic is less important than others. It is omitted because of a lack of space and because the health hazards associated with dusts are dealt with competently in the many texts on Industrial or Occupational Hygiene which are now available. Students need to be aware however, that even chemically inert dusts or "nuisance dust" can be a major health hazard. Particularly where products contain a significant proportion of particles under 10 µm and where there is a possibility of the material becoming airborne during handling and processing. The engineering approach to the health hazard of fine powders should be strategic wherever possible; aiming to reduce dustiness by agglomeration, to design equipment for containment of material and to minimise exposure of workers.

The topics included demonstrate how the behaviour of powders is often quite different from the behaviour of liquids and gases. Behaviour of particulate solids may be surprising and often counter-intuitive when intuition is based on our experience with fluids. The following are examples of this kind of behaviour:

When a steel ball is placed at the bottom of a container of sand and the container is vibrated in a vertical plane, the steel ball will rise to the surface.

A steel ball resting on the surface of a bed of sand will sink swiftly if air is passed upward through the sand causing it to become fluidized.

Stirring a mixture of two free-flowing powders of different sizes may result in segregation rather than improved mixture quality.

Engineers and scientist are used to dealing with liquids and gases whose properties can be readily measured, tabulated and even calculated. The boiling point of pure benzene at one atmosphere pressure can be safely relied upon to remain at 80.1°C. The viscosity of water at 20°C can be confidently predicted to be 0.001 Pas. The thermal conductivity of copper at 100°C is 377 W/m.K. With particulate solids, the picture is quite different. The flow properties of sodium bicarbonate powder, for example, depends not only on the particle size distribution, the particle shape and surface properties, but also on the humidity of the atmosphere and the state of compaction of the powder. These variables are not easy to characterise and so their influence on the flow properties is difficult to predict with any confidence.

In the case of particulate solids it is almost always necessary to rely on performing appropriate measurements on the actual powder in question rather than relying on tabulated data. The measurements made are generally measurements of bulk properties, such as shear stress, bulk density, rather than measurements of fundamental properties such as particle size, shape and density. Although this is the present situation, in the not too distant future, we will be able to rely on sophisticated computer models for simulation of particulate systems. Mathematical modelling of particulate solids behaviour is a rapidly developing area of research around the world, and with increased computing power and better visualisation software, we will soon be able to link fundamental particle properties directly to bulk powder behaviour. It will even be possible to predict, from first principles, the influence of the presence of gases and liquids within the powder or to incorporate chemical reaction.

Particle technology is a fertile area for research. Many phenomena are still unexplained and design procedures rely heavily on past experience rather than on fundamental understanding. This situation presents exciting challenges to researchers from a wide range of scientific and engineering disciplines around the world. Many research groups have web sites which are interesting and informative at levels ranging from primary schools to serious researchers. Students are encouraged to visit these sites to find out more about particle technology. Our own web site at Monash University can be accessed via the Chemical Engineering Department web page at

http://www.eng.monash.edu.au/chemeng/

Martin Rhodes, Mount Eliza, May 1998

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### 1

### Single Particles in a Fluid

This chapter deals with the motion of single solid particles in fluids. The objective here is to develop an understanding of the forces resisting the motion of any such particle and provide methods for the estimation of the steady velocity of the particle relative to the fluid. The subject matter of the chapter will be used in subsequent chapters on the behaviour of suspensions of particles in a fluid, fluidization, gas cyclones and pneumatic transport.

#### 1.1 MOTION OF SOLID PARTICLES IN A FLUID

The drag force resisting very slow steady relative motion (creeping motion) between a rigid sphere of diameter x and a fluid of infinite extent of viscosity  $\mu$  is composed of two components (Stokes, 1851):

a pressure drag force, 
$$F_p = 2\pi x \mu U$$
 (1.1)

a shear stress drag force, 
$$F_s = \pi x \mu U$$
 (1.2)

Total drag force resisting motion, 
$$F_D = 3\pi x \mu U$$
 (1.3)

where *U* is the relative velocity.

This is known as Stokes' law. Experimentally, Stokes' law is found to hold almost exactly for single particle Reynolds number,  $Re_p \le 0.1$ , within 1% for  $Re_p \le 0.3$ , within 3% for  $Re_p \le 0.5$  and within 9% for  $Re_p \le 1.0$ , where the single particle Reynolds number is defined in Equation (1.4).

Single particle Reynolds number, 
$$Re_p = xU\rho_f/\mu$$
 (1.4)

A drag coefficient, 
$$C_D$$
 is defined as  $C_D = R'/(\frac{1}{2}\rho_f U^2)$  (1.5)

where R' is the force per unit projected area of the particle.

Thus, for a sphere: 
$$R' = F_D / \left(\frac{\pi x^2}{4}\right)$$
 (1.6)

and, Stokes' law, in terms of this drag coefficient, becomes:

$$C_{\rm D} = 24/Re_{\rm p} \tag{1.7}$$

At higher relative velocities, the inertia of the fluid begins to dominate (the fluid must accelerate out of the way of the particle). Analytical solution of the Navier–Stokes equations is not possible under these conditions. However, experiments give the relationship between the drag coefficient and the particle Reynolds number in the form of the so-called standard drag curve (Figure 1.1). Four regions are identified; the Stokes' law region, the Newton's law region in which drag coefficient is independent of Reynolds number, an intermediate region between the Stokes and Newton regions and the boundary layer separation region. The Reynolds number ranges and drag coefficient correlations for these regions are given in Table 1.1.

The expression given for  $C_D$  in the intermediate region in Table 1.1 is that of Dallavalle (1948). An alternative is that of Schiller and Naumann (1933) (Equa-

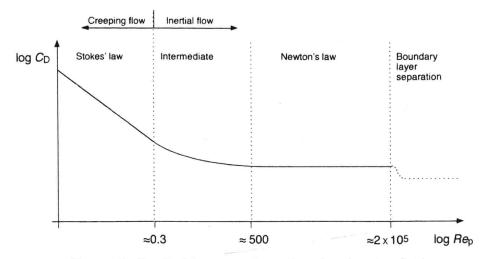


Figure 1.1 Standard drag curve for motion of a sphere in a fluid

Table 1.1 Reynolds number ranges for single particle drag coefficient correlations

Region	Stokes	Intermediate	Newton's Law
Rep range	< 0.3	$0.3 < Re_{\rm p} < 500$	$500 < Re_p > 2 \times 10^5$
$C_{\mathrm{D}}$	$24/Re_{\rm p}$	$\approx 24/Re_p + 0.44$	$\approx 0.44$

tion (1.8)), which fits the data with an accuracy of around  $\pm 7\%$  in the intermediate range.

$$C_{\rm D} = \frac{24}{Re_{\rm p}} (1 + 0.15 Re_{\rm p}^{0.687}) \tag{1.8}$$

#### 1.2 PARTICLES FALLING UNDER GRAVITY THROUGH A FLUID

The relative motion under gravity of particles in a fluid is of particular interest. In general, the forces of buoyancy, drag and gravity act on the particle:

$$gravity - buoyancy - drag = acceleration force$$
 (1.9)

A particle falling from rest in a fluid will initially experience a high acceleration as the shear stress drag, which increases with relative velocity, will be small. As the particle accelerates the drag force increases, causing the acceleration to reduce. Eventually a force balance is achieved when the acceleration is zero and a maximum or terminal relative velocity is reached. This is known as the single particle terminal velocity.

For a spherical particle, Equation (1.9) becomes

$$\frac{\pi x^3}{6} \rho_p g - \frac{\pi x^3}{6} \rho_f g - R' \frac{\pi x^2}{4} = 0$$
 (1.10)

Combining Equation (1.10) with Equation (1.5),

$$\frac{\pi x^3}{6} (\rho_p - \rho_f) g - C_{D_2} \frac{1}{2} \rho_f U_T^2 \frac{\pi x^2}{4} = 0$$
 (1.11)

where  $U_T$  is the single particle terminal velocity. Equation (1.11) gives the following expression for the drag coefficient under terminal velocity conditions:

$$C_{\rm D} = \frac{4}{3} \frac{gx}{U_{\rm T}^2} \left( \frac{(\rho_{\rm p} - \rho_{\rm f})}{\rho_{\rm f}} \right)$$
 (1.12)

Thus in the Stokes' law region, with  $C_D = 24/Re_p$ , the single particle terminal velocity is given by

$$U_{\rm T} = \frac{x^2(\rho_{\rm p} - \rho_{\rm f})g}{18\mu} \tag{1.13}$$

Note that in the Stokes' law region the terminal velocity is proportional to the square of the particle diameter.

In the Newton's law region, with  $C_D=0.44$ , the terminal velocity is given by

$$U_{\rm T} = 1.74 \left( \frac{x(\rho_{\rm p} - \rho_{\rm f})g}{\rho_{\rm f}} \right)^{1/2} \tag{1.14}$$

Note that in this region the terminal velocity is independent of the fluid viscosity and proportional to the square root of the particle diameter.

In the intermediate region no explicit expression for  $U_T$  can be found. However, in this region, the variation of terminal velocity with particle and fluid properties is approximately described by the following:

$$U_{\rm T} \propto x^{1.1}$$
,  $(\rho_{\rm p} - \rho_{\rm f})^{0.7}$ ,  $\rho_{\rm f}^{-0.29}$ ,  $\mu^{-0.43}$ 

Generally, when calculating the terminal velocity for a given particle or the particle diameter for a given velocity, it is not known which region of operation is relevant. One way around this is to formulate the dimensionless groups,  $C_D Re_p^2$  and  $C_D / Re_p$ :

To calculate U<sub>T</sub>, for a given size x. Calculate the group

$$C_{\rm D}Re_{\rm p}^2 = \frac{4}{3} \frac{x^3 \rho_{\rm f}(\rho_{\rm p} - \rho_{\rm f})g}{\mu^2}$$
 (1.15)

which is independent of  $U_T$ 

(Note that  $C_D Re_p^2 = \frac{4}{3} Ar$ , where Ar is the Archimedes number) For given particle and fluid properties,  $C_D Re_p^2$  is a constant and will therefore produce a straight line of slope -2 if plotted on the logarithmic coordinates (log  $C_D$  versus log  $Re_p$ ) of the standard drag curve. The intersection of this straight line with the drag curve gives the value of Rep and hence  $U_{\rm T}$  (Figure 1.2).

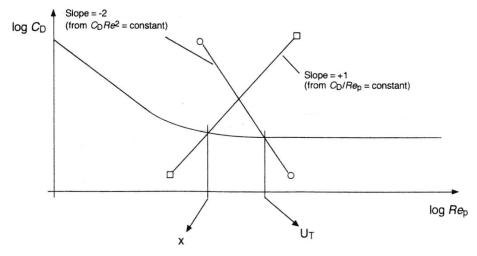


Figure 1.2 Method for estimating terminal velocity for a given size of particle and vice versa (Note:  $Re_p$  is based on the equivalent volume sphere diameter,  $x_v$