Student's Solutions Manual

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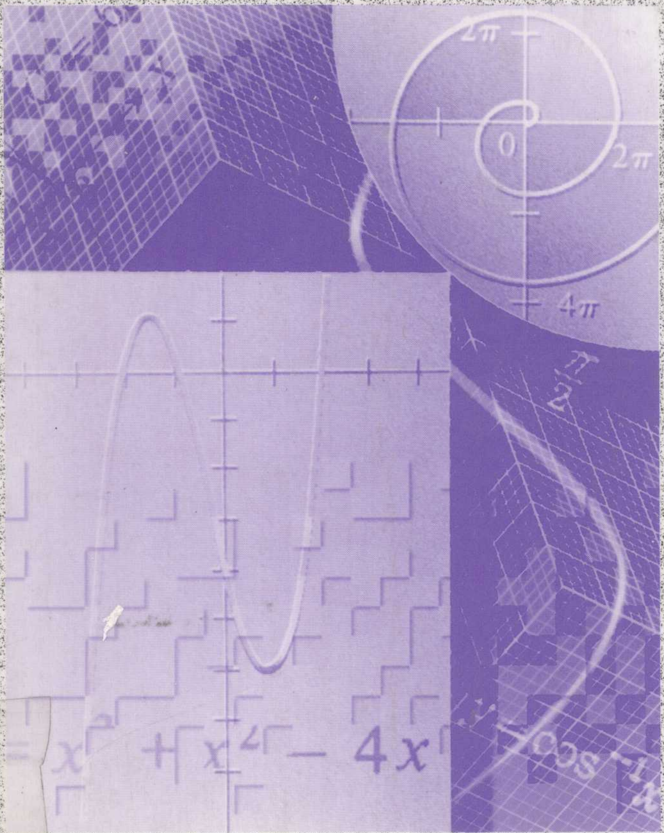

PRECALCULUS
Functions and Graphs

SEVENTH EDITION

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PREFACE

This *Student's Solutions Manual* contains selected solutions and strategies for solving typical exercises in the text, *Precalculus: Functions and Graphs, Seventh Edition*, by Earl W. Swokowski and Jeffery A. Cole. No particular problem pattern has been followed in this manual, but the emphasis has been placed on the solutions of the applied "word" problems. I have tried to illustrate enough solutions so that the student will be able to obtain an understanding of all types of problems in each section.

A significant number of today's students are involved in various outside activities, and find it difficult, if not impossible, to attend all class sessions. This manual should help meet the needs of these students. In addition, it is my hope that this manual's solutions will enhance the understanding of all readers of the material and provide insights to solving other exercises.

I would appreciate any feedback concerning errors, solution correctness, solution style, or manual style. These and any other comments may be sent directly to me at the address below or in care of the publisher.

I would like to thank: Editor David Dietz, of PWS Publishing Company, for his guidance and continued support; Gary Rockswold, of Mankato State University, for supplying solutions for many of the new applied problems and calculator exercises; Thomas Vanden Eynden, of Thomas More College, for doing a detailed accuracy check of the exercises; George Morris, of Scientific Illustrators, for creating the mathematically precise art package; and Sally Lifland, of Lifland et al. Bookmakers, for assembling the final manuscript. I dedicate this book to my children, Becky and Brad.

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Notations

The following notations are used in the manual.

Note:	{ Notes to the student pertaining to hints on solutions, common mistakes, or conventions to follow. }
{ }	{ comments to the reader are in braces }
LS	{ Left Side of an equation }
RS	{ Right Side of an equation }
$\approx$	{ approximately equal to }
$\Rightarrow$	{ implies, next equation, logically follows }
$\Leftrightarrow$	{ if and only if, is equivalent to }
•	{ bullet, used to separate problem statement from solution or explanation }
★	{ used to identify the answer to the problem }
§	{ *section* references }
$\forall$	{ For all, i.e., $\forall x$ means "for all x". }
$\mathbb{R} - \{a\}$	{ The set of all real numbers except a. }
$\therefore$	{ therefore }
QI–QIV	{ quadrants I, II, III, IV }

To the Student

This manual is a text supplement and should be read along *with* the text. Read all exercise solutions in this manual since explanations of concepts are given and then appear in subsequent solutions. All concepts necessary to solve a particular problem are not reviewed for every exercise. If you are having difficulty with a previously covered concept, look back to the section where it was covered for more complete help. The writing style I have used in this manual reflects the way I explain concepts to my own students. It is not as mathematically precise as that of the text, including phrases such as "goes down" or "touches and turns around." My students have told me that these terms help them understand difficult concepts with ease.

The most common complaint about solutions manuals that I receive from my students is that there are not enough exercise solutions in them. I believe there is a sufficient number of solutions in this manual, with more than one-third of the exercises solved in every section—most of them are odd-numbered exercises.

Lengthier explanations and more steps are given for the more difficult problems. Additional information that my students have found helpful is included—see page 10. The guidelines given in the text are followed for some solutions—see page 80.

In the review sections, the solutions are somewhat abbreviated since more detailed solutions were given in previous sections. However, this is not true for the word problems in these sections since they are unique. In easier groups of exercises, representative solutions are shown. Occasionally, alternate solutions are also given.

All figures are new for this edition. They have been plotted using computer software, offering a high degree of precision. The calculator graphs are from the TI-81 screen, and any specific instructions are for the TI-81. When possible, each piece of art was made with the same scale to show a realistic and consistent graph.

This manual was done using EXP: *The Scientific Word Processor.* I have used a variety of display formats for the mathematical equations, including centering, vertical alignment, and flushing text to the right. I hope that these make reading and comprehending the material easier for you.

Table of Contents

Chapter 1: Topics from Algebra

1.1 Exercises

1 (a) Since x and y have opposite signs, the product xy is negative.

(b) Since $x^2 > 0$ and $y > 0$, $x^2y > 0$.

(c) Since $x < 0$ { x is negative} and $y > 0$ { y is positive}, $\frac{x}{y}$ is negative.

Thus, $\frac{x}{y} + x$ is the sum of two negatives, which is *negative.*

(d) Since $y > 0$ and $x < 0$, $y - x > 0$.

3 (a) Since -7 is to the left of -4 on a coordinate line, $-7 \boxed{<} -4$.

(b) Using a calculator, we see that $\frac{\pi}{2} \approx 1.5708$. Hence, $\frac{\pi}{2} \boxed{>} 1.57$.

(c) $\sqrt{225} \boxed{=} 15$ *Note:* $\sqrt{225} \neq \pm 15$

Note: An informal definition of absolute value that may be helpful is

$$|\,something\,| = \begin{cases} itself & \text{if } itself \text{ is positive or zero} \\ -(itself) & \text{if } itself \text{ is negative} \end{cases}$$

9 (a) $|-3-2| = |-5| = -(-5)$ {since $-5 < 0$} $= 5$

(b) $|-5| - |2| = -(-5) - 2 = 5 - 2 = 3$

(c) $|7| + |-4| = 7 + [-(-4)] = 7 + 4 = 11$

13 (a) Since $(4 - \pi)$ is positive, $|4 - \pi| = 4 - \pi$.

(b) Since $(\pi - 4)$ is negative, $|\pi - 4| = -(\pi - 4) = 4 - \pi$.

(c) Since $(\sqrt{2} - 1.5)$ is negative, $|\sqrt{2} - 1.5| = -(\sqrt{2} - 1.5) = 1.5 - \sqrt{2}$.

17 (a) $d(A,\, B) = |1 - (-9)| = |10| = 10$ (b) $d(B,\, C) = |10 - 1| = |9| = 9$

(c) $d(C,\, B) = d(B,\, C) = 9$ (d) $d(A,\, C) = |10 - (-9)| = |19| = 19$

Note: Exer. 19–24: Since $|a| = |-a|$, the answers could have a different form.

For example, $|-3 - x| \geq 8$ is equivalent to $|x + 3| \geq 8$.

21 $d(A,\, B) = |-3 - x| \Rightarrow |-3 - x| \geq 8$

25 Pick an arbitrary value for x that is less than -3, say -5.

Since $3 + (-5) = -2$ is negative, we conclude that if $x < -3$, then $3 + x$ is negative.

Hence, $|3 + x| = -(3 + x) = -x - 3$.

29 If $a < b$, then $a - b < 0$, and $|a - b| = -(a - b) = b - a$.

31 Since $x^2 + 4 > 0$ for every x, $|x^2 + 4| = x^2 + 4$.

33 $\text{LS} = \dfrac{ab + ac}{a} = \frac{ab}{a} + \frac{ac}{a} = b + c \boxed{\neq} \text{RS } (b + ac)$.

37 $\text{LS} = (a \div b) \div c = \frac{a}{b} \cdot \frac{1}{c} = \frac{a}{bc}$. $\text{RS} = a \div (b \div c) = a \div \frac{b}{c} = a \cdot \frac{c}{b} = \frac{ac}{b}$. $\text{LS} \boxed{\neq} \text{RS}$

39 $\text{LS} = \frac{a - b}{b - a} = \dfrac{-(b - a)}{b - a} = -1 \boxed{=} \text{RS}$.

[45] (a) Since the decimal point is 5 places to the right of the first nonzero digit,

$$427{,}000 = 4.27 \times 10^5.$$

(b) Since the decimal point is 8 places to the left of the first nonzero digit,

$$0.000\ 000\ 098 = 9.8 \times 10^{-8}.$$

[47] (a) Moving the decimal point 4 places to the left, we have $85{,}200 = 8.52 \times 10^4$.

(b) Moving the decimal point 6 places to the right, we have $0.000\ 005\ 5 = 5.5 \times 10^{-6}$.

[51] It is helpful to write the units of any fraction, and then "cancel" those units to determine the units of the final answer.

$$\frac{186{,}000 \text{ miles}}{\text{second}} \cdot \frac{60 \text{ seconds}}{1 \text{ minute}} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} \cdot \frac{24 \text{ hours}}{1 \text{ day}} \cdot \frac{365 \text{ days}}{1 \text{ year}} \cdot 1 \text{ year} \approx 5.87 \times 10^{12} \text{ mi}$$

[53] $$\frac{\dfrac{1.01 \text{ grams}}{\text{mole}}}{\dfrac{6.02 \times 10^{23} \text{ atoms}}{\text{mole}}} \cdot 1 \text{ atom} = \frac{1.01 \text{ grams}}{6.02 \times 10^{23}} \approx 0.1678 \times 10^{-23} \text{ g} = 1.678 \times 10^{-24} \text{ g}$$

[57] (a) $\text{IQ} = \dfrac{\text{mental age (MA)}}{\text{chronological age (CA)}} \times 100 = \frac{15}{12} \times 100 = 125.$

(b) $\text{CA} = 15$ and $\text{IQ} = 140 \Rightarrow 140 = \text{MA}/15 \times 100 \Rightarrow \text{MA} = 21.$

1.2 Exercises

[1] $(-\frac{2}{3})^4 = (-\frac{2}{3}) \cdot (-\frac{2}{3}) \cdot (-\frac{2}{3}) \cdot (-\frac{2}{3}) = \frac{16}{81}$

Note: Do not confuse $(-x)^4$ and $-x^4$ since $(-x)^4 = x^4$ and $-x^4$ is the negative of x^4.

[5] $-2^4 + 3^{-1} = -16 + \frac{1}{3} = -\frac{48}{3} + \frac{1}{3} = \frac{-47}{3}$

[7] $16^{-3/4} = 1/16^{3/4} = 1/(\sqrt[4]{16})^3 = 1/2^3 = \frac{1}{8}$

[9] $(-0.008)^{2/3} = (\sqrt[3]{-0.008})^2 = (-0.2)^2 = 0.04 = \frac{4}{100} = \frac{1}{25}$

[13] A common mistake is to write $x^3x^2 = x^6$, and another is to write $(x^3)^2 = x^5$.

The following solution illustrates the proper use of the exponent rules.

$$\frac{(6x^3)^2}{(2x^2)^3} = \frac{6^2(x^3)^2}{2^3(x^2)^3} = \frac{36(x^{3\cdot 2})}{8(x^{2\cdot 3})} = \frac{36x^6}{8x^6} = \frac{36}{8} = \frac{9}{2}$$

[15] $(8x^4y^{-3})(\frac{1}{2}x^{-5}y^2) = (8 \cdot \frac{1}{2})(x^{4+(-5)})(y^{(-3)+2}) = 4x^{-1}y^{-1} = \frac{4}{xy}$

[17] $(\frac{1}{3}x^4y^{-3})^{-2} = (\frac{1}{3})^{-2}(x^4)^{-2}(y^{-3})^{-2} = (\frac{3}{1})^2x^{-8}y^6 = 3^2x^{-8}y^6 = \dfrac{9y^6}{x^8}$

[19] $(3y^3)^4(4y^2)^{-3} = 3^4(y^3)^4(4)^{-3}(y^2)^{-3} = 81y^{12} \cdot 4^{-3}y^{-6} = 81y^6 \cdot \frac{1}{64} = \frac{81}{64}y^6$

[25] $\left(\dfrac{3x^5y^4}{x^0y^{-3}}\right)^2$ {remember that $x^0 = 1$} $= \dfrac{9x^{10}y^8}{y^{-6}} = 9x^{10}y^{14}$

27 $(3x^{5/6})(8x^{2/3}) = 24x^{(5/6)+(4/6)} = 24x^{9/6} = 24x^{3/2}$

29 $(27a^6)^{-2/3} = 27^{-2/3}a^{-12/3} = \dfrac{1}{27^{2/3}a^{12/3}} = \dfrac{1}{(\sqrt[3]{27})^2a^4} = \dfrac{1}{9a^4}$

33 $\left(\dfrac{-8x^3}{y^{-6}}\right)^{2/3} = \dfrac{(-8)^{2/3}(x^3)^{2/3}}{(y^{-6})^{2/3}} = \dfrac{(\sqrt[3]{-8})^2\,x^{(3)(2/3)}}{y^{(-6)(2/3)}} = \dfrac{(-2)^2x^2}{y^{-4}} = \dfrac{4x^2}{y^{-4}} = 4x^2y^4$

37 $\dfrac{(x^6y^3)^{-1/3}}{(x^4y^2)^{-1/2}} = \dfrac{(x^6)^{-1/3}(y^3)^{-1/3}}{(x^4)^{-1/2}(y^2)^{-1/2}} = \dfrac{x^{-2}y^{-1}}{x^{-2}y^{-1}} = 1$

41 $\sqrt[3]{(a+b)^2} = [(a+b)^2]^{1/3} = (a+b)^{2/3}$

45 (a) $4x^{3/2} = 4x^1x^{1/2} = 4x\sqrt{x}$

(b) $(4x)^{3/2} = (4x)^1(4x)^{1/2} = (4x)^1\,4^{1/2}x^{1/2} = 4x\cdot 2\cdot x^{1/2} = 8x\sqrt{x}$

51 $\sqrt[5]{-64} = \sqrt[5]{-32}\,\sqrt[5]{2} = \sqrt[5]{(-2)^5}\,\sqrt[5]{2} = -2\sqrt[5]{2}$

55 $\sqrt{9x^{-4}y^6} = (9x^{-4}y^6)^{1/2} = 9^{1/2}(x^{-4})^{1/2}(y^6)^{1/2} = 3x^{-2}y^3 = \dfrac{3y^3}{x^2}$

Note: For exercises similar to those in 59–62, pick a multiplier that will make all of the exponents of the terms in the denominator a multiple of the index.

59 The index is 2. Choose the multiplier to be $\sqrt{2y}$ so that the denominator contains only terms with even exponents. $\sqrt{\dfrac{3x}{2y^3}} = \sqrt{\dfrac{3x}{2y^3}}\cdot\dfrac{\sqrt{2y}}{\sqrt{2y}} = \sqrt{\dfrac{6xy}{4y^4}} = \dfrac{\sqrt{6xy}}{2y^2}$, or $\dfrac{1}{2y^2}\sqrt{6xy}$

61 The index is 4. Choose the multiplier to be $\sqrt[4]{3x^2}$ so that the denominator contains only terms with exponents that are multiples of 4.

$$\sqrt[4]{\frac{5x^8y^3}{27x^2}} = \sqrt[4]{\frac{5x^8y^3}{27x^2}}\cdot\frac{\sqrt[4]{3x^2}}{\sqrt[4]{3x^2}} = \frac{\sqrt[4]{15x^{10}y^3}}{\sqrt[4]{81x^4}} = \frac{\sqrt[4]{x^8}\,\sqrt[4]{15x^2y^3}}{3x} = \frac{x^2\,\sqrt[4]{15x^2y^3}}{3x} = \frac{x}{3}\sqrt[4]{15x^2y^3}$$

63 $\sqrt[4]{(3x^5y^{-2})^4} = 3x^5y^{-2} = \dfrac{3x^5}{y^2}$

65 $\sqrt{5xy^7}\,\sqrt{10x^3y^3} = \sqrt{50x^4y^{10}} = \sqrt{25x^4y^{10}}\,\sqrt{2} = 5x^2y^5\sqrt{2}$

67 $\sqrt{x^6y^4} = \sqrt{(x^3)^2(y^2)^2} = \sqrt{(x^3)^2}\,\sqrt{(y^2)^2} = |x^3|\,|y^2| = |x^3|\,y^2$ since y^2 is always nonnegative. *Note:* $|x^3|$ could be written as $x^2\,|x|$.

69 $\sqrt[4]{x^8(y-1)^{12}} = \sqrt[4]{(x^2)^4((y-1)^3)^4} = |x^2|\,|(y-1)^3| = x^2\,|(y-1)^3|$, or $x^2(y-1)^2\,|(y-1)|$

71 $(a^r)^2 = a^{2r}$ $\boxed{\neq}$ $a^{(r^2)}$ since $2r \neq r^2$ for every r.

79 $W = 230$ kg $\Rightarrow L = 0.46\sqrt[3]{W} = 0.46\sqrt[3]{230} \approx 2.82$ m

[81] $b = 75$ and $w = 180 \Rightarrow W = \dfrac{w}{\sqrt[3]{b-35}} = \dfrac{180}{\sqrt[3]{75-35}} \approx 52.6.$

$b = 120$ and $w = 250 \Rightarrow W = \dfrac{w}{\sqrt[3]{b-35}} = \dfrac{250}{\sqrt[3]{120-35}} \approx 56.9.$

It is interesting to note that the 75-kg lifter can lift 2.4 times his/her body weight and the 120-kg lifter can lift approximately 2.08 times his/her body weight, but *the formula ranks the 120-kg lifter as the superior lifter.*

1.3 Exercises

[1] $(2u+3)(u-4)+4u(u-2) = (2u^2-5u-12)+(4u^2-8u) = 6u^2-13u-12$

[3] Divide the denominator into each term of the numerator.

$$\frac{8x^2y^3-10x^3y}{2x^2y} = \frac{8x^2y^3}{2x^2y} - \frac{10x^3y}{2x^2y} = 4y^2-5x$$

[5] We recognize this product as the difference of two squares.

$$(2x+3y)(2x-3y) = (2x)^2-(3y)^2 = 4x^2-9y^2$$

[7] When squaring binomials, try to become very efficient at determining the three terms—mentally think of the phrase "square of the first term, twice the product of the two terms, square of the second term," until it becomes second nature.

$$(3x+2y)^2 = (3x)^2+2(3x)(2y)+(2y)^2 = 9x^2+12xy+4y^2$$

[9] $(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y}) = (\sqrt{x})^2-(\sqrt{y})^2 = x-y$

[11] $(x-2y)^3 = (x)^3+3(x)^2(-2y)+3(x)(-2y)^2+(-2y)^3$

$= x^3+3(x^2)(-2y)+3(x)(4y^2)-8y^3$

$= x^3-6x^2y+12xy^2-8y^3$

[13] We recognize this as a trinomial that may be able to be factored into the product of two binomials. $8x^2-53x-21 = (8x+3)(x-7)$

[15] The factors for x^2+3x+4 would have to be of the form $(x+\_)$ and $(x+\_)$. The factors of 4 are 1 & 4 and 2 & 2, but their sums are 5 and 4, respectively.

Thus, x^2+3x+4 is *irreducible.*

[17] $4x^2-20x+25 = (2x-5)(2x-5) = (2x-5)^2$

[19] $x^4-4x^2 = x^2(x^2-4) = x^2(x^2-2^2) = x^2(x+2)(x-2)$

[21] We recognize this expression as the difference of two cubes.

$64x^3-y^6 = (4x)^3-(y^2)^3$

$= (4x-y^2)\,[(4x)^2+(4x)(y^2)+(y^2)^2]$

$= (4x-y^2)(16x^2+4xy^2+y^4)$

[23] We recognize this expression as the sum of two cubes.

$$\begin{aligned}64x^3+27 &= (4x)^3+(3)^3\\ &= (4x+3)\left[(4x)^2-(4x)(3)+(3)^2\right]\\ &= (4x+3)(16x^2-12x+9)\end{aligned}$$

[25] First we will factor out the greatest common factor, 3.

$$3x^3+3x^2-27x-27=3(x^3+x^2-9x-9)$$

Since there are more than 3 terms, we will next try to factor by grouping.

$$\begin{aligned}3(x^3+x^2-9x-9) &= 3[x^2(x+1)-9(x+1)]\\ &= 3(x^2-9)(x+1)\\ &= 3(x+3)(x-3)(x+1)\end{aligned}$$

[27] We could treat this expression as the difference of two squares or the difference of two cubes. Factoring as the difference of two squares and then as the sum and difference of two cubes leads to the following:

$$\begin{aligned}a^6-b^6 &= (a^3)^2-(b^3)^2\\ &= (a^3+b^3)(a^3-b^3)\\ &= (a+b)(a-b)(a^2-ab+b^2)(a^2+ab+b^2)\end{aligned}$$

[29] We might first try to factor this expression by grouping since it has more than 3 terms, but this would prove to be unsuccessful. Instead, we will group the terms containing x and the constant term together, and then proceed as in Example 2 part (c).

$$\begin{aligned}x^2+4x+4-9y^2 &= (x^2+4x+4)-9y^2\\ &= (x+2)^2-(3y)^2\\ &= (x+2+3y)(x+2-3y)\end{aligned}$$

[33] $$\begin{aligned}\frac{9x^2-4}{3x^2-5x+2}\cdot\frac{9x^4-6x^3+4x^2}{27x^4+8x} &= \frac{(3x+2)(3x-2)}{(3x-2)(x-1)}\cdot\frac{x^2(9x^2-6x+4)}{x(27x^3+8)}\\ &= \frac{(3x+2)(3x-2)}{(3x-2)(x-1)}\cdot\frac{x^2(9x^2-6x+4)}{x(3x+2)(9x^2-6x+4)}=\frac{x}{x-1}\end{aligned}$$

[37] $$\frac{2}{x}+\frac{3x+1}{x^2}-\frac{x-2}{x^3}=\frac{2x^2+(3x+1)x-x+2}{x^3}=\frac{5x^2+2}{x^3}$$

[39] $$\frac{3t}{t+2}+\frac{5t}{t-2}-\frac{40}{t^2-4}=\frac{3t}{t+2}+\frac{5t}{t-2}-\frac{40}{(t+2)(t-2)}$$
$$=\frac{3t(t-2)}{(t+2)(t-2)}+\frac{5t(t+2)}{(t+2)(t-2)}-\frac{40}{(t+2)(t-2)}$$
$$=\frac{3t^2-6t+5t^2+10t-40}{(t+2)(t-2)}$$
$$=\frac{8t^2+4t-40}{(t+2)(t-2)}$$
$$=\frac{4(2t+5)(t-2)}{(t+2)(t-2)}=\frac{4(2t+5)}{t+2}$$

[43] $$\frac{2x}{x+2}-\frac{8}{x^2+2x}+\frac{3}{x}=\frac{2x(x)-8+3(x+2)}{x(x+2)}=\frac{2x^2+3x-2}{x(x+2)}=\frac{(2x-1)(x+2)}{x(x+2)}=\frac{2x-1}{x}$$

[45] $$3+\frac{5}{u}+\frac{2u}{3u+1}=\frac{3u(3u+1)+5(3u+1)+2u(u)}{u(3u+1)}$$
$$=\frac{9u^2+3u+15u+5+2u^2}{u(3u+1)}=\frac{11u^2+18u+5}{u(3u+1)}$$

[47] $$\frac{2x+1}{x^2+4x+4}-\frac{6x}{x^2-4}+\frac{3}{x-2}=\frac{(2x+1)(x-2)-6x(x+2)+3(x^2+4x+4)}{(x+2)^2(x-2)}=$$
$$\frac{-x^2-3x+10}{(x+2)^2(x-2)}=-\frac{x^2+3x-10}{(x+2)^2(x-2)}=-\frac{(x+5)(x-2)}{(x+2)^2(x-2)}=-\frac{x+5}{(x+2)^2}$$

[49] The lcd of the entire expression is ab.

Thus, we will multiply both the numerator and denominator by ab.

$$\frac{\frac{b}{a}-\frac{a}{b}}{\frac{1}{a}-\frac{1}{b}}=\frac{\left(\frac{b}{a}-\frac{a}{b}\right)\cdot ab}{\left(\frac{1}{a}-\frac{1}{b}\right)\cdot ab}=\frac{b^2-a^2}{b-a}=\frac{(b+a)(b-a)}{b-a}=a+b$$

[51] $$\frac{y^{-1}+x^{-1}}{(xy)^{-1}}=\frac{\frac{1}{y}+\frac{1}{x}}{\frac{1}{xy}}=\frac{\left(\frac{1}{y}+\frac{1}{x}\right)\cdot xy}{\left(\frac{1}{xy}\right)\cdot xy}\{\text{ lcd is } xy\}=\frac{\frac{1}{y}\cdot xy+\frac{1}{x}\cdot xy}{\frac{1}{xy}\cdot xy}=\frac{x+y}{1}=x+y$$

[55] $$\frac{(x+h)^2-3(x+h)-(x^2-3x)}{h}=\frac{x^2+2xh+h^2-3x-3h-x^2+3x}{h}=\frac{2xh+h^2-3h}{h}=$$
$$\frac{h(2x+h-3)}{h}=2x+h-3$$

[57] $$\frac{\frac{3}{x-1}-\frac{3}{a-1}}{x-a}=\frac{\frac{3(a-1)-3(x-1)}{(x-1)(a-1)}}{x-a}=\frac{\frac{3a-3-3x+3}{(x-1)(a-1)}}{x-a}=\frac{3a-3x}{(x-1)(a-1)(x-a)}=$$
$$\frac{3(a-x)}{(x-1)(a-1)(x-a)}=-\frac{3}{(x-1)(a-1)}\left\{\text{Note that, in general, } \frac{a-x}{x-a}=-1.\right\}$$

59 $\dfrac{\dfrac{1}{(x+h)^3}-\dfrac{1}{x^3}}{h}=\dfrac{\dfrac{x^3-(x+h)^3}{(x+h)^3x^3}}{h}=\dfrac{x^3-(x+h)^3}{hx^3(x+h)^3}=$

$$\frac{[x-(x+h)][x^2+x(x+h)+(x+h)^2]}{hx^3(x+h)^3}=\frac{-h(3x^2+3xh+h^2)}{hx^3(x+h)^3}=-\frac{3x^2+3xh+h^2}{x^3(x+h)^3}$$

61 $\dfrac{\sqrt{t}+5}{\sqrt{t}-5}=\dfrac{\sqrt{t}+5}{\sqrt{t}-5}\cdot\dfrac{\sqrt{t}+5}{\sqrt{t}+5}=\dfrac{t+10\sqrt{t}+25}{t-25}$

63 $\dfrac{1}{\sqrt[3]{a}-\sqrt[3]{b}}=\dfrac{1}{\sqrt[3]{a}-\sqrt[3]{b}}\cdot\dfrac{\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}}{\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}}=\dfrac{\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}}{a-b}$

67 $\dfrac{\sqrt{2(x+h)+1}-\sqrt{2x+1}}{h}=\dfrac{\sqrt{2(x+h)+1}-\sqrt{2x+1}}{h}\cdot\dfrac{\sqrt{2(x+h)+1}+\sqrt{2x+1}}{\sqrt{2(x+h)+1}+\sqrt{2x+1}}$

$$=\frac{(2x+2h+1)-(2x+1)}{h(\sqrt{2(x+h)+1}+\sqrt{2x+1})}=\frac{2}{\sqrt{2(x+h)+1}+\sqrt{2x+1}}$$

71 $\dfrac{(x^2+2)^2}{x^5}=\dfrac{x^4+4x^2+4}{x^5}=\dfrac{x^4}{x^5}+\dfrac{4x^2}{x^5}+\dfrac{4}{x^5}=x^{-1}+4x^{-3}+4x^{-5}$

Note: Exercises 73–90 are worked using the factoring concept given as the third method of simplification in Example 7.

73 The *smallest* exponent that appears on the factor x is -3.

$$x^{-3}+x^2\ \{\text{factor out } x^{-3}\}\ =x^{-3}(1+x^{2-(-3)})=x^{-3}(1+x^5)=\frac{1+x^5}{x^3}$$

79 The smallest exponent that appears on the factor (x^2-4) is $-\frac{1}{2}$ and the smallest exponent that appears on the factor $(2x+1)$ is 2. Thus, we will factor out $(x^2-4)^{-1/2}(2x+1)^2$.

$(x^2-4)^{1/2}(3)(2x+1)^2(2)+(2x+1)^3(\frac{1}{2})(x^2-4)^{-1/2}(2x)$

$$=(x^2-4)^{-1/2}(2x+1)^2[6(x^2-4)+x(2x+1)]$$

If you are unsure of this factoring, it is easy to visually check at this stage by merely multiplying the expression—that is, we mentally add the exponents on the factor (x^2-4), $-\frac{1}{2}$ and 1, and we get $\frac{1}{2}$, which is the exponent we started with.

Proceeding, we may simplify as follows:

$$=\frac{(2x+1)^2(8x^2+x-24)}{(x^2-4)^{1/2}}$$

81 $(3x+1)^6(\frac{1}{2})(2x-5)^{-1/2}(2)+(2x-5)^{1/2}(6)(3x+1)^5(3)=$

$$(3x+1)^5(2x-5)^{-1/2}[(3x+1)+18(2x-5)]=\frac{(3x+1)^5(39x-89)}{(2x-5)^{1/2}}$$

[83] $$\frac{(6x+1)^3(27x^2+2)-(9x^3+2x)(3)(6x+1)^2(6)}{(6x+1)^6}=$$

$$\frac{(6x+1)^2[(6x+1)(27x^2+2)-18(9x^3+2x)]}{(6x+1)^6}=\frac{27x^2-24x+2}{(6x+1)^4}$$

[85] $$\frac{(x^2+2)^3(2x)-x^2(3)(x^2+2)^2(2x)}{[(x^2+2)^3]^2}=\frac{(x^2+2)^2(2x)\left[(x^2+2)^1-x^2(3)\right]}{(x^2+2)^6}=$$

$$\frac{2x(x^2+2-3x^2)}{(x^2+2)^4}=\frac{2x(2-2x^2)}{(x^2+2)^4}=\frac{4x(1-x^2)}{(x^2+2)^4}$$

[89] $$\frac{(4x^2+9)^{1/2}(2)-(2x+3)(\frac{1}{2})(4x^2+9)^{-1/2}(8x)}{[(4x^2+9)^{1/2}]^2}=$$

$$\frac{(4x^2+9)^{-1/2}[2(4x^2+9)-4x(2x+3)]}{(4x^2+9)}=\frac{18-12x}{(4x^2+9)^{3/2}}=\frac{6(3-2x)}{(4x^2+9)^{3/2}}$$

[91] The dimensions of I are (x) and $(x-y)$. The area of I is $(x-y)x$, and the area of II is $(x-y)y$. The area $A=\underline{x^2-y^2}=(x-y)x+(x-y)y=\underline{(x-y)(x+y)}.$

1.4 Exercises

[3] $(3x-2)^2=(x-5)(9x+4)\Rightarrow 9x^2-12x+4=9x^2-41x-20\Rightarrow 29x=-24\Rightarrow$

$x=-\frac{24}{29}$

Note: You may have solved equations such as those in Exercise 5 using a process called *cross-multiplication* in the past. This method is sufficient for problems of the form $\frac{P}{Q}=\frac{R}{S}$, but the following guidelines for solving an equation containing rational expressions apply to rational equations of a more complex form.

Guidelines for Solving an Equation Containing Rational Expressions

(1) Determine the lcd of the rational expressions.

(2) Find the values of the variable that make the lcd zero. These are *not* solutions, because they yield at least one zero denominator when substituted into the given equation.

(3) Multiply each term of the equation by the lcd and simplify, thereby eliminating all of the denominators.

(4) Solve the equation obtained in guideline 3.

(5) The solutions of the given equation are the solutions found in guideline 4, with the exclusion of the values found in guideline 2.

5 $\left[\frac{3x+1}{6x-2}=\frac{2x+5}{4x-13}\right]\cdot(6x-2)(4x-13) \Rightarrow 12x^2-35x-13=12x^2+26x-10 \Rightarrow$

$-3=61x \Rightarrow x=-\frac{3}{61}$ { note that $x \neq \frac{1}{3}, \frac{13}{4}$ }

7 $\left[\frac{4}{x+2}+\frac{1}{x-2}=\frac{5x-6}{x^2-4}\right]\cdot(x+2)(x-2) \Rightarrow 4(x-2)+x+2=5x-6 \Rightarrow 0=0.$

This is an identity, and the solutions consist of every number in the domains of the given expressions. Thus, the solutions are all real numbers except ± 2, which we denote by $\mathbb{R}-\{\pm 2\}$.

9 $\left[\frac{5}{2x+3}+\frac{4}{2x-3}=\frac{14x+3}{4x^2-9}\right]\cdot(2x+3)(2x-3) \Rightarrow 5(2x-3)+4(2x+3)=14x+3 \Rightarrow$

$18x-3=14x+3 \Rightarrow 4x=6 \Rightarrow x=\frac{3}{2},$

which is not in the domain of the given expressions. No solution

11 $75x^2+35x-10=0$ { factor out the gcf, 5 } $\Rightarrow$

$5(15x^2+7x-2)=0$ { divide by 5 } $\Rightarrow$

$15x^2+7x-2=0 \Rightarrow (3x+2)(5x-1)=0 \Rightarrow x=-\frac{2}{3}, \frac{1}{5}$

13 $\left[\frac{2x}{x+3}+\frac{5}{x}-4=\frac{18}{x^2+3x}\right]\cdot x(x+3) \Rightarrow 2x(x)+5(x+3)-4(x^2+3x)=18 \Rightarrow$

$0=2x^2+7x+3 \Rightarrow (2x+1)(x+3)=0 \Rightarrow$

$x=-\frac{1}{2}$ { -3 is not in the domain of the given expressions }

15 $25x^2=9 \Rightarrow x^2=\frac{9}{25} \Rightarrow x=\pm\sqrt{\frac{9}{25}}=\pm\frac{3}{5}$

19 To solve the equation $x^2+4x+2=0$, use the quadratic formula,

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, with $a=1$, $b=4$, and $c=2$.

$$x=\frac{-(4)\pm\sqrt{(4)^2-4(1)(2)}}{2(1)}=\frac{-4\pm\sqrt{16-8}}{2}=\frac{-4\pm\sqrt{8}}{2}=\frac{-4\pm 2\sqrt{2}}{2}=-2\pm\sqrt{2}$$

Note: A common mistake is to not divide 2 into both terms of the numerator.

21 $|3x-2|+3=7 \Rightarrow |3x-2|=4 \Rightarrow 3x-2=4$ or $3x-2=-4 \Rightarrow$

$3x=6$ or $3x=-2 \Rightarrow x=2$ or $x=-\frac{2}{3}$

23 $3|x+1|-2=-11 \Rightarrow 3|x+1|=-9 \Rightarrow |x+1|=-3.$

Since the absolute value of an expression is nonnegative, $|x+1|=-3$ has no solution.

25 $9x^3-18x^2-4x+8=0 \Rightarrow 9x^2(x-2)-4(x-2)=0 \Rightarrow (9x^2-4)(x-2)=0 \Rightarrow$

$x=\pm\frac{2}{3}, 2$