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Martin van Hees

Rights and Decisions

Formal Models of Law and Liberalism

Kluwer Academic Publishers

MARTIN VAN HEES
Department of Public Administration and Public Policy,
University of Twente,
Enschede, The Netherlands

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KLUWER ACADEMIC PUBLISHERS
DORDRECHT / BOSTON / LONDON

A C.I.P. Catalogue record for this book is available from the Library of Congress

ISBN 0-7923-3754-9

Published by Kluwer Academic Publishers,
P.O. Box 17, 3300 AA Dordrecht, The Netherlands.

Kluwer Academic Publishers incorporates
the publishing programmes of
D. Reidel, Martinus Nijhoff, Dr W. Junk and MTP Press.

Sold and distributed in the U.S.A. and Canada
by Kluwer Academic Publishers,
101 Philip Drive, Norwell, MA 02061, U.S.A.

In all other countries, sold and distributed
by Kluwer Academic Publishers Group,
P.O. Box 322, 3300 AH Dordrecht, The Netherlands.

Printed on acid-free paper

Printed in the Netherlands

Law and Philosophy Library

VOLUME 23

For Anneke and Thomar

ACKNOWLEDGEMENTS

This book is based on my doctoral thesis which was written at the University of Nijmegen. I thank all those people who have contributed to its realization. In particular, I would like to thank Ad van Deemen, Grahame Lock and Harrie de Swart for their encouragements and support. I am also very grateful to Marcel Wissenburg for his nagging questions and helpful suggestions. Furthermore, I would like to thank Jos de Beus, Wulf Gaertner, Prasanta Pattanaik, Huib Pellikaan, Dick Ruiter and Jeroen van Rijen for their contributions to different stages of the research.

Some of the results of chapter 8 have been published in *Social Choice and Welfare* vol. 12 (1995), pp. 155-164. I am grateful to the publishers for their permission to use this material. Furthermore, financial support by the Netherlands Organization for Scientific Research (NWO) is gratefully acknowledged.

Above all, I am indebted to Anneke. I dedicate this book to her and to our son.

TABLE OF CONTENTS

CHAPTER 1

THE FORMAL ANALYSIS OF RIGHTS

Amartya Sen has made many important contributions to the development of the theory of rational choice.[1] One of those contributions was his introduction, in the early seventies, of individual rights into the formal analysis of processes of collective decision making.[2] Sen formulated individual rights in terms of properties of specific decision procedures and showed that it is impossible to define decision procedures which satisfy both a very mild assumption about the rights of individuals and the Pareto condition - the condition which states that an alternative may not be chosen whenever there is another alternative unanimously preferred to it. The assumption about the rights of individuals was defended as a necessary requirement of any theory of liberalism. The impossibility theorem became therefore known as the 'impossibility of the Paretian liberal' or simply as 'Sen's liberal paradox'.[3] Alan Gibbard extended Sen's framework in an interesting way. In (Gibbard 1974) he defined conditions of liberalism which are logically stronger than Sen's but which, Gibbard argued, are perfectly in line with Sen's notion of individual rights. He showed that these conditions cannot be satisfied by any decision procedure, not even when the Pareto

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1 We use the term *rational choice theory* to refer to formal theories of individual and collective decision making. Rational choice theory encompasses *social choice theory*, which focuses especially on the procedural aspects of collective decision making, as well as *game theory* which is concerned with the strategic aspects of decision making. For introductions to and reviews of the theory of social choice, see (Kelly 1978; Sen 1986). (Luce and Raiffa 1957) is the classic introduction to game theory. (Fudenberg and Tirole, 1991) is a more recent introduction. (Pattanaik 1978 ; Moulin 1983; Peleg 1984) border on the frontiers between game theory and social choice theory.

2 In our exposition of Sen's contribution to the study of the concepts of freedom and liberalism we concentrate on his formal analysis.

3 We shall follow the terminology of the original statement of the paradox and speak about 'liberalism'. However, Sen later preferred the term 'libertarianism' (Sen 1976). In (Sen 1983) the theorem is formulated as a tension between liberty and the Pareto condition.

condition is dropped. This result became known as 'Gibbard's paradox'. In this chapter we describe the theoretical background of our study. Since the two liberal paradoxes are important parts of that background, we present Sen's liberal paradox in section 1 and Gibbard's paradox in section 2.

The two liberal paradoxes provoked many reactions.[4] First of all, it has been argued that they are not only relevant to the theory of liberalism, but also to other topics. For example, Gärdenfors and Pettit broadened the meaning of Sen's liberal paradox by showing that it can also be applied to the theory of representation (Gärdenfors and Pettit 1989). Batra and Pattanaik have extended the framework to incorporate group rights in the context of theories of federalism (Batra and Pattanaik 1972). Others have explored ways to escape the impossibility results by weakening one or more of the assumptions. A fruitful line of research in this direction has been one in which restrictions on the preferences of the individuals were examined.[5] Whereas these two lines of investigation - extensions and restrictions - take the conceptual framework introduced by Sen as their basis, there have also been authors who maintained that the framework of the theory of social choice, in which individual rights are defined in terms of the preferences of individuals, does not adequately capture the notion of individual rights and that a game-theoretic framework would be more suitable. A forceful and cogent argument for the game-theoretic approach has been given in an article written jointly by Wulf Gaertner, Prasanta Pattanaik and Kotaro Suzumura (Gaertner *et al.* 1992). We outline their main arguments in section 3. In a discussion of Sen's response to those arguments we argue that it can be concluded that the game-theoretic approach is in many circumstances indeed a more appropriate way of analysing individual rights. However, as we shall see in sections 4 and 5, those game-theoretic approaches also have their disadvantages. In particular, they lack a firm foundation. One of the objects of this study is to provide such a foundation. In doing so we shall use insights from deontic logic, which is the formal study of the logical relationships between such concepts as permissions, obligations, rights, etc.

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4 (Sen 1976; Sen 1983; Wriglesworth 1985) contain reviews of the literature.

5 For an extensive review of this research see (Wriglesworth 1985).

With a few exceptions, theorists in the field of rational choice have ignored the logicians' formal studies of individual rights, and, conversely, the debate among rational choice theorists did not have much impact on the development of deontic logic. In section 6 it is hypothesized that the insights offered by deontic logic may be used to solve the problems connected with the game-theoretic approach to the analysis of rights, and, furthermore, that the insights offered by game-theoretic analyses can be fruitfully incorporated in deontic logic. Whereas the first six sections of this chapter provide the theoretical background to the problems which we investigate, the last section describes the objects of our study and the plan of this monograph.

1. THE IMPOSSIBILITY OF THE PARETIAN LIBERAL

Sen's notion of liberalism is based on the idea that each individual has a *protected sphere*: there are things in the life of a person with regard to which that individual should be free to do whatever he or she likes.[6] If John wants to read a book, eat candy, play soccer, watch a movie, etc., then John may do those things *regardless* of the opinions which the other members of society have about those activities. In the social choice theoretic framework of Sen, this idea is expressed in terms of the *decisiveness* of individuals with respect to *social states.*

A social state is a complete description of a state of affairs. Let X_+ denote the set of all social states. The set is assumed to be finite. Furthermore, it is assumed that individuals do not always make decisions about the set of all social states, but that sometimes a proper subset of X_+ is presented for choice. We shall call the set of social states which are presented for choice the *issue*; its elements are called *available* social states. Any non-empty subset of X_+ can be an issue. A *Group Decision Procedure (GDP)* is a decision mechanism which assigns to an issue and an n-tuple of preference orderings ($n \geq 2$) over the set of all social states (one ordering for each

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[6] 'Liberal values seem to require that there are choices that are personal and the relevant person should be free to do what he likes.' (Sen 1970, p. 87)

individual) at least one element of the issue.[7] If the GDP can take any logically possible issue and any n-tuple of individual preference orderings as its input, it is said to satisfy the condition of universal domain (*UD*).

On the basis of the individual preferences regarding the set of social states X_+ a social choice is made, i.e., one of the available social states is chosen.[8] A group of individuals is called *decisive* over a pair $\{x,y\}$ of social states ($x \neq y$) if the social state y will never be chosen when x is available and when all members of the group strictly prefer x to y and, conversely, if x will never be chosen when y is available and when all group members strictly prefer y to x. Sen's condition of liberalism says that every individual is decisive over at least one pair of social states.

S *For every individual i there are social states x and y such that {i} is decisive over {x,y}.*

The liberal paradox formulated by Sen expresses a tension between principle *S* and the (weak) Pareto condition:

PAR *The group of all individuals is decisive over every pair of social states.*

According to the Pareto condition, if everybody strictly prefers a social state x to a social state y, then y should not be chosen when x is available.

We can now formulate Sen's liberal paradox. It states that

There does not exist a GDP which satisfies UD, S and PAR.[9]

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[7] An *ordering* is a complete and transitive binary relation (see definition 7*4).

[8] We shall talk about collective decisions in terms of social *choices* instead of social *preferences*. There are several ways of defining social preferences on the basis of social choices (Kelly 1978, p. 20). See also (Sen 1993).

[9] The proof is straightforward, see (Sen 1970). Since the idea of the proof is contained in the example, we omit the proof.

Example. To illustrate the paradox let society consist of two individuals, denoted 1 and 2.[10] Person 1 is a conformist, whereas person 2 is a non-conformist. They have to make a choice about the set of social states $\{x,y,z,v\}$. The elements of this issue differ only with respect to the colour of the shirts the individuals are wearing: in x person 1 wears blue and person 2 white, in y they both wear white, in z person 1 wears white and person 2 wears blue and in v they both wear blue. For obvious reasons, we write $x = (b,w)$, $y = (w,w)$, $z = (w,b)$ and $v = (b,b)$. We assume that condition S is satisfied: say person 1 is decisive over $\{(w,w),(b,w)\}$ and person 2 over $\{(w,w),(w,b)\}$. The preferences of the individuals 1 and 2 regarding the social states belonging to the issue are (in decreasing order of strict preference),

1	2
(w,w)	(b,w)
(b,b)	(w,b)
(b,w)	(w,w)
(w,b)	(b,b)

In other words, person 1 - the conformist - wants to wear white if person 2 wears white and blue if person 2 wears blue. Person 2 wants to wear a shirt with a different colour than 1's: if person 1 wears white he would rather wear blue and if 1 wears blue he prefers white.

Since person 1 is decisive over $\{(w,w),(b,w)\}$ and since person 1 strictly prefers (w,w) to (b,w), the social state (b,w) cannot be chosen. For similar reasons (w,w) cannot be chosen: person 2 is decisive over $\{(w,w),(w,b)\}$ and strictly prefers (w,b) to (w,w). Finally, since both individuals strictly prefer (b,w) to (w,b) and (w,w) to (b,b) the Pareto condition precludes (w,b) and (b,b) from being chosen. Hence we see that every social state is rejected: none of the available social states can be chosen. In other words, at least one of the conditions *UD*, *S* or *PAR* is violated by the procedure. If the preference profile does not belong to the domain of the group decision procedure, then condition *UD* is violated. If it does, *S* or *PAR* will be violated.

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[10] The example is taken from (Gaertner *et al.* 1992).

Sen emphasizes that the condition of liberalism is a *necessary* requirement of any theory of liberalism; it is *not* a sufficient condition (Sen 1970, p. 79; Sen 1992, p. 140). Thus a proposition which states that a decision procedure exists that satisfies condition *S* does not establish the possibility of liberalism; there may be other conditions deemed to be relevant from a liberal point of view which the procedure does not satisfy. If the notion of decisiveness correctly represents an individual right, then principle *S* is very weak indeed. A theory of liberalism implies that individuals have several rights and therefore probably also several pairs of social states with respect to which an individual is decisive. On the other hand, a proposition about the *im*possibility of defining a decision procedure which satisfies condition *S* and some other normative principle has important consequences for the theory of liberalism: if liberalism indeed entails condition *S*, then such an impossibility result states that there is an intrinsic conflict between liberalism and this other normative principle, in our case the Pareto condition.

Before we discuss a critique of Sen's analysis of individual rights, we present the Gibbard paradox which uses a stronger notion of individual rights than Sen's, but which is also based on the idea of modelling rights in terms of the decisiveness of individuals.

2. GIBBARD'S LIBERAL PARADOX

Allan Gibbard (1974) follows Sen's idea that liberalism demands that each individual should have the right to decide some matters on his or her own. Like Sen, he assumes that this implies that individuals are decisive with respect to certain pairs of social states. However, whereas Sen left the set of social states unspecified and therefore left it unclear over which pairs of social states an individual is decisive, Gibbard characterizes the set of all social states in a certain way and makes a specific assumption regarding the pairs of social states over which individuals are decisive.

The example has already given an indication of the approach taken by Gibbard: there, a social state was written as a pair of features. One feature specified the colour of 1's shirt, the other that of 2's. In the example we assumed that the social states were identical with respect to all further

features. Hence there was no need to further specify those other features. More generally, a social state x can be described as a vector $(x_0, x_1, \ldots, x_n)$ consisting of a public feature characteristic, x_0, and n private feature characteristics: one for each individual.[11] A private feature characteristic represents the things about which the individual alone decides, e.g., the colour of his or her shirt. The public feature describes those things which do not belong to the protected spheres of individuals. If, for any individual i, M_i denotes the set of all features of type i, then X_+ is defined as the Cartesian product $M_0 \times \ldots \times M_n$. Gibbard's condition of liberalism consists of two parts:

G *For every individual i: (a) M_i contains at least two elements, and (b) for all distinct social states x, y $\in$ X_+: if x and y only differ with respect to the private feature characteristic of i, then {i} is decisive over {x,y}.*

Obviously, condition G is stronger than condition S: if condition G is satisfied, then S is satisfied as well. The converse implication need not be true. Nevertheless, the justification of condition S also seems to supports G: it invokes the idea that each individual has a protected sphere in which he or she alone has the right to decide what happens.

Gibbard's paradox states that

There does not exist a GDP satisfying UD and G.

To illustrate the Gibbard paradox we take the same example of the right to determine the colour of one's shirt. To facilitate matters, we assume that a social state is *completely* described by the colour of the shirts of the individuals 1 and 2, i.e., M_0 is empty, and that there are exactly two individuals. Hence we can still write those social states as ordered pairs in which the first feature represents the colour of person 1's shirt and the

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[11] We have simplified Gibbard's specification of a social state somewhat. See (Gibbard 1974, pp. 390-1).

second the colour of person 2's shirt. Applying part (b) of the condition, we see that person 1 is decisive over $\{(w,w),(b,w)\}$ and $\{(b,b),(w,b)\}$, and that the other individual is decisive over $\{(b,w),(b,b)\}$ and $\{(w,b),(w,w)\}$. Looking at the preference profile described in the example, one can easily check that none of the social states can be chosen. Social state (w,w) is rejected since person 2 strictly prefers (w,b) to it and since he is decisive over $\{(w,w),(w,b)\}$. Similarly, his decisiveness for $\{(b,b),(b,w)\}$ and his preferences exclude (b,b). Furthermore, person 1's decisiveness over the sets $\{(w,w),(b,w)\}$ and $\{(b,b),(w,b)\}$ implies in this situation that neither (b,w) nor (w,b) can be chosen: person 1, the conformist, strictly prefers (w,w) to (b,w) and (b,b) to (w,b). Hence none of the social states can be chosen. Or, stated differently, one cannot choose an element from this issue without violating condition *G*.

3. A CRITIQUE OF THE SOCIAL CHOICE APPROACH

Over the more than two decades that have passed since Sen first published his liberal paradox, there has been much discussion of Sen's and Gibbard's formalization of individual rights. The critique has culminated in an important article written by Wulf Gaertner, Prasanta Pattanaik and Kotaro Suzumura (Gaertner *et al.* 1992). Like many authors before them, they argue that the decisiveness concept does not adequately capture the notion of individual rights and that a game-theoretic approach would be more appropriate.[12] One of their arguments questions the reasonableness of condition *S* and, as a consequence, also the reasonableness of the stronger condition *G*. Their other arguments are concerned with condition *G*. Since condition *G* is logically stronger than condition *S*, these arguments do not directly go against Sen's condition. However, Gaertner *et al.* maintain, like Gibbard himself, that Gibbard's condition is based on the same idea as Sen's, i.e., the notion of a protected sphere, and that, consequently, their critique of *G* also entails a critique of *S*.

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[12] Cf. (Fine 1975; Aldrich 1977; Gärdenfors 1981; Gibbard 1982; Sugden 1985; Barry 1986; Gigliotti 1988; Deb 1990; Suzumura 1991; Gaertner 1992).