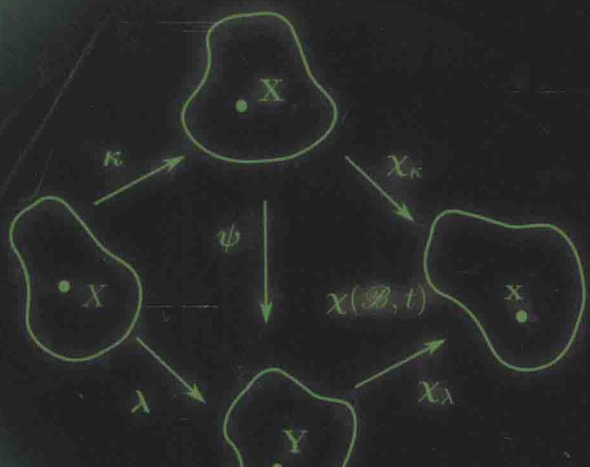
CONTINUUM MECHANICS

The Birthplace of Mathematical Models



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The Birthplace of Mathematical Models

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CONTINUUM MECHANICS

To Adele

PREFACE

American universities often relegate continuum mechanics to advanced graduate courses, mainly in engineering. In this setting, the subject remains obscure to other important audiences. I hope that this semester-length book will help to introduce the rudiments of continuum mechanics to first-year graduate students in engineering, physics, and geophysics, as well as to advanced mathematics undergraduates. In all of these fields, continuum mechanics lies at the foundation of prominent mathematical models. And in my experience the concepts are accessible to these students.

The title deserves a comment. This is a textbook on continuum mechanics. It includes derivations of commonly used mathematical models based on continuum mechanical concepts, to illustrate the philosophy that good mathematical models come from rigorous foundations. I distinguish this approach from that employed in many mathematical modeling textbooks, where the range of models may also include molecular, biological, financial, and stochastic systems. In those settings, methodologies beyond those normally associated with continuum physics often appear; see [23], [24], and [42] for examples. While I applaud trend toward teaching mathematical modeling in this broader sense, this book focuses on continua.

Anyone who teaches a first-level course in continuum mechanics must choose among several stylistic and conceptual approaches. I've tried to adopt an approach that meshes with undergraduate courses on vector calculus and linear algebra in the following respects:

- Wherever possible, I use direct notation instead of index notation. From a pedagogic point of view, I think this choice is easier on newcomers. It also minimizes reliance on particular coordinate systems. That said, index notation appears in several places, and I don't shy away from using Cartesian coordinates for examples and problems.

- I use infinitesimals sparingly. When they appear, it is largely for heuristic purposes. Most undergraduate mathematics courses emphasize parametrizations, Taylor approximations, and limits in lieu of infinitesimals.

- Students increasingly encounter partial differential equations (PDEs)—Nature's mother tongue—early in their college careers. I try to show how many of the most commonly encountered PDEs arise from continuum mechanical principles. In places, the book introduces qualitative attributes of these equations that I believe promote physical understanding but that students typically encounter in other courses.

The order of topics in continuum mechanics varies widely according to authors' tastes. To me it seems natural to start with the geometric, algebraic, and analytic foundations—especially the facts about vectors and tensors that the standard undergraduate curriculum does not reliably cover. Chapter 1 reviews this material. With this foundation in place, Chapters 2 and 3 cover topics in kinematics, the descriptive language for continuum motions. Chapter 4 covers balance laws, from which arise many of the governing equations of interest in applied mathematics and physical science. Chapter 5 introduces constitutive laws through a short tour of common mathematical models encountered in heat flow, fluid dynamics, and solid mechanics. This chapter includes an optional section on potential theory. Chapter 6 views constitutive laws from a more theoretical vantage point, outlining an elementary treatment of constitutive axioms, in response to the laudable trend toward including this material in introductory texts. This discussion of constitutive theory really amounts to a first glimpse, illustrating a handful of concepts and techniques for a particular type of material, thermoviscous fluids.

In Chapter 7, the book attempts something unusual for texts at this level, introducing multiconstituent continua. I base the treatment on mixture theory, which has limitations, unsettled issues, and much more depth than a single chapter can probe. Nevertheless, Chapter 7 provides a streamlined way to introduce diffusion, a topic of immense importance in environmental science and engineering, and it enables rational derivations of the field equations most commonly used to model flows in porous media.

Even a book this slender contains more material than many instructors will try to cover in a semester. It may help to know that one can skip the following sections without missing essential material used later:

- Section 2.3: Pathlines, Streamlines, and Streaklines.
- Section 3.4: Vorticity and Circulation (if one skips Section 5.3).
- Section 4.6: Jump Conditions.
- Any sections in Chapter 5; they are largely independent of each other.
- Chapter 7; however, later sections in this chapter depend on earlier ones.

One can also consider skipping Chapter 1, if the audience is unusually well prepared.

The book has at least three quirks.

1. Although more general settings exist, for a course at this level I restrict attention to the three-dimensional Euclidean space of geometric vectors. I refer to this space as $\mathbb{E}$, reserving the notation $\mathbb{R}^3$ for the space of ordered triples of real numbers. The difference may seem fussy to some, but I want to distinguish 3-vectors from the coordinates used to represent them. In examples and exercises, where concrete numerical representations of vectors are indispensible, I typically fix a basis and then use $\mathbb{R}^3$ freely.
2. For examples and exercises, I stick to Cartesian coordinates. Curvilinear and even nonorthogonal coordinate systems arise frequently in applications. But from a modeler's perspective continuum mechanics serves primarily as a framework for deriving governing equations. At this level direct notation, with occasional reference to Cartesian coordinates, seems pedagogically sufficient.
3. The treatment of PDEs is far from systematic. That subject deserves courses of its own. But I have selected from the theory of PDEs a small number of largely qualitative topics, in an effort to promote intuition about the physics. I hope these topics will either refresh material learned in previous courses on PDEs or prompt students to take such courses.

Interspersed throughout the text are over 200 exercises. Most ask the reader to fill in logical connections that the narrative elides. Some provide glimpses of related topics. I don't intend for these exercises to be difficult, but in case my judgment is flawed I've included an appendix with hints and solutions.

I owe thanks to several people at John Wiley & Sons and their affiliates for guidance during the production of the book. In particular, I'm grateful to Susanne Steitz-Filler, Sari Friedman, Katrina Maceda, and Lincy Priya for their expertise and for the time they spent on the manuscript after it was under contract.

Figure 3.9, adapted from [50, Figure 111, p. 341], appears with kind permission of Springer Science+Business Media.

Anyone who studies continuum mechanics owes a tremendous debt to the giants. I learned a great deal from the work of Ray Bowen, Bernard Coleman, Jerald Ericksen, A. Cemal Eringen, A.E. Green, Morton Gurtin, Paul Naghdi, Walter Noll, Ronald Rivlin, and Clifford Truesdell, among many other masters of the field.

I also owe to my teachers George F. Pinder and William G. Gray the conviction that these concepts lie at the proper foundation of any efforts at mathematical modeling. I'm indebted to Aleksey Telyakovskiy, Li Wu, and Stefan Heinz for reviewing the manuscript. I owe many thanks to several students at the University of Wyoming who endured preliminary notes and early drafts of this material. They asked many questions that helped me correct errors, and they challenged me to develop clearer explanations. Special thanks go to Stephen Bagley, Yi-Hung Kuo, Kevin Lenth, and Xiaoban Wu, who, in addition, developed many hints for the exercises.

Finally, I owe an enormous debt of gratitude to my colleagues at the University of Wyoming, who helped shape my views of this subject over many years and convinced me that it deserves more attention. Special among them is my friend and colleague, Andy Hansen, for more improvements, insight, and encouragement than he knows.

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