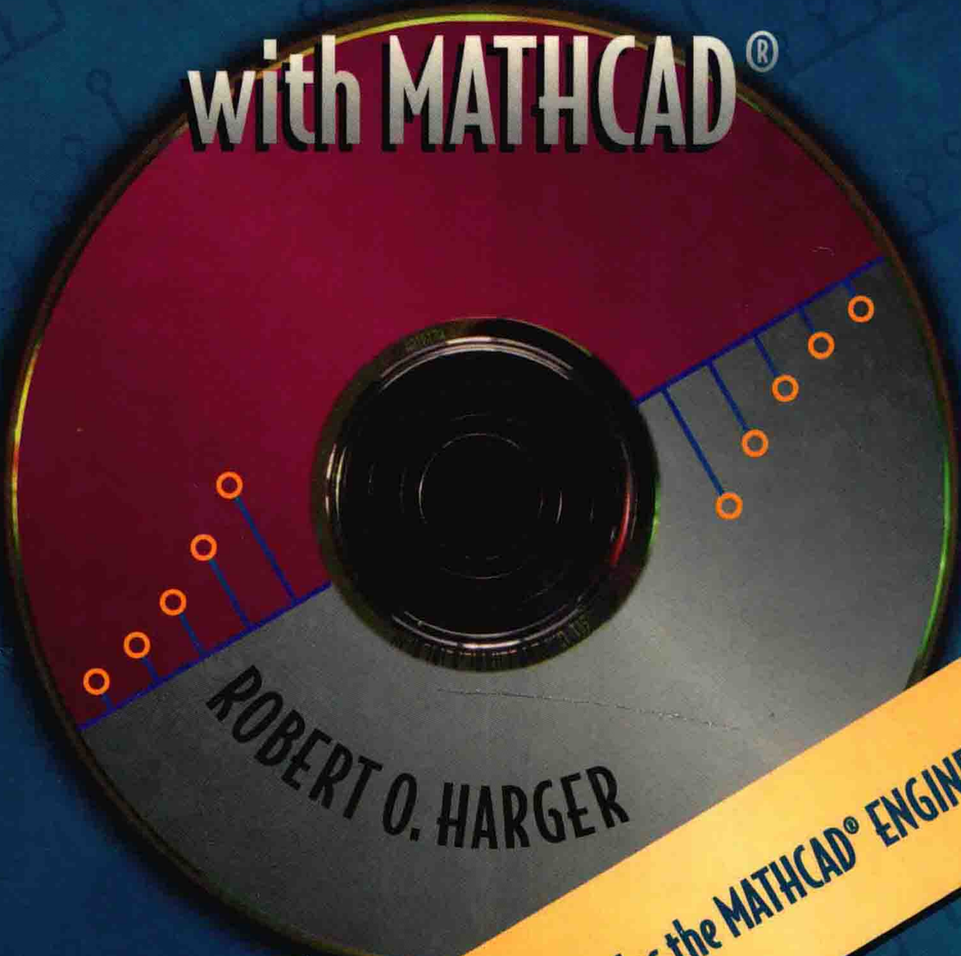
An Introduction to DIGITAL SIGNAL PROCESSING with MATHCAD®



ROBERT O. HARGER

Includes the MATHCAD® ENGINE 7.0

An Introduction to

Digital Signal Processing
with Mathcad®

A Mathcad Electronic Book

Robert O. Harger
University of Maryland

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International Thomson Publishing Japan
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About Mathcad Electronic Books

Mathcad Electronic Books are described in the *Mathcad User's Guide*. We point out here five special features that facilitate their use.

The Electronic Book Controls Palette

A Mathcad Electronic Book is a sequence of Mathcad documents. Each document corresponds to a section of a traditional book. To enable flexible access to these sections, there is a **Controls Palette** at the upper left of the Mathcad applications window. It consists of seven buttons that allow direct access to the Table of Contents, the Index, the preceding and succeeding sections, the previously accessed section and the previous and next page in the accessed section.

Hyperlinks

Hyperlinks from a text region of a document to another document are indicated by the presence of **underlined and boldfaced text**. If you **click once** on a hyperlinked region, you will see a **message** at the bottom of the application window. (Try it!) The message will tell you what **action** will happen if you activate the hyperlink: e.g., "Go to Chapter 13". To execute the action, **double-click** on the region. The associated message will often indicate a page of the linked section. After double-clicking on the linked region, you can then use **Go to Page...** under the **Edit** menu to get to the page of the linked section.

The **hyperlink icon** shown at the left will result in a special **Book Popup Window**. It is used in this book to provide amplifying remarks to exercises, problems and projects. The **Book Popup Window** contains a special Mathcad document that can only be copied to the clipboard.

Special Pasting Action

Mathcad documents are made up of regions containing text, mathematics or graphics. A region can be easily copied from the Electronic Book to a user-created document by dragging. Multiple regions may be more easily copied using the Windows Clipboard.

The Full-Text Search

Every word in an Electronic Book can be located using the Full-Text Search feature, which is accessed by choosing **Search Book** in the **Books** menu.

Annotation

While the original Electronic Book is always accessible, the user can make, highlight, save and delete changes for a personal copy This feature is accessed by choosing **Annotate Book** in the **Books** menu.

Preface

This interactive book is an introduction to the analysis, modeling, design and simulation of digital signal processing systems directly on a digital computer using the high-level mathematical language Mathcad. The use of a high-level mathematical language, with symbolic, numeric, graphic and text capabilities, enables relatively easy implementation of all the topics covered in an introductory course in digital signal processing systems. Interactivity, "what if" trial-and-error solutions and graphic visualization are powerful learning methods.

A Mathcad document has a worksheet interface familiar to scientists and engineers. The syntax is relatively easy to learn and a productive facility can be obtained in a few hours. A brief Mathcad tutorial is included at the end of the book to help you get started. The active document is itself a self-checking report, easily distributed electronically.

This book is a set of hyperlinked Mathcad documents. Each document is completely interactive and composed of text, mathematics and graphics regions that can be edited and copied into new documents created by the user.

Such an environment naturally enforces some of the reality of digital signal processing (DSP) systems. A successfully implemented model is an efficiently produced prototype to guide implementation in the lower-level mathematical languages of real DSP systems.

Introduction

Digital signal processing (DSP) is an area of ever-increasing importance and interest. Its diverse applications include systems that transmit and store information - for example, digital communication systems such as wireless networks; systems that process data - for example, radar and sonar systems; and personal entertainment systems - for example, the digital compact cassette. Spurred by the ever-increasing power of microchip digital computer and the availability of high-level software for analysis and design, DSP has rapidly matured into a distinct subject based on numerical computation algorithms. Software systems are used in all stages of digital system design, from algorithm design to manufacturing. Mathematical software such as Mathcad is an economical and powerful tool for DSP algorithm conception, understanding and design.

Our primary objective in this book is to introduce the reader to digital signal processing on the computer. A secondary objective is to move reasonably rapidly to interesting, motivating applications of DSP. Collaterally, the reader will attain a facility with Mathcad, a high-level mathematical programming language. A background tha would be helpful to the reader is a previous exposure to the concepts of linear systems and the use of a computer with a Windows operating system. However, much of the material is developed from basic ideas.

Contents

A linear system that is defined by the finite convolution sum, with a finite length impulse response (FIR), has many applications, including imaging and adaptive systems, and is a natural starting point for discussion of DSP systems. The natural frequency decomposition, in which many filter and system concepts are phrased and best understood, is given by the discrete Fourier transform (DFT). The DFT is implemented via what is arguably the most important algorithm in DSP, the fast Fourier transform (FFT). Then signal modeling, generation, filtering and simulation can be discussed with the FFT at hand as a built-in function in Mathcad. Chapters 1 through 4 cover this basic DSP material.

An introduction to digital image processing is accessible at this point and is given in Chapter 27. Chapter 5 applies these basic concepts to the practically important problem of processing long signals.

The frequency decomposition appropriate for arbitrarily long signals is the discrete-time Fourier transform (DTFT), a theoretical construct described in Chapters 6 and 7. The DTFT is introduced as a limiting case of the DFT. The DTFT has periodicity 2π in the frequency (ω) and is, in the cases of interest here, a rational function of $\exp(i\omega)$. The poles and zeros are of considerable interest as they provide insight into the nature of a linear system and even yield a crude design technique. The poles and zeros generally are not of the form $\exp(i\omega_k)$; in other words, they may lie elsewhere in the complex (z) plane. If one replaces $\exp(i\omega)$ with z in the DTFT, the resulting form is called the z transform (ZT). As numerical algorithms are used in DSP systems, the algebraic DTFTs and ZTs needed here are computed knowing the sum of a geometric series and using the symbolic mathematics capability of Mathcad.

FIR filters can have a linear phase characteristic, which is of considerable practical importance. A nonlinear phase characteristic causes, for example, the dispersion of pulses which can cause intersymbol interference and hence errors in digital communication systems. This class of FIR filters is described in Chapters 8 and 9.

Chapters 10 through 14 describe several FIR filter design techniques. The frequency sampling method, discussed in Chapter 10, also helps to clarify the nature of FIR filters. An application of this filter design method in a sophisticated communication system that generates a "single-sideband waveform" is discussed at length in Chapter 11. The classical window design methods are covered in Chapter 12. Chapter 13 discusses design with an optimal window and demonstrates the power of high-level software in DSP design. Here the relation to the Kaiser approximation is pointed out. The equiripple approximation, a popular design criterion for a filter's frequency response, is given an accessible treatment in Chapter 14. Advanced algorithms that compute such filters are a standard feature of DSP design software.

In many applications the ultimate source of digital signals is a sequence of samples of an analog signal. Chapter 15 addresses the critical relation between the two types of signals and explores related topics such as aliasing, reconstruction, truncation and spectral estimation. Finally, because the ability to change sample rates of digital signals is necessary, Chapters 16 and 17 treat the interpolation and decimation operations whose effects are most readily understood by associating the digital signal with a real or virtual analog signal.

A linear difference equation is a natural, efficient computational algorithm, and its implementation can be a preferred realization of a DSP system. When it has an autoregressive part, its impulse response will generally be of infinite length. Hence such filters are called infinite impulse response (IIR) filters, and the complications of stability, or summability, of sequences arises. Chapters 18 through 20 discuss these complications and demonstrate that the DTFT, necessarily a rational function of $\exp(i\omega)$, provides the needed theoretical tool for frequency domain analysis of these systems. Its extension to the ZT gives the pole-zero portrait of an IIR filter and leads to the geometric view, which provides insight into simple design methods.

Chapter 21 introduces a classical design method for IIR filters: mapping an analog filter design into a digital filter design with the bilinear transformation. The design of digital filters of the Butterworth and Chebyshev families is discussed in Chapters 22 and 23, respectively. The digital filter is directly designed for a specified squared-magnitude frequency response.

Chapters 24 through 26 introduce spectral density estimation. The spectral density can be a defining characterization of a digital signal for DSP. We require some background on random sequences, which is provided in Chapter 24, and statistical estimation, which is provided in Chapter 25. Chapter 26 discusses the common periodogram estimator and a modification of it to provide a computationally effective spectral density estimator.

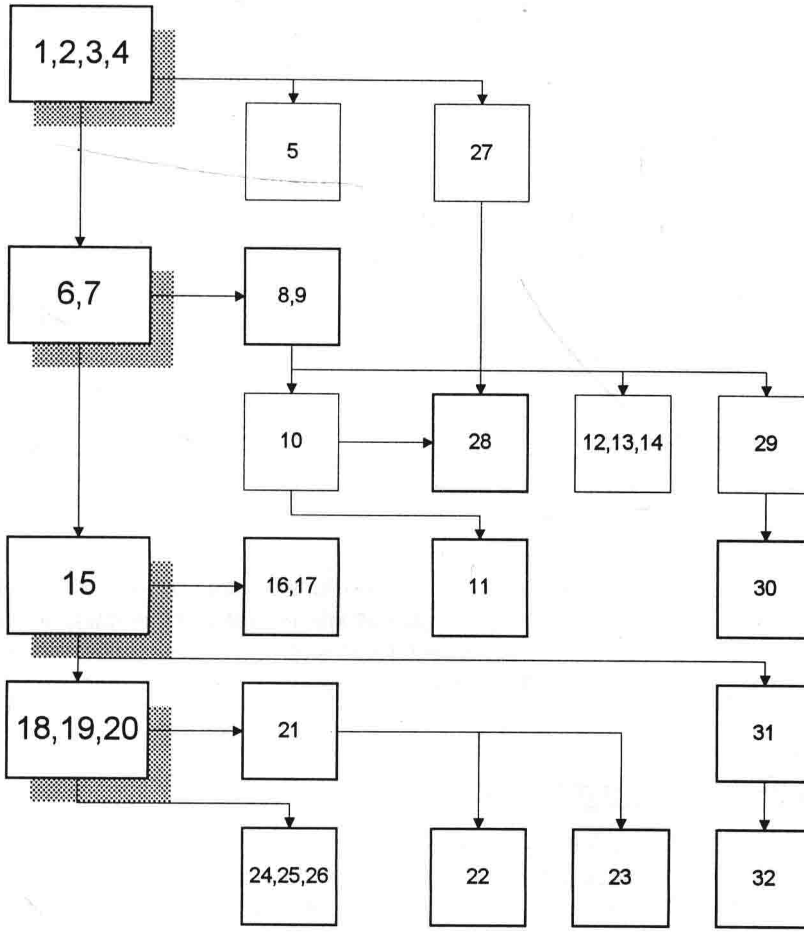
Chapter 27 introduces digital images and their processing with simple FIR filters. The large computational demands of image processing are immediately evident. Chapter 28 introduces two-dimensional FIR filter design by the frequency sampling method.

Chapter 29 discusses the increasingly important topic of adaptive filtering. Such filters are useful when the data characterization is unknown or temporally varying. System identification and noise cancellation with the LMS algorithm are discussed. Then Chapter 30 considers a sophisticated application, the important problem of adaptive filtering, or equalization, for digital communication through bandlimited channels.

Radar signal processing was historically one of the first and most technologically demanding applications for DSP. Chapter 31 discusses the classic range-finding radar and illustrates the use of symbolic mathematics in a system analysis. Chapter 32 discusses imaging synthetic aperture radar, which is itself a sampling system. Its data rate is so great that DSP has only gradually replaced analog processing.

The reader should select and order the material in this book according to his or her own needs and interest. Interdependencies of the sections are sketched below for reference. Column flow on the left indicates a sequence through the basic ideas of DSP. This material can be augmented at various points. Notice, for example, that a basic introduction to image processing (in Chapter 27) can be taken up near the outset. Important augmentations to the basic material are Chapters 8 and 9, which provide additional material on FIR filters with linear phase, and Chapters 10 and 12, which discuss simple but useful design methods. Following the discussion of FIR filters and FIR design via the frequency sampling method, the reader may refer to Chapter 28 for information on the design of image-processing filters. Alternatively, the reader could progress to Chapter 29 for a discussion of adaptive filtering, or to Chapter 30, which explores digital communication, a key DSP application.

Section Dependencies



Suggestions on Use

The book has been written in an informal style to invite active involvement. Theorem - proof style has been deliberately avoided, but assertions are supported appropriately, with references given. Definitions are indicated with boldface. Examples are often used to aid in introducing new ideas.

Three types of specific interactivity are suggested: exercises, problems and projects, in order of increasing difficulty. Exercises are an integral part of the development of the material and generally request interactivity, such as changing a parameter and noting the result. Most of these are meant to be done as a chapter is studied.

Problems may suggest supplementary work and may require construction of a proof or a Mathcad document, perhaps after consulting a reference. The projects are meant to show the application of the material and, especially here, the reader should follow his or her own interest. They range in difficulty from straightforward design projects to those worthy of a final course project.

Each chapter is a Mathcad document that can be rapidly computed to enable efficient trial-and-error learning. Each also contains more than enough material for a learning unit corresponding to a one-hour-and-fifteen-minute interactive session.

After diligent interaction with the core material of this book, the reader will have developed a working familiarity with the basic ideas of digital signal processing and the principal methods of digital filter design. This interactive study also introduces the important DSP applications of digital image processing, spectral density estimation, adaptive filtering, radar data processing and digital communications. The reader will also have developed an independent skill with a high-level mathematical programming language with which he or she may efficiently develop realistic DSP models, algorithms, designs and simulations.

Acknowledgements

I would like to thank the students who have helped me develop the book and its use in a computer classroom over many semesters at the University of Maryland. Some of their suggestions are cited in the book. I also thank the editors at PWS Publishing for supplying the core of the tutorial chapter and for their guidance, patience and endurance in this novel venture. I and the publisher also wish to thank the following reviewers: Guoxiang Gu, Louisiana State University, Yu Hen Hu, University of Wisconsin, Kurt Kosbar, University of Missouri - Rolla, Bisi Oladipupo, Hampton University and Ravi Warrier, GMI Engineering and Management Institute.

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Digital signals as sequences; examples of digital signals; basic operations on sequences; basic DSP operations on digital signals

2 Digital (FIR) Filters

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3 Discrete Fourier Transform

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4 Discrete Fourier Transform Properties

Basic properties; DFT and periodic convolution; fast Fourier transform; a digital signal processing application

5 Fast Convolution

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6 Discrete-Time Fourier Transform

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7 Discrete-Time Fourier Transform Properties

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