GRAY and ULM

APPLICATIONS OF COLLEGE MATHEMATICS

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Preface

This textbook is one volume of a two-volume series in mathematics for college students. The two volumes constitute a course of study for the student whose paramount interests do not lie in fields that require a profound knowledge of mathematics. They may be used either sequentially or independently, for no specific cross-references are made to either volume.

The material presented is a nontheoretical approach to the study of applied mathematics. Theory has been included where necessary to facilitate understanding of applications. Chapter 1 develops the concepts of sets, relations, and functions only to the extent needed to understand the mathematics presented in succeeding chapters.

Topics have been selected that are appropriate and appeal to those students planning careers in business, finance, education, and the social sciences. We hesitate to recommend the study of specific chapters for the several major fields of study. However, it seems that the entire textbook, with the possible exception of chapter 8, is suitable for a course in applied business mathematics and finance. The education major may find chapters 1, 2, 3, 4, 5, and 8 most appropriate for a course of study. Chapters 1, 2, 3, 5, and 9 should be of particular interest to the students who are majoring in the social sciences.

Students entering this course should have a minimum of three years of high school mathematics or equivalent preparation at the college level. For those students who require a review of the real number system and the coordinate plane, we have included a brief discussion of each in the appendices.

We wish to express our thanks to our colleagues for their assistance and encouragement. Our special thanks go to our typist Mrs. John McAvoy for her excellent work and to our wives, Maggie and Jody, for their patience and understanding.

St. Petersburg, Florida

A. WILLIAM GRAY
OTIS M. ULM

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1.
Sets, Relations, and Functions

1.1 INTRODUCTION

The student of today finds that the rate of change in his environment is increasing at a rapid pace. In our changing society, for a citizen to be informed, for a manager to be able to supervise and control his business, or for an employee to perform his tasks successfully, a working knowledge of mathematics is desirable if not essential. We are aware of the necessity for the scientist and the engineer to have a profound knowledge of the concepts of mathematics, but we may not be aware of the extent to which mathematics is employed in all phases of modern life.

The objectives of the authors are to develop an appreciation of the extent to which mathematics permeates modern society. Our civilization depends heavily on mathematics and could not long exist, let alone make progress, without this important tool. Such being the case, we can live and cope more effectively with today's problems if we have some knowledge of the way mathematics is applied. Many of the topics in the succeeding chapters are familiar to you, for you may have studied them in some form in a previous mathematics course. However, in some cases, this may well be the first time that you have encountered some topics that are included. Today, it is quite fashionable to think of the computer as the answer to all problems dealing with mathematics. We live in the computer age, but the computer can perform only those mathematical tasks that man instructs it to do.

Your introduction to this course of study begins with a study of some of the profound notions of mathematics, that is the study of sets, relations, and functions. These notions are presented as an aid to the understanding of the mathematical applications as presented in the succeeding chapters.

1.2 SETS AND SUBSETS

We think of a *set* as a collection of objects such as dishes, pencils, books, or numbers. The individual components that make up the set are called *elements* and each individual component is an element of the set. The terms "set" and "element" may be described in this way but, mathematically speaking, they are *undefined terms*. When the number of elements in a set is zero or a natural number we say the set is *finite*. A set is *infinite* when the number of elements cannot be denoted by zero or a natural number.

The use of sets is helpful in clarifying many of the concepts in elementary mathematics and in unifying different but related ideas. There are several ways in which sets are denoted. The three more commonly used ways of describing sets are by "verbal description," "the roster method," and "the defining rule." When the verbal description method is used, the set is defined by a sentence that indicates the elements that compose the set. When the roster method is used, the elements of the set are listed and enclosed in braces. When the defining rule is used, the set is denoted by a descriptive phrase that is enclosed in brackets and only those elements which have this desired property are elements of the set.

The three methods are used in the following illustrations.

Illustration 1.

(a) By the verbal description method. The set A is the set of even integers greater than zero but less than ten.

(b) By the roster method. The set $A = \{2,4,6,8\}$.

(c) By the defining rule method. The set $A = \{x$, such that x is an even integer greater than zero but less than ten$\}$.

A shorter and more efficient notation for the defining rule is

$$A = \{x \mid 2 \leq x \leq 8 \text{ and } x \in \text{even integers}\},$$

where the bar is read "such that" and ε is read "is an element of." This latter method is sometimes called "set builder notation."

Illustration 2.

(a) The set B is the set of letters in the English alphabet.

(b) $B = \{a,b,c,\cdots,z\}$. (The three dots denote the missing elements).

(c) $B = \{x \mid x$ is a letter of the English alphabet$\}$. (Note that x is used as an arbitrary element of the set, not as a letter of the alphabet).

It is often difficult to describe a set by a particular method. Consider the following set A.

$$A = \{\text{pony, bird, pebble, spinach}\}.$$

The defining rule would be difficult if not impossible to determine. On the other hand, it would be impossible to use the roster method to indicate a set S when $S = \{x \mid 1 \leq x \leq 2$ and x is a number$\}$. Thus, we see that it is important that sets may be described in more than one way.

RELATIONSHIP BETWEEN SETS AND SUBSETS

There are several types of relationships that exist between sets. The first of these that we shall consider is that of identical sets.

Definition 1.1: Two sets, A and B, are said to be *identical* when every element of set A is an element of set B and every element of set B is an element of set A.

When two sets, A and B, are identical the relationship is written symbolically as $A = B$.

Illustration. If $A = \{1,2,3\}$ and $B = \{2,3,1\}$ then $A = B$

When set A is not identical to set B it is still possible for each element of A to be an element of B. In such cases we say that set A is a *proper* subset of set B.

Definition 1.2: The set A is a *proper subset* of set B when every element of set A is an element of set B and set B contains one or more elements that are not in set A.

When set A is a proper subset of set B the relation is written symbolically as $A \subset B$.

Illustration 1. If $A = \{1,2\}$ and $B = \{1,2,3\}$ then $A \subset B$.

Illustration 2. If $A = \{a,b,c,\cdots,z\}$ and $B = \{a,e,i,o,u\}$ then $B \subset A$.

When we are not sure that sets A and B are identical but we know that every element of A is an element of B, then we say that A is a subset of B, that is, A is a proper subset of B or A is identical to B.

Definition 1.3: The set A is a *subset* of set B when every element of set A is an element of set B.

This relationship is symbolized by $A \subseteq B$ (read "A is a subset of B").

Illustration 1. If A is the set of people on campus and B is the set of students on campus then $B \subseteq A$.

Illustration 2. If A is the set of students who register for a course and B is the set of students who will pass the course then $B \subseteq A$.

The next relationship that we shall consider is that of a one–to–one correspondence between sets. Suppose there are 25 chairs and 25 students in a classroom. When the students are seated there is a chair for each student and a student for each chair and we say that a one–to–one relationship exists between the two sets.

Definition 1.4: Two sets, A and B, are said to be in a *one–to–one correspondence* when each element of A is paired with one and only one element of B and each element of B is paired with one and only one element of A.

Illustration 1. If $A = \{1,2,3,\cdots\}$ and $B = \{-1,-2,-3,\cdots\}$ then the two sets may be paired in the following way.

$$A = \{\ 1 \quad 2 \quad 3 \quad \cdots \quad n \quad \cdots\}$$

$$B = \{-1 \quad -2 \quad -3 \quad \cdots \quad -n \quad \cdots\}$$

Illustration 2. If set $A =$ {Mary, Jane, Sue} and $B =$ {Tom, Bill, Joe} then the two sets may be paired in the following way.

$A =$ {Mary, Jane, Sue}

$B =$ {Tom, Bill, Joe}

When two finite sets can be put into a one–to–one correspondence then we say that the two sets are equivalent.

Definition 1.5: Two finite sets, A and B, are *equivalent* when they can be put into a one–to–one correspondence.

This equivalence relationship between two sets is written symbolically as $A \longleftrightarrow B$.

Illustration 1. If $A = \{a,b,c,d\}$ and $B = \{1,2,3,4\}$ then $A \longleftrightarrow B$.

Illustration 2. If $A = \{\text{cat,car,penny}\}$ and $B = \{\text{Sue,Jo,Elmer}\}$ then $A \longleftrightarrow B$.

The sets which we have discussed and illustrated are such that we know immediately whether or not an element is a member of the set. Such sets are called *well–defined sets.* To avoid confusion it must be possible to determine whether a given element belongs to a set by checking it against the list of elements in the set or against the rule governing membership in the set. The set of letters in the English alphabet is well defined, but the set of "good people" is not well defined, for there are no established criteria for determining "good people."

The set that contains all of the elements under discussion is called the universal set or simply the "universe."

Definition 1.6: The set containing all of the elements for a particular discussion is called the *universal set.* This set is denoted by U.

The universal set may be finite or infinite.

Illustration 1. If $U = \{a,b,c,\cdots,z\}$ then U is a finite set.

Illustration 2. If $U = \{1,2,3,\cdots\}$ then U is an infinite set.

A set with no elements is called a null set and is defined as follows:

Definition 1.7: A set which contains no elements is the *null set* or *empty set.* This set is symbolized by ϕ or $\{\}$.

There is only one null set and it is a proper subset of every set except itself.

Illustration. If W is the set of women who have been president of the United States then the set W is empty.

Symbolically this set is written as $W = \phi$ or $W = \{\}$.

A comparison between the two sets U and ϕ shows that they have no elements in common, but the combined elements are the same as the universe. When the combined elements of two sets that have no elements in common are the same as the universe, we say that the two sets are complementary.

Definition 1.8: Given a universal set U and a subset A, then the *complement of* A (written A') is the set of elements in U that are not in A.

Illustration 1. If $U = \{1,2,3,4,5\}$ and $A = \{1,3,5\}$ then $A' = \{2,4\}$.

Illustration 2. If $U = \{x \mid x$ is a letter of the English alphabet$\}$ and $A = \{y \mid y$ is a vowel$\}$, then $A' = \{z \mid z$ is a consonant$\}$

In the illustrations the set A is a proper subset of U and A' is a proper subset of U, thus the relationship $A' \subset U$ is always true unless $A = \phi$.

SET OPERATIONS FOR FORMING NEW SETS

A new set can be derived from two initial sets, A and B, by forming the intersection of the two sets. The set formed by the intersection is the set of elements that is common to both.

Definition 1.9: The *intersection* of sets A and B, denoted by $A \cap B$, is the set of elements that belong to both A and B. Symbolically, $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.

The expression $\{x \mid x \in A \text{ and } x \in B\}$ is read, "the set of all x such that x is an element of A and x is an element of B."

Illustration 1. If $A = \{1,2,3\}$ and $B = \{2,4,6,8\}$ then $A \cap B = \{2\}$.

Illustration 2. If $A = \{1,3,5,7,\cdots\}$ and $B = \{2,4,6,8,\cdots\}$ then $A \cap B = \phi$.

When the intersection of two sets is the null set, then the two sets have no common elements and are said to be disjoint.

Definition 1.10: Two sets, A and B, are *disjoint* if there is no element common to both.

The new set formed from the intersection of disjoint sets is always the null set, that is, $A \cap B = \phi$.

Illustration 1. If $A = \{1,3,5,7,\cdots\}$ and $B = \{2,4,6,\cdots\}$ then $A \cap B = \phi$ and sets A and B are disjoint sets.

Illustration 2. If $A = \{x \mid x$ is a male$\}$ and $B = \{y \mid y$ is a female$\}$ then $A \cap B = \phi$.

Complementary sets are, by definition, disjoint sets for they have no elements that are common to both.

A new set derived by combining two initial sets, that is, forming a set from the elements that are in either set, is called the union of the two sets.

Definition 1.11: The *union* of sets A and B, denoted $A \cup B$, is the set of elements that are in A or B. Symbolically, $A \cup B = \{x \mid x \varepsilon A \text{ or } x \varepsilon B\}$.

The expression on the right of the identity sign is read, "the set of all x such that x is an element of A or x is an element of B."

Illustration 1. If $A = \{1,2,3,4\}$ and $B = \{4,5,6,7\}$ then $A \cup B = \{1,2,3,4,5,6,7\}$

Illustration 2. If $A = \{x \mid x \text{ is a vowel}\}$ and $B = \{y \mid y \text{ is a consonant}\}$ then $A \cup B = \{z \mid z \text{ is a letter of the English alphabet}\}$.

There is another way of deriving a new set from initial sets. This operation is not as obvious as that of forming the union, intersection, and complement but is just as important. This operation differs from the others because it is based on the idea of *ordered pairs.* Ordered pairs, simply stated, are pairs of elements taken according to some fixed order. This ordered pair concept is utilized in defining the Cartesian product of two sets.

Definition 1.12: The *Cartesian product* of two sets, A and B, is the set of all ordered pairs (a,b) such that $a \varepsilon A$ and $b \varepsilon B$.

The Cartesian product is denoted by $A \times B$ (read "A cross B") and is not to be confused with A times B.

Illustration 1. If $A = \{1,2,3\}$ and $B = \{\text{a,b,c}\}$ then $A \times B = \{(1,\text{a}),(1,\text{b}),(1,\text{c}),(2,\text{a}),(2,\text{b}),(2,\text{c}),(3,\text{a}),(3,\text{b}),(3,\text{c})\}$.

In Illustration 1 the number of elements in A is 3, the number of elements in B is 3, and there are $3 \cdot 3 = 9$ elements in the product set.

Illustration 2. A stationery company makes three colors of stationery, blue, pink, and white. The stationery is scented either lilac, gardenia, magnolia, Arpege, or Chanel. If we let A = {blue,pink,white} and B = {lilac,gardenia,magnolia,Arpege,Chanel} then $A \times B$ = {(blue, lilac), (blue,gardenia), (blue,magnolia), (blue,Arpege), (blue,Chanel), (pink,lilac), (pink,gardenia), (pink,magnolia), (pink, Arpege), (pink,Chanel), (white,lilac), (white,gardenia), (white,magnolia), (white,Arpege), (white,Chanel)}.

In Illustration 2 there are three elements in the set of colors and five elements in the set of perfumes. Thus there are $3 \cdot 5 = 15$ elements in the product set.

Exercise 1.1

1. Write the set $A = \{a,b,c, \cdots, z\}$ in
 (a) Set builder notation.
 (b) By giving its verbal description.

2. Distinguish between a subset and a proper subset.

3. True or false: $\phi = \{\phi\}$. Explain.

4. Explain the statement "Two identical sets are necessarily equivalent but two equivalent sets are not necessarily identical."

5. Which of the following sets are well defined?
 (a) The set of Ford cars.
 (b) The set of students in your mathematics class.
 (c) The set of blonde students.
 (d) The set of well-mannered people.
 (e) The set of good books.

6. In how many ways can the sets A and B be put into a one-to-one correspondence if $A = \{1,2,3\}$ and $B = \{4,5,6,7\}$?

7. Suppose that $A \cup B = A \cap B$. What can you conclude about sets A and B?

8. Suppose that $A' \cap B' = \phi$. What can you conclude about sets A and B?

9. If possible, find sets P, Q, and R such that $P \cup (Q \cap R) = (P \cup Q) \cap R$.

10. If possible find sets P and Q such that $P' \cup Q' \neq (P \cap Q)'$.

11. If $A \subseteq B$ then which of the following is correct?
 (a) $B \subseteq A$
 (b) $B \nsubseteq A$
 (c) All $x \in A$ are contained in B.
 (d) Some $x \in A$ are not contained in B.

12. Mary is giving a party. She invites two girl friends, Jane and Sue, and three boys, Tom, John, and Bill.
 (a) List the different couples that can be formed for dancing (assuming boy dances with girl).
 (b) The set formed in part (a) is a Cartesian product set. Why?

13. In the following problems determine whether $A \subset B$, $B \subset A$, or $A = B$.
 (a) $A = \{1,2,3, \cdots, 10\}$ $\quad B = \{2,4,6,8,10\}$
 (b) $A = \{x \mid x \in \text{natural numbers}\}$ $\quad B = \{x \mid x \in \text{positive integers}\}$
 (c) $A =$ The set of prime numbers $\quad B = \{x \mid x \in \text{rational numbers}\}$
 (d) $A = \{x \mid 10 > x > 0, x \in \text{positive integers}\}$ $\quad B = \{x \mid x \in \text{prime numbers}\} \cap \{x \mid x \text{ is an even integer}\}$