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H. Kolanoski

Two-Photon Physics at e^+e^- Storage Rings



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With 123 Figures



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Preface

The investigation of two-photon interactions is a relatively young field of experimental particle physics and is closely connected to the advent of high energy electron-positron storage rings. In a storage ring the interactions between the clouds of virtual photons accompanying both beams can be used to study photon-photon reactions over a wide kinematical range. The majority of experimental results in this field has been obtained only in the last five years, in particular at the storage ring PETRA in Hamburg. In two-photon reactions with leptonic final states quantum electrodynamics can be tested up to the fourth order of the fine structure constant α . In the last few years, the experimental activities have concentrated on studies of two-photon production of hadrons and results have been obtained on the two-photon coupling of meson resonances, on the production of hadrons with large transverse momenta and on the structure of the photon as measured in deep-inelastic electron-photon scattering. The simple and controllable initial state in two-photon interactions allows hadron dynamics to be probed cleanly and allows tests of quantum chromodynamics, the theory describing strong interactions among hadrons.

This report reviews the experimental status of two-photon physics and discusses to what extent the theoretical predictions for two-photon reactions have been corroborated by experiment.

Bonn, January 1984

H.Kolanoski

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The work on the review originally started from lectures which I gave at the XXII. Cracow School of Theoretical Physics at Zakopane in June 1982. I would like to thank the organizers of this school, in particular Profs. A. Białyas and K. Zalewski, for the kind invitation and for the hospitality I received during my stay in Poland.

Finally, I would like to thank my wife Ria and my daughters Saskia, Julia and Martina for their patience and for allowing me the time and space within the family to do this work.

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1. Introduction

According to Maxwell's classical linear equations, electromagnetic waves cross each other without disturbance. However, the quantized theory of electromagnetism, quantum electrodynamics, predicts that interactions occur between the field quanta, the photons, by way of quantum fluctuations of the vacuum. For small photon energies, as in the optical range, the cross sections are extremely small. Therefore, photon-photon scattering has only been observed at large energies, i.e. above the threshold for the production of electron-positron pairs.

Very intense photon sources are provided by storage rings for highly energetic electron and positron beams. These rings were built to investigate the annihilation of electrons and positrons. In lowest order of the electromagnetic coupling constant α , $O(\alpha^2)$, that process can be viewed as the annihilation of e^+e^- into a virtual (timelike) photon which then couples to a final state X of leptons or hadrons (Fig.1.1a).

The two-photon production of a final state X,

$$e^+e^- \rightarrow e^+e^- X,$$

as shown in Fig.1.1b is of order α^4 . Each of the two leptons radiates a photon and the two photons produce the final state X. In this case the produced system X has even charge conjugation, whereas in the one-photon annihilation process the final state has odd charge conjugation. Involving two electromagnetic currents, two-photon reactions exhibit a rich dynamical structure which supplies additional and often complementary physics information to that obtained from one-photon annihilation processes.

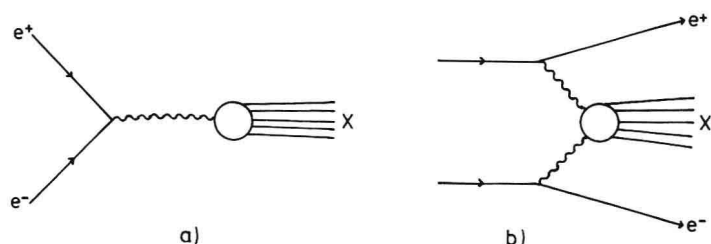


Fig.1.1. Production of a final state X in e^+e^- collisions: a) by one photon and b) by two photons.

1.1 Physics with Two Photons

The cross sections arising from diagram b) increase with the beam energy E like $(\log E/m_e)^2$, while those from a) decrease at least like $1/E^2$. Thus it turns out that the two-photon process, although of order α^4 , already dominates over the order α^2 annihilation process at beam energies of a few GeV. However, most of the events from the two-photon process have low invariant masses of the produced system due to the typical bremsstrahlung spectrum, $\sim 1/E_\gamma$, of the radiated photons.

The photon propagators in diagram b) cause the bulk of the photons to be radiated nearly on-mass-shell at small angles relative to the beam. This effectively provides two colliding beams of quasi-real photons, with a small beam divergence of order m_e/E . Hence we talk about photon-photon scattering, regarding the electron and positron beams just as sources for the "photon beams". The high energy machines available today, like PETRA and PEP, provide powerful photon beams with high two-photon luminosities¹. Integrating over invariant photon-photon masses above the pion pair threshold, the two-photon luminosities at beam energies around 15 GeV are only an order of magnitude smaller than the e^+e^- luminosities. Whereas for a given beam energy the e^+e^- kinematics of an annihilation process is fixed, the continuous spectra of the photon beams allow simultaneous measurements at different $\gamma\gamma$ invariant masses and for different momentum transfers and polarisations of the photons.

Quantum electrodynamics (QED) can be tested up to order α^4 by measuring two-photon production of lepton pairs. The investigation of two-photon production of hadronic final states provides the possibility of probing hadron dynamics with a simple, calculable initial state. Our main interest lies in the study of the coupling of the photon to hadrons, which in the framework of the quark model means the coupling to quarks as the constituents of hadrons. In the resonance region the quark model of bound quark-antiquark systems can be tested by measuring the two-photon resonance couplings. A large fraction of the experimental results obtained in the last years deals with these couplings. Enough information is now available to allow meaningful tests of SU(3) symmetry and of models with multiquark or gluonium states. The possible existence of bound states of gluons (gluonia) has been discussed extensively in recent years and the measurements of the $\gamma\gamma$ widths of resonances provide important inputs to this debate.

The study of hard scattering processes in two-photon reactions was made possible when beam energies up to about 20 GeV became available at the e^+e^- storage rings PETRA at Hamburg and PEP at Stanford. The investigation of the production of particles with high transverse momenta, jet production, and scattering of highly virtual photons (deep-inelastic scat-

¹ The luminosity L of interacting beams determines the rate \dot{N} at which a reaction with a cross section σ occurs according to the equation: $\dot{N} = L \cdot \sigma$.

tering), allows tests of quantum chromodynamics (QCD), the widely accepted theory of strong interactions.

Two-photon production of hadrons proceeds in lowest order as in Fig.1.2a where the two photons have a pointlike QED coupling to a quark pair and the quarks subsequently fragment into hadrons. However, this pointlike behaviour of the photon is observable only at high momentum transfer, in interactions at small distances. We know that the vector meson dominance (VMD) model works rather well in most processes involving real or quasi-real photons. In the VMD picture the photons transform into virtual vector mesons (ρ , ω , ϕ , ...) which then interact strongly with hadrons (Fig.1.2b). According to this model photon-photon scattering should exhibit the characteristic features of hadronic interactions observed in hadron-hadron scattering.

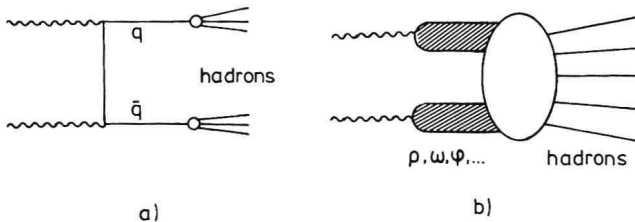


Fig.1.2. a) Pointlike and b) hadron-like two-photon interactions.

Since the "hadron-like" and the "pointlike" pieces arise from the same photon, we expect a smooth transition between the two pictures. The scale-free coupling in Fig.1.2a is reached only asymptotically for high transverse momenta of the quarks or for a high momentum transfer of at least one of the photons to the quarks. As the momentum transfer becomes smaller, the interaction is spread over larger distances and longer times, so that the quarks can interact via gluon exchange and form bound states carrying the quantum numbers of the photon (vector mesons) as sketched in Fig.1.3.

In two-photon reactions this dual nature of the photon can be investigated: In the regime of low four-momentum transfer Q^2 of the photons and low transverse momenta of the produced hadrons the hadronic character of the photon will dominate (total cross section of quasi-real photons). At large Q^2 and/or high transverse momenta of the hadrons the elementary, pointlike nature of the photon is emphasized (high- p_T jets, structure functions).

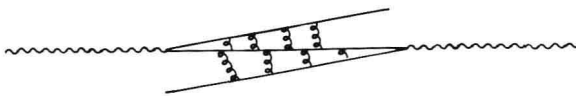


Fig.1.3. Sketch of $\gamma\gamma$ coupling to quarks with low transverse momentum.

1.2 The Historical Development of Two-Photon Physics

The first calculations of two-photon reactions, specifically elastic photon-photon scattering, were published about 50 years ago /1/. Much later, in 1960, the first theoretical papers relevant to two-photon physics at e^+e^- storage rings appeared. F.Low /2/ suggested measuring the two-photon coupling of the π^0 at storage rings via

$$e^+e^- \rightarrow e^+e^-\pi^0$$

and F.Calogero and C.Zemach /3/ calculated the process

$$e^+e^- \rightarrow e^+e^-\pi^+\pi^-.$$

A large number of papers appeared at the end of the sixties and early seventies, when the storage rings in Novosibirsk, Frascati, Orsay, Stanford and Hamburg became available. Most of the theoretical work on low energy and resonance physics with two photons stems from that time and excellent reviews are available /4/. For a rather complete bibliography of the theoretical work from that period see under ref. /4/ the review article of Budnev et al..

At about the same time the first experimental results on two-photon QED reactions were obtained in Novosibirsk and Frascati /5, 6/. With the discovery of the charm quark in one-photon annihilation the general interest turned in this direction and the experimental and theoretical activities in two-photon physics were reduced to a minimum.

In view of the high energies which became available at PETRA and PEP, theorists realized that two-photon physics provides an excellent opportunity to test QCD by studying large- p_T hadron production and jet formation /7/ and by measuring the structure function of the photon /8/. The experimental activity was resumed with the measurement of the two-photon production of the η' at the storage ring SPEAR in 1979. In the following years much experimental information was collected on resonance couplings, total cross sections and hadron pair production. Experiments at PETRA started to look for hard scattering processes such as two-photon jet production and particle production at large transverse momenta. Much effort has also been devoted to the measurements of the structure function of the photon and the first attempts to determine from these measurements the QCD scale parameter Λ .

At the moment we seem to be in a time of fruitful activity and continuous progress in two-photon physics. Many new experimental results are expected in the near future.

1.3 Organization of the Review

This article reviews the experimental achievements of two-photon physics at e^+e^- storage rings in the last years. In general, we want to concentrate on published results and neglect preliminary data unless they

contribute new aspects to a topic. The field is still young enough so that an almost complete coverage of the experimental results seems possible. Theoretical ideas and models are discussed in so far as they are relevant for the understanding of the experimental results. No attempt is made to review the whole theoretical work accumulated over the years, references are mainly given to some basic work and to reviews. In preparing this report, the conference talks on two-photon physics in recent years /9/ and particularly the proceedings of the two-photon meetings /10/ were of substantial help.

The article is organized as follows: In Chap.2 the two-photon kinematics and the general form of the cross section are introduced. Chapter 3 gives a short summary of the characteristics of the detectors mentioned later. The review of experimental results starts in Chap.4 with a brief overview of the QED results. Two-photon meson and baryon pair production is discussed in Chap.5. Much room is devoted in Chap.6 to the review of the experimental results on two-photon couplings of resonances and the discussion of their theoretical implications. The following three chapters deal with measurements of the two-photon total cross section: in Chap.7 the total cross section measurements at low Q^2 are discussed, Chap.8 focuses on the production of hadrons with high transverse momenta and in Chap.9 the case is studied where at least one of the photons is highly virtual, allowing the measurement of the structure of the photon. Finally, Chap.10 summarizes the experimental results and lists the problems and open questions which might be tackled in the near future.

2. Kinematics and Cross Section of Two-Photon Reactions

2.1 The Kinematics of the Reaction $e^+e^- \rightarrow e^+e^-X$

Two-photon scattering in e^+e^- storage rings can be observed in the reaction

$$e^+e^- \rightarrow e^+e^-X \quad (2.01)$$

according to the lowest order diagram in Fig.2.1a: an electron and a positron radiate photons, which produce a particle system X with even C parity. Though this is not the only diagram leading to the reaction (2.01), in most cases it is the dominant one. However, in certain kinematical regions the "virtual bremsstrahlung processes" with negative C parity of the final state X (Fig.2.1b,c) can also give a sizeable contribution [11/].

Due to the photon propagators in diagram a) the photons in this reaction are emitted predominantly at small angles, of order m_e/E , with respect to the beam. That leads to small (space-like) momentum transfers

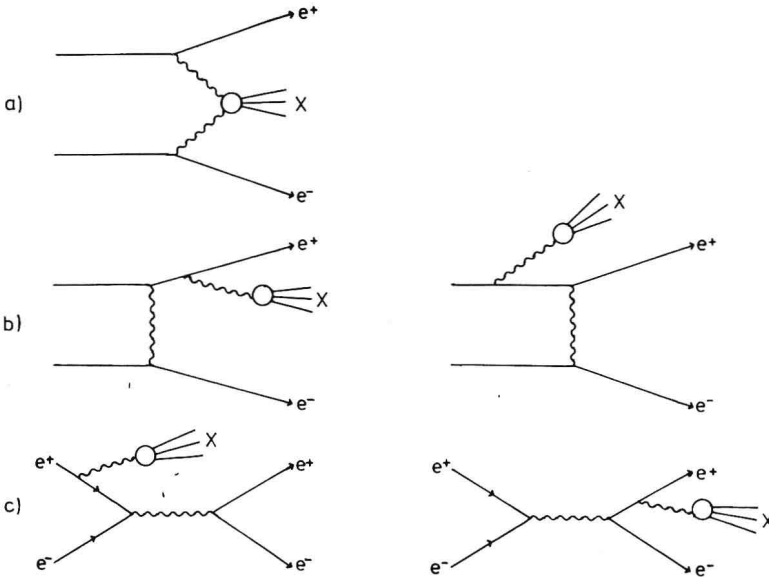


Fig.2.1. Diagrams contributing to the reaction $e^+e^- \rightarrow e^+e^-X$: In diagram a) X is a C -even state, in diagrams b) and c) X is a C -odd state.

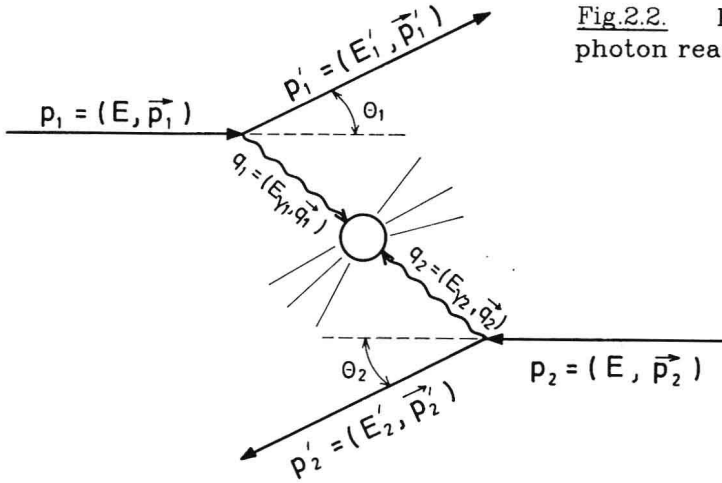


Fig.2.2. Kinematics of the two-photon reaction $e^+e^- \rightarrow e^+e^-X$.

to the system X. The invariant mass of the system X peaks at small values because the photon energies follow roughly the characteristic bremsstrahlung spectrum $\sim 1/E_\gamma$.

For the inclusive detection of X the kinematics of reaction (2.01) is completely determined by the four-momenta of the incoming and of the scattered electron and positron (p_1 , p_2 and p_1' , p_2' , respectively; see Fig.2.2). For unpolarized lepton beams (i.e. if there is no overall azimuthal dependence) five variables are needed to determine the $\gamma\gamma$ system at a given beam energy E. The following variables are defined in the laboratory system:

- the energies E_1' , E_2' of the scattered leptons,
- their angles θ_1 , θ_2 with respect to the beam axis,
- the angle Φ between the two lepton scattering planes.

The kinematical variables of the $\gamma\gamma$ system can be expressed in terms of the variables of the scattered leptons. The energies and normalized energies of the photons are:

$$\begin{aligned} E_{\gamma 1} &= E - E_1' \quad \text{and} \quad E_{\gamma 2} = E - E_2', \\ \omega_1 &= E_{\gamma 1}/E \quad \text{and} \quad \omega_2 = E_{\gamma 2}/E. \end{aligned} \quad (2.02)$$

The squared masses of the (space-like) photons are:

$$\begin{aligned} q_1^2 &= -Q_1^2 = (p_1 - p_1')^2 = 2m_e^2 - 2EE_1'(1 - \sqrt{1 - (m_e/E)^2} \cdot \sqrt{1 - (m_e/E_1')^2} \cdot \cos\theta_1) \\ q_2^2 &= -Q_2^2 = (p_2 - p_2')^2 = 2m_e^2 - 2EE_2'(1 - \sqrt{1 - (m_e/E)^2} \cdot \sqrt{1 - (m_e/E_2')^2} \cdot \cos\theta_2). \end{aligned} \quad (2.03)$$

For $\theta_i \gg m_e/E$ (2.03) reduces to

$$q_i^2 \approx -2EE_i'(1 - \cos\theta_i). \quad (2.04)$$