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Albert C.J. Luo

Discretization and Implicit Mapping Dynamics

离散化与隐映射动力学

HIGHER EDUCATION PRESS

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图书在版编目（CIP）数据

离散化与隐映射动力学 = Discretization and Implicit Mapping Dynamics ：英文 / 罗朝俊著. -- 北京 ：高等教育出版社，2015. 9
（非线性物理科学 / 罗朝俊，（瑞典）伊布拉基莫夫，（墨）阿弗莱诺维奇主编）
ISBN 978-7-04-042835-3

Ⅰ. ①离… Ⅱ. ①罗… Ⅲ. ①非线性力学 - 动力学 - 研究 - 英文 Ⅳ. ① O322

中国版本图书馆 CIP 数据核字（2015）第 196044 号

策划编辑 王丽萍　　责任编辑 王丽萍　　封面设计 杨立新　　责任印制 韩刚

出版发行	高等教育出版社	咨询电话	400-810-0598
社　　址	北京市西城区德外大街4号	网　　址	http://www.hep.edu.cn
邮政编码	100120		http://www.hep.com.cn
印　　刷	北京汇林印务有限公司	网上订购	http://www.landraco.com
开　　本	787mm× 1092mm 1/16		http://www.landraco.com.cn
印　　张	20.25	版　　次	2015 年 9 月第 1 版
字　　数	280 千字	印　　次	2015 年 9 月第 1 次印刷
购书热线	010-58581118	定　　价	89.00 元

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NONLINEAR PHYSICAL SCIENCE

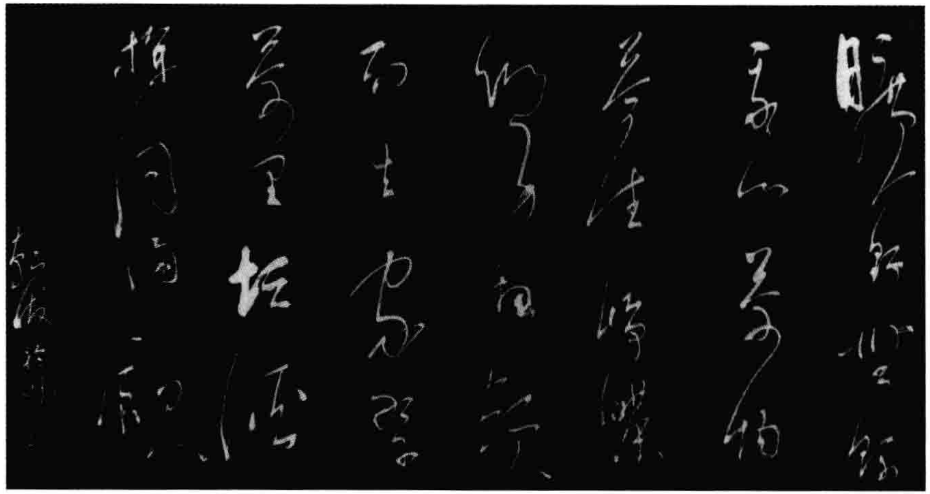
非线性物理科学

NONLINEAR PHYSICAL SCIENCE

Nonlinear Physical Science focuses on recent advances of fundamental theories and principles, analytical and symbolic approaches, as well as computational techniques in nonlinear physical science and nonlinear mathematics with engineering applications.

Topics of interest in *Nonlinear Physical Science* include but are not limited to:

- New findings and discoveries in nonlinear physics and mathematics
- Nonlinearity, complexity and mathematical structures in nonlinear physics
- Nonlinear phenomena and observations in nature and engineering
- Computational methods and theories in complex systems
- Lie group analysis, new theories and principles in mathematical modeling
- Stability, bifurcation, chaos and fractals in physical science and engineering
- Nonlinear chemical and biological physics
- Discontinuity, synchronization and natural complexity in the physical sciences



Preface

This book discusses discretization of differential equations of continuous nonlinear systems and implicit mapping dynamics of periodic flows to chaos. In recent years, approximate analytical solutions for periodic motions to chaos in continuous nonlinear systems were developed by the author through finite Fourier series. However, for many nonlinear dynamical systems, it is difficult to achieve such approximate analytical solutions of periodic motions to chaos. With computer extensive applications in numerical computations, one has used the discrete forms of differential equations of nonlinear systems to obtain numerical solutions via recurrent iterations. The discrete forms in recurrent iterations will cause accumulated computational errors for numerical results. Once the iteration number increases, numerical results given by the discrete forms cannot approximately represent true solutions of nonlinear dynamical systems. To improve the computational accuracy, one has tried to adopt implicit maps as discrete forms to achieve numerical results. However, such implicit mapping forms cannot be iterated directly, which cause the difficulty to extensive applications of discrete implicit maps in continuous nonlinear systems. In this book, the author would like to systematically discuss implicit mapping dynamics of periodic motions to chaos in continuous dynamical systems, and discrete Fourier series based on the discrete nodes of periodic motions will be used to obtain the harmonic responses in frequency space, which can be measured from experiments.

This book includes six chapters. In Chap. 1, a brief literature survey is completed. Chapter 2 reviewed the nonlinear theory for stability and bifurcation of fixed points in discrete nonlinear systems. In Chap. 3, discretization of differential equations is discussed comprehensively. The explicit and implicit discrete schemes in nonlinear dynamical system are discussed through one-step and multi-step discretization of differential equations, and the corresponding stability and convergence of the explicit and implicit discrete maps are discussed. In Chap. 4, implicit mapping dynamics of period-m fixed points in discrete dynamical systems are discussed with positive and negative discrete maps, and the complete solutions of Ying-Yang states of period-m fixed points are presented. In Chap. 5, the methodology for the solutions of periodic motions in continuous dynamical systems with/without time delay is presented through the mapping dynamics of discrete implicit mappings under specific truncated errors. The discrete Fourier series of periodic motions are discussed from discrete nodes of periodic motions, and the corresponding approximate analytical expression can be obtained. Harmonic amplitude quantity levels can be analyzed for periodic motions in continuous nonlinear systems. Chapter 6 discusses the bifurcation trees of periodic motions to chaos in the Duffing oscillator to demonstrate the implicit mapping dynamics of the discretized Duffing oscillator. Such semi-analytical results of periodic motions in the Duffing oscillator are compared with the approximate analytical solutions of periodic motions based on the finite Fourier series solutions.

Finally, I would like to appreciate my former student, Dr. Yu Guo, for completing all numerical computations. Herein, I thank my wife (Sherry X. Huang) and my children (Yanyi Luo, Robin Ruo-Bing Luo, and Robert Zong-Yuan Luo) for their understanding and infinite support.

Albert C.J. Luo

Contents

Chapter 1
Introduction

For solutions of periodic motions in nonlinear dynamical systems, analytical and numerical techniques have been adopted. The analytical methods include the method of averaging, perturbation methods, harmonic balance method, and generalized harmonic balance method. Through the analytical methods, one can obtain the analytical expressions of approximate solutions of periodic motions in dynamical systems. The numerical methods are based on discrete maps obtained by discretization of differential equations for dynamical systems. The discrete maps include explicit and implicit maps. The explicit maps can be directly used to obtain numerical solutions of differential equations for dynamical systems, but the computational errors for the recurrence iteration of explicit maps will be accumulated in numerical results. Once the recurrence iteration times become large, the numerical results may not be adequate for numerical solutions of dynamical systems. Herein, implicit maps will be used to develop mapping structures for periodic motions. The implicit maps cannot be simply used by the recurrence iteration. For periodic flows in nonlinear dynamics, mapping structures based on implicit maps can be developed. Of course, an explicit mapping can be expressed by an implicit map as a special case. Based on the mapping structures, analytical prediction of periodic flows in nonlinear dynamical systems can be completed. The mapping structure gives a set of nonlinear algebraic equations, which can be solved. Without the recurrence iteration, the solution errors of node points of periodic flows are fixed without computational errors caused by iterations. The purpose of this book is to develop a semi-analytical method for periodic flows to chaos in nonlinear dynamical systems with/without time delay through implicit mapping structures.

1.1 A Brief History

To determine periodic flows in nonlinear dynamical systems, existing techniques for periodic motions in nonlinear systems are reviewed briefly. The analytical methods for periodic motions are discussed first. Lagrange (1788) developed the method of averaging for periodic motions in the three-body problem as a perturbation of the two-body problem. The idea is based on the solutions of linear systems. Such an idea was further extended by Poincare in the end of the nineteenth century. Thus, Poincare (1899) developed the perturbation theory for motions of celestial bodies. van der Pol (1920) used the method of averaging for the periodic solutions of oscillation systems in circuits. Such an application caused great interest in the perturbation theory for the approximate analytical solution of periodic motions in nonlinear systems. Until 1928, the asymptotic validity of the method of averaging was not proved. Fatou (1928) gave the proof of the asymptotic validity through the solution existence theorems of differential equations. Krylov and Bogoliubov (1935) further developed the method of averaging, and the detailed presentation was given in Bogoliubov and Mitropolsky (1961). Hayashi (1964) presented the perturbation methods including averaging method and principle of harmonic balance. Barkham and Soudack (1969) extended the Krylov–Bogoliubov method for the approximate solutions of nonlinear autonomous second-order differential equations [also see, Barkham and Soudack (1970)]. Nayfeh (1973) employed the multiple-scale perturbation method to develop approximate solutions of periodic motions in the Duffing oscillators. Holmes and Rand (1976) discussed the stability and bifurcation of periodic motions in the Duffing oscillator. Nayfeh and Mook (1979) used the perturbation method to investigate nonlinear structural vibrations, and Holmes (1979) demonstrated chaotic motions in nonlinear oscillators through the Duffing oscillator with a twin-well potential. Ueda (1980) numerically simulated chaos by period-doubling of periodic motions of Duffing oscillators. A generalized harmonic balance approach was used by Garcia-Margallo and Bejarano (1987) to determine approximate solutions of nonlinear oscillations with strong nonlinearity. Rand and Armbruster (1987) determined the stability of periodic solutions by the perturbation method and bifurcation theory. Yuste and Bejarano (1989) employed the elliptic functions instead of trigonometric functions to extend the Krylov–Bogoliubov method. Coppola and Rand (1990) used the averaging method with elliptic functions for approximation of limit cycle. Wang et al. (1992) used the harmonic balance method and the Floquet theory to investigate the nonlinear behaviors of the Duffing oscillator with a bounded potential well [also see, Kuo et al. (1992)]. Luo and Han (1997) determined the stability and bifurcation conditions of periodic motions of the Duffing oscillator. However, only symmetric periodic motions of the Duffing oscillators were investigated. Luo and Han (1999) investigated the analytical prediction of chaos in nonlinear rods through the Duffing oscillator. Peng et al. (2008) presented the approximate symmetric solution of period-1 motions in the Duffing oscillator by the harmonic balance method with three harmonic terms. Luo (2012a) developed a generalized harmonic balance method for the approximate analytical solutions of periodic motions and chaos in nonlinear dynamical systems. This method used the finite-term Fourier series to approximately express periodic motions, and the coefficients are time-varying. With averaging, a dynamical system of coefficients is obtained, and from such a dynamical system, the approximate solutions of periodic motions are achieved and the corresponding stability and bifurcation analysis are completed. Luo and Huang (2012a) used such a generalized harmonic balance method with finite terms for the analytical solutions of period-1 motions in the Duffing oscillator with a twin-well potential. Luo and Huang (2012b) also employed a generalized harmonic balance method to find analytical solutions of period-m motions in such a Duffing oscillator. The analytical bifurcation trees of periodic motions in the Duffing oscillator to chaos were obtained [also see, (Luo and Huang 2012c, d, 2013a, b, c, d)]. Such analytical bifurcation trees show the connection from periodic solution to chaos analytically. For a better understanding of nonlinear behaviors in nonlinear dynamical systems, analytical bifurcation trees of period-1 motions to chaos in a periodically forced oscillator with quadratic nonlinearity were presented in Luo and Yu (2013a, b, 2015), and period-m motions in the periodically forced van der Pol equation were presented in Luo and Laken (2013). The analytical solutions of periodic oscillations in the van der Pol oscillator can be used to verify the conclusions in Cartwright and Littlewood (1947) and Levinson (1948). The results for the parametric quadratic nonlinear oscillator in Luo and Yu (2014) analytically show the complicated period-1 motions and the corresponding bifurcation structures. The detailed presentation for analytical methods for periodic flows in nonlinear dynamical systems can be found in Luo (2014a, b).

In recent years, time-delayed systems are of great interest since such systems extensively exist in engineering (e.g., Tlusty 2000; Hu and Wang 2002). The infinite dimensional state space causes the significant difficulty to solve such time-delayed problems. Thus, one used numerical methods for the corresponding complicated behaviors. On the other hand, one is interested in the stability and bifurcation of equilibriums of the time-delayed systems (e.g., Stepan 1989; Sun 2009; Insperger and Stepan 2011). In addition, one is also interested in analytical solutions of periodic motions in time-delayed dynamical systems. Perturbation methods have been used for such periodic motions in delayed dynamical systems. For instance, the approximate solutions of the time-delayed nonlinear oscillator were investigated by the method of multiple scales (e.g., Hu et al. 1998; Wang and Hu 2006). The harmonic balance method was also used to determine approximate solutions of periodic motions in delayed nonlinear oscillators [e.g., MacDonald (1995); Liu and Kalmar-Nagy (2010); Lueng and Guo (2014)]. However, such approximate solutions of periodic motions in the time-delayed oscillators are based on one or two harmonic terms, which are not accurate enough. In addition, the corresponding stability and bifurcation analysis of such approximate solutions of periodic motions may not be adequate. Luo (2013) presented an alternative way for the accurate analytical solutions of periodic flows in time-delayed dynamical systems (see also, Luo 2014c). This method is without any small-parameter requirement. In addition, this approach can also be applied to the coefficient varying with time. Luo and Jin (2014a) analytically presented the bifurcation tree of period-1 motions to chaos in a periodically forced, time-delayed quadratic nonlinear oscillator. Luo and Jin (2014b, c, d) discussed the bifurcation trees of period-m motions to chaos in the periodically forced Duffing oscillator with a linear time-delayed displacement.

From the literature survey, for some simple nonlinear systems, the approximate analytical solutions of periodic motions can be obtained. However, for most of the nonlinear dynamical systems, it is very difficult to obtain analytical solutions of periodic motions. Thus, numerical results of periodic motions in complicated nonlinear dynamical systems become very significant in engineering. In fact, human being has a long history as old as human civilization to use numerical algorithms to get approximate numerical results instead of exact results. For instance, the Rhind Papyrus of ancient Egypt describes a root-finding method for solving a simple equation in about 1650 BC, and Archimedes of Syracuse (287–212 BC) used numerical algorithm to approximately compute lengths, areas, and volumes of geometric figures. Based on the ideas and spirits of numerical approximations, Isaac Newton and Gottfried Leibnitz developed the calculus by infinitesimal elements to linear approximation and infinitesimal summarization to integration. Because of calculus development, one can describe more complicated mathematical models for real physical problems, but it is very difficult to solve such accurate mathematical models explicitly. This is an important impetus for one to develop numerical methods to get approximate solutions of the accurate mathematical models. Thus, Newton developed several numerical methods to find approximate solutions. For instance, numerical methods for root-finding and polynomial interpolation were developed by Newton. Since then, Euler (1707–1783), Lagrange (1736–1813), and Gauss (1777–1855) further developed numerical methods for approximate results, such as Euler method for differential equations, Lagrange interpolation method, and Gauss interpolation. The more detailed information about numerical methods can be found in Goldestine (1977).

This book will focus on numerical methods for nonlinear dynamical systems. For this issue, Euler developed an explicit method to achieve approximate solutions numerically. Such Euler method is a one-step discrete method. This method is still used in numerical computation, but its computational accuracy is very low, and numerical solutions are not accurate. Bashforth and Adams (1883) presented a multi-step discrete method for numerical solutions of differential equations. Moulton (1926) extended such a method to the Adams–Moulton method. The Adams–Bashforth method is the explicit method as a predictor, and the Adams–Moulton method is the implicit method as a corrector. In addition, the Adams–Bashforth method can be extended for the practical application of the Taylor series method as presented in Nordsieck (1962). Milne (1949) used the entire interval for integration based on Newton–Cotes quadrature formulas. The recent theory of linear multi-step method was systematically discussed by Dahlquist (1956, 1959). The general formulas were presented, and the corresponding consistency, stability, and convergence were discussed by the linear stability theory. Runge (1895) started modern one-step methods with the order of two and three for numerical solutions of differential equations. Heun (1900) raised the order of the method from two and three to four. Kutta (1901) gave the formulation of the method with the order conditions. Nystrom (1925) made the correction of the fifth-order method of Kutta and showed how to apply the Runge–Kutta method to the second-order differential equations. Butcher (1963) discussed the coefficients of Runge–Kutta method, and the implicit Runge–Kutta methods were presented in Butcher (1964, 1975).

With extensive applications of computers, numerical computations become very popular to obtain numerical results for differential equations through discretization.

Once the discrete maps are obtained for dynamical systems, discrete dynamical systems can be used to investigate nonlinear dynamics of dynamical systems. Based on nonlinear maps, one discovered the existence of chaotic motions in nonlinear dynamical systems through iteration of discrete maps.

In 2005, Luo (2005a, b) presented a mapping dynamics of discrete dynamical systems which is a more generalized symbolic dynamics. The systematical description of mapping dynamics in discontinuous dynamical systems was presented in Luo (2009). The discrete maps can be any implicit and/or explicit functions rather than explicit maps in numerical iterative methods only. From discrete mapping structures, periodic motions in discrete dynamical systems can be predicted analytically, and the stability and bifurcation analysis of periodic motions in nonlinear dynamical systems can be completed. Such an idea was applied to discontinuous dynamical systems in Luo (2009, 2012b, c).

1.2 Book Layout

The main body in this book will discuss discretization of differential equations of nonlinear continuous dynamical systems to obtain implicit maps for periodic flows. The mapping structures will be employed to analytically predict the periodic flows in nonlinear continuous systems, and the corresponding stability and bifurcation can be discussed.

In Chap. 2, a theory for nonlinear discrete systems is reviewed. The local and global theories of stability and bifurcation for nonlinear discrete systems are discussed. The stability switching and bifurcation on specific eigenvectors of the linearized system at fixed points under a specific period are presented. The higher order singularity and stability for nonlinear discrete systems on the specific eigenvectors are discussed.

In Chap. 3, the discretization of continuous systems is presented. The explicit and implicit discrete maps are discussed for numerical predictions of continuous systems. Basic discrete schemes are presented which include forward and backward Euler methods, and midpoint and trapezoidal rule methods. An introduction to Runge–Kutta methods is presented, and the Taylor series method and second-order Runge–Kutta method are introduced. The explicit Runge–Kutta methods for third and fourth order are systematically presented. The implicit Runge–Kutta methods are discussed based on the polynomial interpolation, which include a generalized implicit Runge–Kutta method, Guass method, Radau method, and Lotta methods. In addition to one-step methods, implicit and explicit multi-step methods are discussed, including Adams–Bashforth method, Adams–Moulton methods, and explicit and implicit Adams methods.

In Chap. 4 presented is a Ying–Yang theory for implicit, discrete, nonlinear systems with consideration of positive and negative iterations of discrete iterative maps. In existing analysis, the solutions relative to “Yang” in nonlinear dynamical systems are extensively investigated. However, the solutions pertaining to “Ying” in nonlinear dynamical systems are not discussed too much. A set of concepts on "Ying" and "Yang" in implicit, nonlinear, discrete dynamical systems are introduced. Based on the Ying–Yang theory, the complete dynamics of implicit discrete systems can be discussed. A discrete dynamical system with the Henon map is investigated as an example. Period-m solutions, stability, and bifurcations for multi-step, implicit discrete systems are discussed.

In Chap. 5, periodic flows in continuous nonlinear systems are discussed through discrete implicit mappings. The period-1 flows in nonlinear systems are discussed by the one-step discrete maps, and then, the period-m flows in nonlinear dynamical systems are also discussed through the one-step discrete maps. Multi-step, implicit discrete maps are employed to discuss the period-1 and period-m motions in nonlinear dynamical systems. Periodic flows in nonlinear time-delayed dynamical systems are discussed with time-delay discrete nodes interpolated by two non-delay discrete nodes. In addition, periodic flows in time-delayed nonlinear dynamical systems are also discussed through the delay nodes determined by integration. Through the discrete nodes in periodic flows, the periodic flows are approximated by the discrete Fourier series and the frequency space of the periodic flows can be determined through amplitude spectrums.

In Chap. 6, periodic motions in the Duffing oscillator are discussed through the mapping structures of discrete implicit maps. The discrete implicit maps are obtained from the differential equation of the Duffing oscillator. From mapping structures, bifurcation trees of periodic motions are predicted analytically through nonlinear algebraic equations of implicit maps, and the corresponding stability and bifurcation analysis of periodic motions in the bifurcation trees are presented. The bifurcation trees of periodic motions are also presented through the harmonic amplitudes of the discrete Fourier series. Finally, from the analytical prediction, numerical simulation results of periodic motions are performed to verify the analytical prediction. The harmonic amplitude spectrums are also presented, and the corresponding analytical expression of periodic motions can be obtained approximately.

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