

# Calculations for Molecular Biology and Biotechnology

A Guide to Mathematics in the Laboratory

Frank H. Stephenson



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**Frank H. Stephenson**

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*To my wife, Laurie, and our beautiful daughter, Myla.*

# Foreword

It was sometimes difficult for me to find a common ground with my high school chemistry teacher. We couldn't agree on what was appropriate attire or hair length for a sophomore. He had no respect for the notion that surfing is a viable career goal. He smoked way too much. But it was either in spite of him or because of him that I learned the mathematics involved in calculating the concentration of solutions, and I eventually agreed with him on the importance of numbers in the biological and chemical sciences. By the time I got out of that class, I was feeling as though I could have converted the speed of light into picomoles. As I undertook a career in molecular biology and teaching, it was what I learned in that class, more than in any other, that I have used on almost a daily basis.

Mathematics is a beautiful and elegant way of expressing order. I have heard it called a universal language. If true, then there are many dialects. There are any number of ways to approach a problem, no one of them necessarily more legitimate than another. I have always found interesting the passion and fervor people attach to their particular approach to mathematics. "Why do you do the problem **that** way?" I might be asked. "Clearly," my critic continues, "if you solve the problem **this** way, it's much quicker, more logical, and easier to follow. This is the only way that makes any sense." Well, maybe to them. Not everyone's brain works in the same way. In solving a problem in concentration, for example, it is probably amenable to solution by using a relationship of ratios, or  $C_1V_1 = C_2V_2$ , or the approach most often taken in this book. In actuality, they are all variations on a theme. Any one of them will get you the answer. Therein, I believe, lies the very beauty of mathematics.

It wasn't until this last year that I discovered the approach that I take to most of the problems encountered in the molecular biology laboratory has a name. It is called **dimensional analysis**. I always thought of it as "canceling terms." My brain is comfortable with this method. Many have tried to convert me to the use of the  $C_1V_1 = C_2V_2$  approach, but all have failed. I have been chided and ridiculed by some for the manner in which I solve problems. I have been applauded by others. When I learned that the approach I take has the name **dimensional analysis**, I felt, in a way, like the person who visits the doctor with some inexplicable malady and who is reassured when the doctor attaches some Latin-sounding name to it. At least then, the

individual knows that other people must also have the affliction, that it has been studied, and that there may even be a cure.

To present every possible way that each problem in this book could be solved would have made it too cumbersome. I make no apologies to those who might think that any one problem is better solved in another manner. To those of you who use this book as a companion in the laboratory, if you replace your values with the ones I have used in the example problems, you will do fine. You will have the numbers you need such that, for example, you will not be adding too little or too much salt that will send your cultured cells into osmotic shock or give you *EcoRI* star activity when digesting DNA.

My appreciation goes to my colleague Maria Abilock for bringing her critical eye to bear on this manuscript. She agreed to critique the draft in spite of the fact that she feels her way of solving problems is better than mine.

Frank H. Stephenson, Ph.D.

# Contents

<b>Foreword .....</b>	<b>xiv</b>
<b>Chapter 1: Scientific Notation and Metric Prefixes ..</b>	<b>1</b>
Introduction.....	1
Significant Digits.....	1
Rounding Off Significant Digits in Calculations.....	2
Exponents and Scientific Notation.....	4
Expressing Numbers in Scientific Notation.....	4
Converting Numbers from Scientific Notation to Decimal Notation.....	6
Adding and Subtracting Numbers Written in Scientific Notation.....	8
Multiplying and Dividing Numbers Written in Scientific Notation.....	9
Metric Prefixes.....	13
Conversion Factors and Canceling Terms.....	14
<b>Chapter 2: Solutions, Mixtures, and Media.....</b>	<b>18</b>
Introduction.....	18
Calculating Dilutions: A General Approach.....	18
Concentrations by a Factor of X.....	20
Preparing Percent Solutions.....	22
Diluting Percent Solutions.....	23
Moles and Molecular Weight: Definitions.....	27
Molarity.....	28
Diluting Molar Solutions.....	31
Converting Molarity to Percent.....	32
Converting Percent to Molarity.....	33
Normality.....	34
PH.....	35
$pK_a$ and the Henderson–Hasselbalch Equation.....	39

<b>Chapter 3: Cell Growth.....</b>	<b>42</b>
The Bacterial Growth Curve.....	42
Sample Data.....	45
Manipulating Cell Concentration.....	46
Plotting OD <sub>550</sub> vs. Time on a Linear Graph .....	48
Plotting the Logarithm of OD <sub>550</sub> vs. Time on a	
Linear Graph.....	49
Logarithms .....	49
Sample OD <sub>550</sub> Data Converted to Log Values.....	50
Plotting Log OD <sub>550</sub> vs. Time.....	50
Plotting the Log of Cell Concentration vs. Time.....	51
Determining Log Values.....	51
Calculating Generation Time.....	52
Slope and Growth Constant.....	52
Generation Time.....	53
Plotting Cell Growth Data on a Semilog Graph.....	55
Plotting OD <sub>550</sub> vs. Time on a Semilog Graph.....	55
Estimating Generation Time from a Semilog Plot of OD <sub>550</sub>	
vs. Time.....	56
Plotting Cell Concentration vs. Time on a Semilog Graph.....	57
Determining Generation Time Directly from a	
Semilog Plot of Cell Concentration vs. Time.....	59
Plotting Cell Density versus OD <sub>550</sub> on a Semilog Graph.....	60
The Fluctuation Test.....	61
Fluctuation Test Example.....	63
Variance.....	64
Measuring Mutation Rate.....	66
The Poisson Distribution.....	67
Calculating Mutation Rate by Using the Poisson Distribution.....	68
Using a Graphical Approach to Calculate Mutation Rate	
from Fluctuation Test Data.....	69
Mutation Rate Determined by Plate Spreading.....	74
Measuring Cell Concentration on a Hemocytometer.....	75



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<b>Chapter 4: Working with Bacteriophage.....</b>	<b>77</b>
Introduction.....	77
Multiplicity of Infection.....	77
Probabilities and Multiplicity of Infection .....	79
Measuring Phage Titer .....	85
Diluting Bacteriophage .....	86
Measuring Burst Size.....	87
 <b>Chapter 5: Quantitation of Nucleic Acids.....</b>	 <b>90</b>
Quantitation of Nucleic Acids by Ultraviolet Spectroscopy..	90
Determining the Concentration of Double-Stranded DNA...	91
Using Absorbance and an Extinction Coefficient to	
Calculate Double-Stranded DNA Concentration.....	94
Calculating DNA Concentration as a Millimolar (mM)	
Amount.....	96
Determining the Concentration of Single-Stranded	
DNA Molecules.....	97
Single-Stranded DNA Concentration Expressed in $\mu\text{g/mL}$ .....	97
Determining the Concentration of High-Molecular-Weight	
Single-Stranded DNA in $\text{pmol}/\mu\text{L}$ .....	98
Expressing ssDNA Concentration as a Millimolarity Amount.....	98
Oligonucleotide Quantitation.....	99
Optical Density (OD) Units.....	99
Expressing an Oligonucleotide's Concentration in $\mu\text{g/mL}$ .....	100
Oligonucleotide Concentration Expressed in $\text{pmol}/\mu\text{L}$ .....	100
Measuring RNA Concentration.....	103
Molecular Weight, Molarity, and Nucleic Acid Length .....	104
Estimating DNA Concentration on an Ethidium Bromide–	
Stained Gel.....	108

## Chapter 6: Labeling Nucleic Acids with

<b>Radioisotopes.....</b>	<b>109</b>
Introduction.....	109
Using Radioactivity: The Curie .....	109
Estimating Plasmid Copy Number.....	110
Labeling DNA by Nick Translation.....	112
Determining Percent Incorporation of Radioactive Label from Nick Translation.....	113
Calculating Specific Radioactivity of a Nick Translation Product..	114
Random Primer Labeling of DNA.....	114
Random Primer Labeling – Percent Incorporation.....	115
Random Primer Labeling – Calculating Theoretical Yield.....	116
Random Primer Labeling – Calculating Actual Yield.....	117
Random Primer Labeling – Calculating Specific Activity of the Product.....	118
Labeling 3' Termini with Terminal Transferase.....	119
3'-End Labeling with Terminal Transferase – Percent Incorporation.....	119
3'-End Labeling with Terminal Transferase – Specific Activity of the Product.....	120
cDNA Synthesis.....	121
First Strand cDNA Synthesis.....	121
Second Strand cDNA Synthesis.....	125
Homopolymeric Tailing.....	128
<i>In Vitro</i> Transcription.....	133

## Chapter 7: Oligonucleotide Synthesis..... 136

Introduction.....	136
Synthesis Yield.....	136
Measuring Stepwise and Overall Yield by the DMT Cation Assay.....	139
Overall Yield.....	139

---

Stepwise Yield.....	140
Calculating Micromoles of Nucleoside Added at Each Base Addition Step.....	142
<b>Chapter 8: The Polymerase Chain Reaction.....</b>	<b>143</b>
Introduction.....	143
Template and Amplification.....	143
Exponential Amplification.....	145
PCR Efficiency.....	147
Calculating the $T_m$ of the Target Sequence.....	151
Primers.....	153
Primer $T_m$ .....	158
Calculating $T_m$ Based on Salt Concentration, G/C Content, and DNA Length.....	159
Calculating $T_m$ Based on Nearest-Neighbor Interactions.....	160
dNTPs.....	165
DNA Polymerase.....	168
Calculating DNA Polymerase's Error Rate.....	169
Quantitative PCR.....	171
<b>Chapter 9: Recombinant DNA.....</b>	<b>186</b>
Introduction.....	186
Restriction Endonucleases.....	186
The Frequency of Restriction Endonuclease Cut Sites.....	188
Calculating the Amount of Fragment Ends.....	189
The Amount of Ends Generated by Multiple Cuts.....	190
Ligation.....	192
Ligation Using Lambda-Derived Vectors.....	194
Packaging of Recombinant Lambda Genomes.....	200
Ligation Using Plasmid Vectors.....	203
Transformation Efficiency.....	207
Genomic Libraries: How Many Clones Do You Need?.....	208

cDNA Libraries: How Many Clones Are Enough?.....	210
Expression Libraries.....	211
Screening Recombinant Libraries by Hybridization to DNA Probes.....	212
Oligonucleotide Probes.....	214
Hybridization Conditions.....	216
Hybridization Using Double-Stranded DNA Probes.....	223
Sizing DNA Fragments by Gel Electrophoresis.....	224
Generating Nested Deletions Using Nuclease BAL 31.....	237
<b>Chapter 10: Protein.....</b>	<b>242</b>
Introduction.....	242
Protein Quantitation by Measuring Absorbance at 280 nm... Using Absorbance Coefficients and Extinction Coefficients to Estimate Protein Concentration.....	242
Relating Absorbance Coefficient to Molar Extinction Coefficient.....	243
Determining a Protein's Extinction Coefficient.....	245
Relating Concentration in Milligrams per Milliliter to Molarity.....	246
Protein Quantitation Using $A_{280}$ When Contaminating Nucleic Acids Are Present.....	248
Protein Quantitation at 205 nm.....	249
Protein Quantitation at 205 nm When Contaminating Nucleic Acids Are Present.....	250
Measuring Protein Concentration by Colorimetric Assay – The Bradford Assay.....	251
Using $\beta$ -Galactosidase to Monitor Promoter Activity and Gene Expression.....	252
Assaying $\beta$ -Galactosidase in Cell Culture.....	254
Specific Activity.....	255
	257

---

Assaying $\beta$ -Galactosidase from Purified Extracts.....	258
The CAT Assay.....	260
Calculating Molecules of CAT.....	263
Use of Luciferase in a Reporter Assay.....	265
<i>In Vitro</i> Translation – Determining Amino Acid Incorporation.....	266
<b>Chapter 11: Centrifugation.....</b>	<b>270</b>
Introduction.....	270
Relative Centrifugal Force ( <i>g</i> Force).....	270
Converting <i>g</i> Force to Revolutions per Minute.....	272
Determining <i>g</i> Force and Revolutions per Minute by Use of a Nomogram.....	273
Calculating Sedimentation Times.....	275
<b>Chapter 12: Forensic Science.....</b>	<b>278</b>
Introduction.....	278
Alleles and Genotypes.....	278
Calculating Genotype Frequencies.....	280
Calculating Allele Frequencies.....	281
The Hardy–Weinberg Equation and Calculating Expected Genotype Frequencies.....	282
The Chi-Square Test: Comparing Observed to Expected Values.....	286
Sample Variance.....	290
Sample Standard Deviation.....	291
$P_i$ : The Power of Inclusion.....	292
$P_d$ : The Power of Discrimination.....	293
DNA Typing and a Weighted Average.....	294
The Multiplication Rule.....	295
<b>Index.....</b>	<b>297</b>

# Scientific Notation and Metric Prefixes

# 1

## Introduction

There are some 3,000,000,000 base pairs making up human genomic DNA within a haploid cell. If that DNA is isolated from such a cell, it will weigh approximately 0.0000000000035 grams. To amplify a specific segment of that purified DNA using the polymerase chain reaction (PCR), 0.00000000001 moles of each of two primers can be added to a reaction that can produce, following some 30 cycles of the PCR, over 1,000,000,000 copies of the target gene.

On a day-to-day basis, molecular biologists work with extremes of numbers far outside the experience of conventional life. To allow them to more easily cope with calculations involving extraordinary values, two shorthand methods have been adopted that bring both enormous and infinitesimal quantities back into the realm of manageability. These methods use scientific notation and metric prefixes. They require the use of exponents and an understanding of significant digits.

## Significant Digits

Certain techniques in molecular biology, as in other disciplines of science, rely on types of instrumentation capable of providing precise measurements. An indication of the level of precision provided by any particular instrument is given by the number of digits expressed in its readout. The numerals of a measurement representing actual limits of precision are referred to as **significant digits**.

Although a zero can be as legitimate a value as the integers 1 through 9, significant digits are usually nonzero numerals. Without information on how a measurement was made or on the precision of the instrument used to make it, zeros to the left of the decimal point trailing one or more nonzero numerals are assumed not to be significant. For example, in stating that the human genome is 3,000,000,000 base pairs in length, the only significant digit in the number is the 3. The nine zeros are not significant. Likewise, zeros to the right of the decimal point preceding a set of nonzero numerals are assumed not to be significant. If we determine that the DNA within a sperm cell weighs 0.0000000000035 grams, only the 3 and the 5 are significant digits. The 11 zeros preceding these numerals are not significant.

**Problem 1.1** How many significant digits are there in each of the following measurements?

a) 3,001,000,000 bp (base pairs)

b) 0.00304 grams

c) 0.000210 liters\*

\* Volume delivered with a calibrated micropipettor.

**Solution 1.1**

<u>Given Number</u>	<u>Number of Significant Digits</u>	<u>The Significant Digits Are:</u>
a) 3,001,000,000 bp	4	3001
b) 0.00304 grams	3	304
c) 0.000210 liters	3	210

## Rounding Off Significant Digits in Calculations

When two or more measurements are used in a calculation, the result can only be as accurate as the least precise value. To accommodate this necessity, the number obtained as solution to a computation should be rounded off to reflect the weakest level of precision. The following guidelines will help determine the extent to which a numerical result should be rounded off.

### Guidelines for Rounding Off Significant Digits

1. When adding or subtracting numbers, the result should be rounded off so that it has the same number of significant digits to the right of the decimal as that number used in the computation with the fewest significant digits to the right of the decimal.
2. When multiplying or dividing numbers, the result should be rounded off so that it contains only as many significant digits as that number in the calculation with the fewest significant digits.

**Problem 1.2** Perform the following calculations, and express the answer using the guidelines for rounding off significant digits described in the preceding box.

- a)  $0.2884 \text{ g} + 28.3 \text{ g}$
- b)  $3.4 \text{ cm} \times 8.115 \text{ cm}$
- c)  $1.2 \text{ L} \div 0.155 \text{ L}$

**Solution 1.2**

a)  $0.2884 \text{ g} + 28.3 \text{ g} = 28.5884 \text{ g}$

The sum is rounded off to show the same number of significant digits to the right of the decimal point as that number in the equation with the fewest significant digits to the right of the decimal point. (In this case, the value 28.3 has one significant digit to the right of the decimal point.)

28.5884 g is rounded off to 28.6 g.

b)  $3.4 \text{ cm} \times 8.115 \text{ cm} = 27.591 \text{ cm}^2$

The answer is rounded off to two significant digits since there are as few as two significant digits in one of the multiplied numbers (3.4 cm).

27.591 cm<sup>2</sup> is rounded off to 28 cm<sup>2</sup>.

c)  $1.2 \text{ L} \div 0.155 \text{ L} = 7.742 \text{ L}$

The quotient is rounded off to two significant digits since there are as few as two significant digits in one of the values (1.2 L) used in the equation.

7.742 L is rounded off to 7.7 L.



## Exponents and Scientific Notation

An **exponent** is a number written above and to the right of (and smaller than) another number (called the **base**) to indicate the power to which the base is to be raised. Exponents of base 10 are used in scientific notation to express very large or vary small numbers in a shorthand form. For example, for the value  $10^3$ , 10 is the base and 3 is the exponent. This means that 10 is multiplied by itself three times ( $10^3 = 10 \times 10 \times 10 = 1000$ ). For numbers less than 1.0, a negative exponent is used to express values as a reciprocal of base 10. For example,

$$10^{-3} = \frac{1}{10^3} = \frac{1}{10 \times 10 \times 10} = \frac{1}{1000} = 0.001$$

## Expressing Numbers in Scientific Notation

To express a number in scientific notation:

1. Move the decimal point to the right of the leftmost nonzero digit. Count the number of places the decimal has been moved from its original position.
2. Write the new number to include all numbers between the leftmost and rightmost significant (nonzero) figures. Drop all zeros lying outside these integers.
3. Place a multiplication sign and the number 10 to the right of the significant integers. Use an exponent to indicate the number of places the decimal point has been moved.
  - a. For numbers greater than 10 (where the decimal was moved to the left), use a positive exponent.
  - b. For numbers less than 1 (where the decimal was moved to the right), use a negative exponent.

**Problem 1.3** Write the following numbers in scientific notation.

- a) 3,001,000,000
- b) 78
- c)  $60.23 \times 10^{22}$