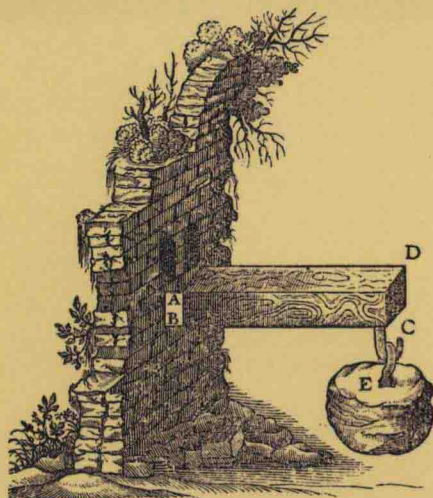


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A HISTORY OF
THE THEORY
OF ELASTICITY
AND OF
THE STRENGTH
OF MATERIALS

VOLUME 1:
FROM GALILEI TO SAINT-VENANT

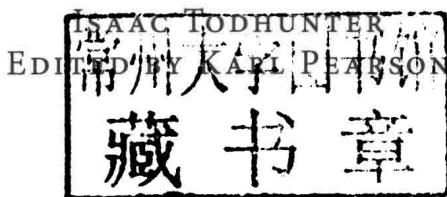
ISAAC TODHUNTER
EDITED BY KARL PEARSON



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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS

University Printing House, Cambridge, CB2 8BS, United Kingdom

Published in the United States of America by Cambridge University Press, New York

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www.cambridge.org

Information on this title: www.cambridge.org/9781108070423

© in this compilation Cambridge University Press 2014

This edition first published 1886
This digitally printed version 2014

ISBN 978-1-108-07042-3 Paperback

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A distinguished mathematician and notable university teacher, Isaac Todhunter (1820–84) became known for the successful textbooks he produced as well as for a work ethic that was extraordinary, even by Victorian standards. A scholar who read all the major European languages, Todhunter was an open-minded man who admired George Boole and helped introduce the moral science examination at Cambridge. His many gifts enabled him to produce the histories of mathematical subjects which form his lasting memorial. First published between 1886 and 1893, the present work was the last of these. Edited and completed after Todhunter's death by Karl Pearson (1857–1936), another extraordinary man who pioneered modern statistics, these volumes trace the mathematical understanding of elasticity from the seventeenth to the late nineteenth century. Volume 1 (1886) begins with Galileo Galilei and extends to the researches of Saint-Venant up to 1850.

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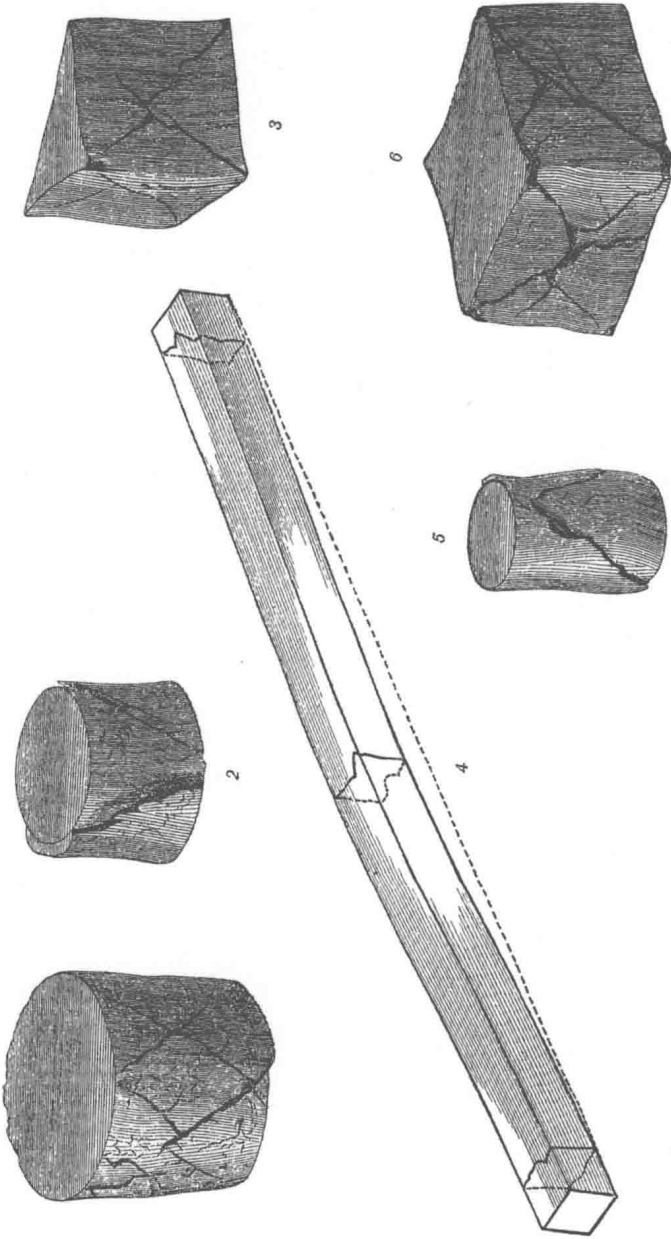
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London: C. J. CLAY AND SONS,
CAMBRIDGE UNIVERSITY PRESS WAREHOUSE,
AVE MARIA LANE.



CAMBRIDGE: DEIGHTON, BELL, AND CO.
LEIPZIG: F. A. BROCKHAUS.



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Frontispiece

A HISTORY OF
THE THEORY OF ELASTICITY

AND OF
THE STRENGTH OF MATERIALS

FROM GALILEI TO THE PRESENT TIME.

BY THE LATE
ISAAC TODHUNTER, D.Sc., F.R.S.

EDITED AND COMPLETED
FOR THE SYNDICS OF THE UNIVERSITY PRESS

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VOL. I. GALILEI TO SAINT-VENANT
1639—1850.

CAMBRIDGE:
AT THE UNIVERSITY PRESS.
1886

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Cambridge:

PRINTED BY C. J. CLAY, M.A. AND SONS,
AT THE UNIVERSITY PRESS.

TO THE MEMORY OF

M. BARRÉ DE SAINT-VENANT

THE FOREMOST OF MODERN ELASTICIANS

THE EDITOR DEDICATES HIS LABOUR

ON THE PRESENT VOLUME.

Si ces imperfections sont malheureusement nombreuses cela vient de ce que la science appliquée est jeune et encore pauvre ; avec ses ressources actuelles, elle peut déjà rendre de grands services, mais ses destinées sont bien plus hautes : elle offre un champ immense au zèle de ceux qui voudront l'enrichir, et beaucoup de parties de son domaine semblent même n'attendre que des efforts légers pour produire des résultats d'une grande utilité.

Saint-Venant.

Jedenfalls sieht man aus den angeführten Thatsachen, dass die Theorie der Elasticitat noch durchaus nicht als abgeschlossen zu betrachten ist, und es wäre zu wünschen, dass recht viel Physiker sich mit diesem Gegenstande beschäftigten, um durch vermehrte Beobachtungen die sichere Grundlage zu einer erweiterten Theorie zu schaffen.

Clausius.

Ceux qui, les premiers, ont signalé ces nouveaux instruments, n'existeront plus et seront complètement oubliés ; à moins que quelque géomètre archéologue ne ressuscite leurs noms. Eh ! qu'importe, d'ailleurs, si la science a marché !

Lamé.

PREFACE.

IN the summer of 1884 at the suggestion of Dr Routh the Syndics of the University Press placed in my hands the manuscript of the late Dr Todhunter's History of Elasticity, in order that it might be edited and completed for the Press. That the publication might not be indefinitely delayed, it was thought advisable to print off chapter by chapter as the work of revision progressed. That this arrangement has accelerated the publication of the first volume is certain, but at the same time it has introduced some disadvantages to which it is necessary for me to refer. In the first place it was impossible to introduce in the earlier cross-references to later portions of the work; this I have endeavoured to rectify by adding a copious index to the whole volume. In the next place I must mention, that it was not till I had advanced some way into the work that I felt convinced that the reproduction in the analysis of a memoir of the individual writer's terminology and notation must be abandoned and a uniform terminology and notation adopted for the whole book. This was absolutely needful if the book was to be available for easy reference, and not merely of interest to the historical student. The choice, however, of such terminology and notation—considering the enormous diversity, I will even say confusion, on this point to be found in the writings of British and continental elasticians—was an extremely venturesome task. To evolve a really scientific terminology which shall stand any chance of universal adoption from a number of words, which

each individual writer has used in his own sense, is no easy matter. If I have in some cases dispensed with such well-worn words as tension, pressure, extension, contraction and so forth, it has been from no desire for novelty, but in order to avoid a conflict of definitions. That the notation and terminology proposed in this work will extend beyond it I hardly venture to hope, I shall be content if they be intelligible to those who may consult this book. They will be found fully discussed in Notes B—D of the Appendix, which I would ask the reader to examine before passing to the text. As I have said, it was unfortunately only after I had made some progress in the work, that I became convinced of the need of terminological and notational uniformity. I think, however, consistency in these points will be found after the middle of the chapter devoted to Poisson. The introduction of this uniform system of symbols and terms has itself involved a considerable amount of additional work on the manuscript. The symbols and terms used in the manuscript are occasionally those of the original memoirs, occasionally those of Lamé or of Saint-Venant. The want of uniformity in the first two chapters will perhaps not be considered a disadvantage, the memoirs being of historical rather than scientific interest, and their language often the most characteristic part of their historical value.

The disadvantages which I have pointed out in this first volume will I trust be obviated in the second by the revision and completion of the whole manuscript before the work of printing is commenced. The second volume will contain an analysis of all researches in elasticity from 1850 to the present time. From 1850 to 1870 most but not all of the chief mathematical memoirs have been already analysed by Dr Todhunter; there is but little of a later date completed. Considering the amount of work to be done, considering that it is advisable to avoid revision and printing being carried on simultaneously, and finally noting the very limited time, which the teaching duties of my present post allow me to spend in a library where it is possible to carry on historical work of this kind, I fear the publication of the second volume would be much delayed were the task of editing it entrusted to me. I lay

stress upon this point as, although I have endeavoured to make the first volume complete in itself, much of its usefulness will be realised only on the appearance of the second. Indeed, in the interests of the reader as well as of the work, I think the Syndics will have to consider the question of appointing another editor, who has more of the needful leisure.

It is proper that I should explain with some detail the manner in which I have performed my task as editor. Dr Todhunter's manuscript consists of two distinct parts, the first contains a purely mathematical treatise on the theory of the 'perfect' elastic solid; the second a history of the theory of elasticity. The treatise based principally on the works of Lamé, Saint Venant and Clebsch is yet to a great extent historical, that is to say many paragraphs are composed of analyses of important memoirs. Thus in the History-manuscript after the title of a memoir there is occasionally only a mere reference to the paragraph of the Theory-manuscript, where it will be found discussed. Certain portions also of the manuscript have inscribed upon them in Dr Todhunter's handwriting 'History or Theory?' The Syndics having determined to publish in the first place the History only, it became necessary to determine how the gaps in the 'History' which were covered by mere reference to the 'Theory' should be filled up. With the sanction of the Syndics I have adopted the following principle: the analysis of a memoir wherever possible is to be Dr Todhunter's. Thus certain, on the whole not very considerable, portions of the Theory-manuscript are incorporated in the History, while all portions of the manuscript marked doubtful have been made use of when required.

Dr Todhunter's manuscript contains two versions, a first writing and a revision. The revision has been again read through by the author, but the principal alterations made are notes or suggestions for further consideration; in some cases the note is merely a statement that a criticism must be either modified or entirely reconstructed, in other cases, it involves a valuable cross-reference. One of the most important of these notes is that

referred to in my footnote on p. 250 ; it led to the only considerable excision which I have thought it proper to make before printing Dr Todhunter's manuscript.

The changes I have made in that manuscript are of the following character ; the introduction of a uniform terminology and notation, the correction of clerical and other obvious errors, the insertion of cross-references, the occasional introduction of a remark or of a footnote. The remarks are inclosed in square brackets. With this exception any article in this volume the number of which is *not included in square brackets* is due entirely to Dr Todhunter. So far as the arrangement of the memoirs is concerned there was little if anything to guide me in the manuscript. Dr Todhunter had evidently intended to give each of the principal elasticians chapters to themselves, and to group the minor memoirs together into periods. This method although it destroys the strict chronological treatment, and to some extent obscures the order of development, yet possesses such advantages, in that it groups together the researches of one man following his own peculiar lines of thought, that I have followed it without hesitation as the best possible. I even regret that I have not devoted special chapters to such elasticians as Hodgkinson, Wertheim and F. E. Neumann ; in the latter case the regret is deepened by the recent publication of his lectures on elasticity.

Turning to my own share in the completing of the work, I fear that at first sight I may appear to have exceeded the duty of an editor. For all the Articles in this volume whose numbers are enclosed in square brackets I am alone responsible, as well as for the corresponding footnotes, and the Appendix with which the volume concludes. The principle which has guided me throughout the additions I have made has been to make the work, so far as it lay in my power, a standard work of reference for its own branch of science. The use of a work of this kind is twofold. It forms on the one hand the history of a peculiar phase of intellectual development, worth studying for the many side lights it throws on